Light propagation in the Solar System for high-precision astrometry at the sub-micro-arcsecond level

Sven Zschocke

Lohrmann – Observatorium / TU Dresden

September 2018

1. Introduction

2. Astrometry at milli - arcsecond (mas) level of accuracy

3. Astrometry at micro - arcsecond (μas) level of accuracy

4. Astrometry at sub-micro - arcsecond (sub-μas) level

5. Relativistic theory of light propagation

6. Integration of geodesic equation in 1.5PN approximation

7. The state-of-the-art before our investigation

8. Light-trajectory in the field of N moving bodies with M_L, S_L

9. Summary and Outlook

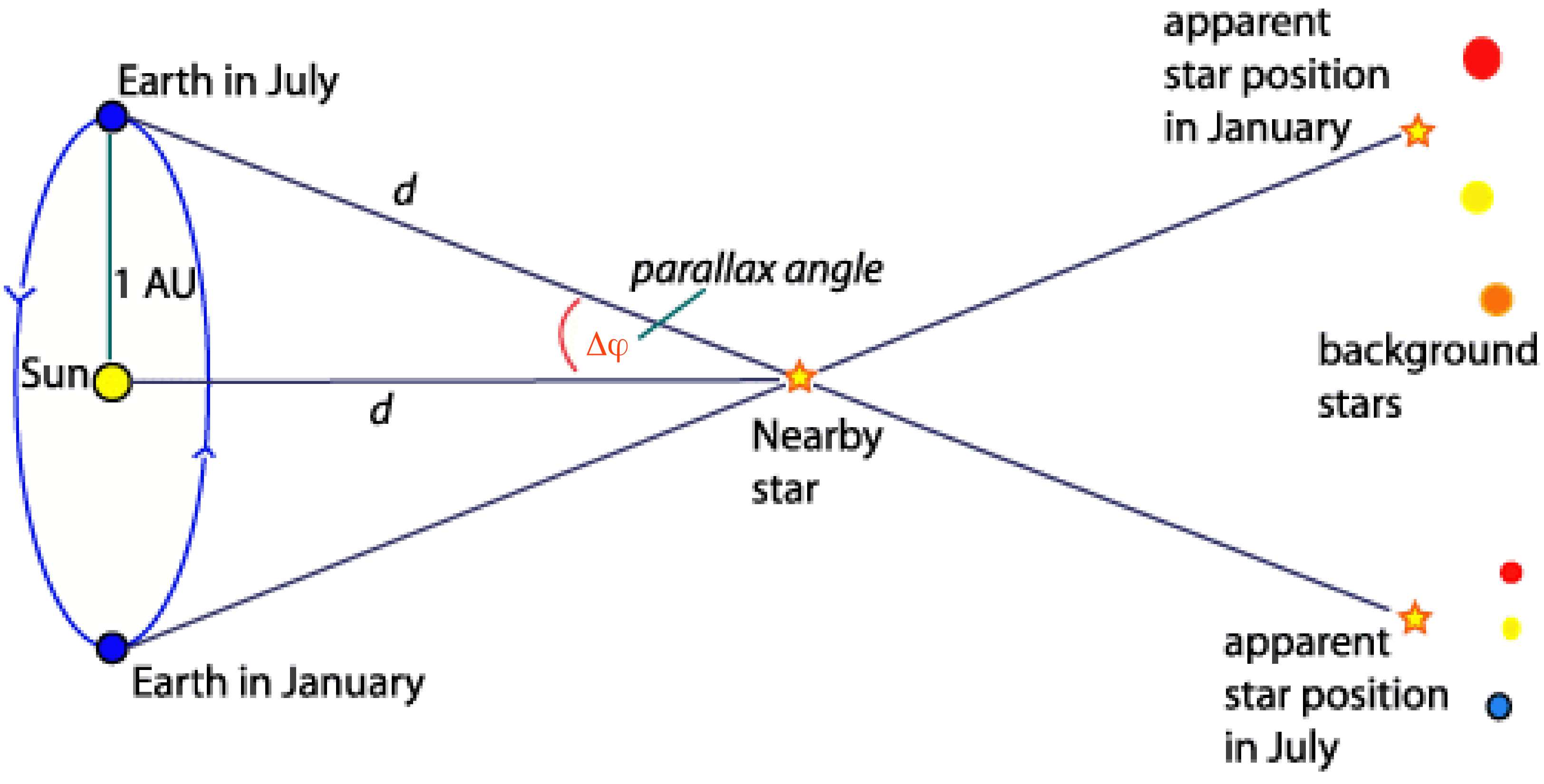
1. Introduction

Why is the distance of the stars so important ?

1. stellar distance reveals luminosity of stars
2. stellar distance reveals mass of stars
3. stellar distance reveals radius of stars
4. stellar distance reveals age of stars

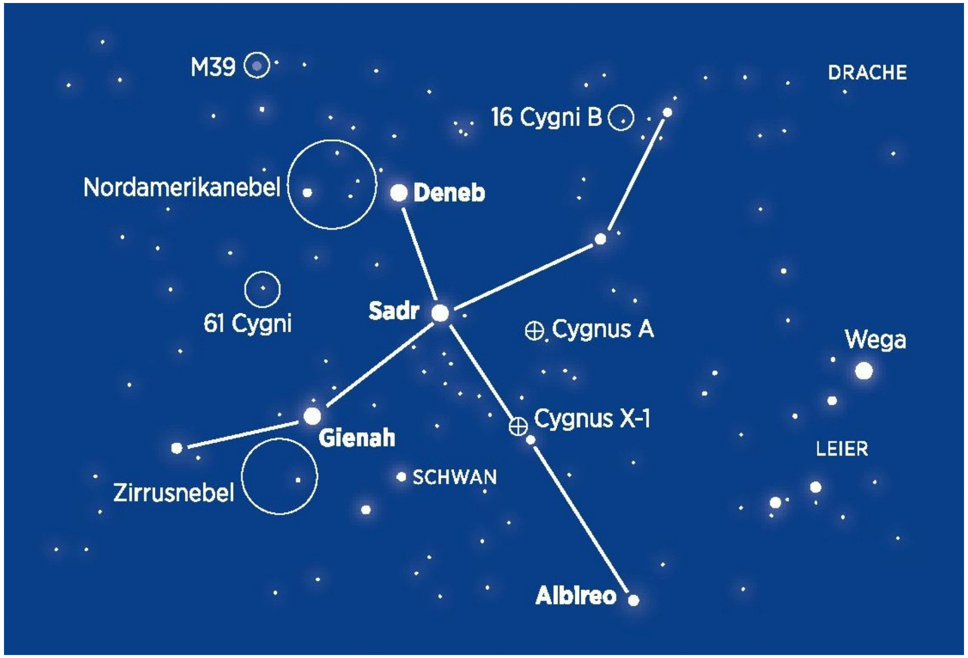
But how to determine the distance to the stars ?

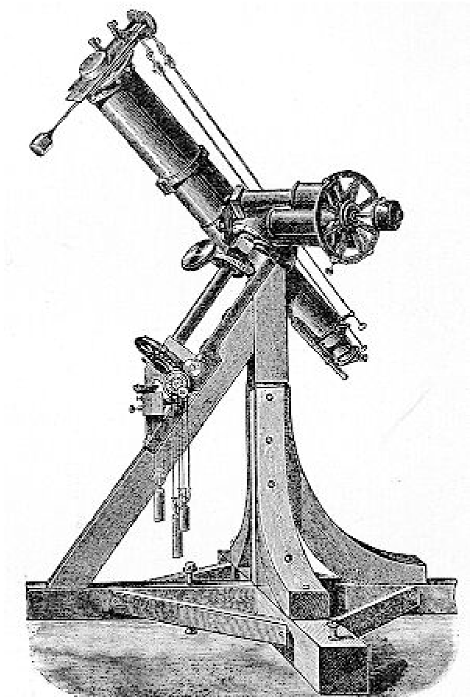
Distance measurement of stars by parallax



Parallax of nearby star

Johann Wilhelm Bessel





Determination of parallax of star (61 Cygni)

1838

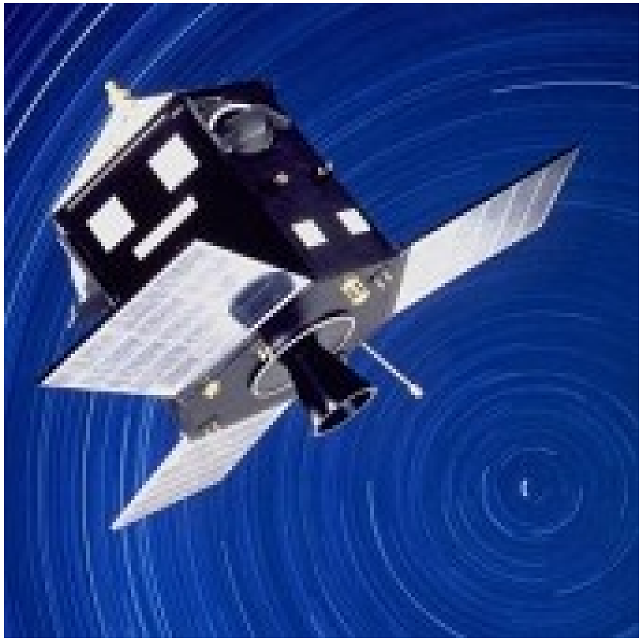
$\Delta\varphi$ = 0.3 as (11 light-years)

(as = arcsecond ~ 5 x 10^{-6} rad)

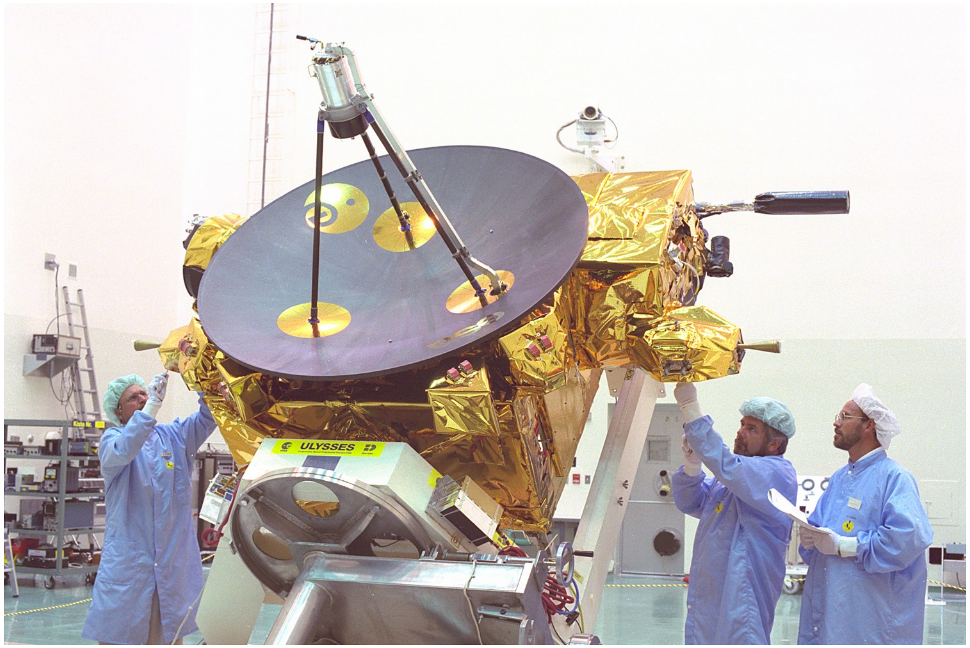
Heliometer of Koenigsberg Observatory

2. Astrometry at milli - arcsecond (mas) level of accuracy

The **Hipparcos** Space Astrometry Mission (ESA, 1989)



Spacecraft orbits
around the Earth



one telescope (diameter: 29 cm)

Results of **Hipparcos** Space Astrometry Mission

1. 120.000 stars with astrometric precision up to $\Delta\varphi$ = 1 mas for stars with apparent magnitude V = 7 mag (Hipparcos Catalogue, 1997)

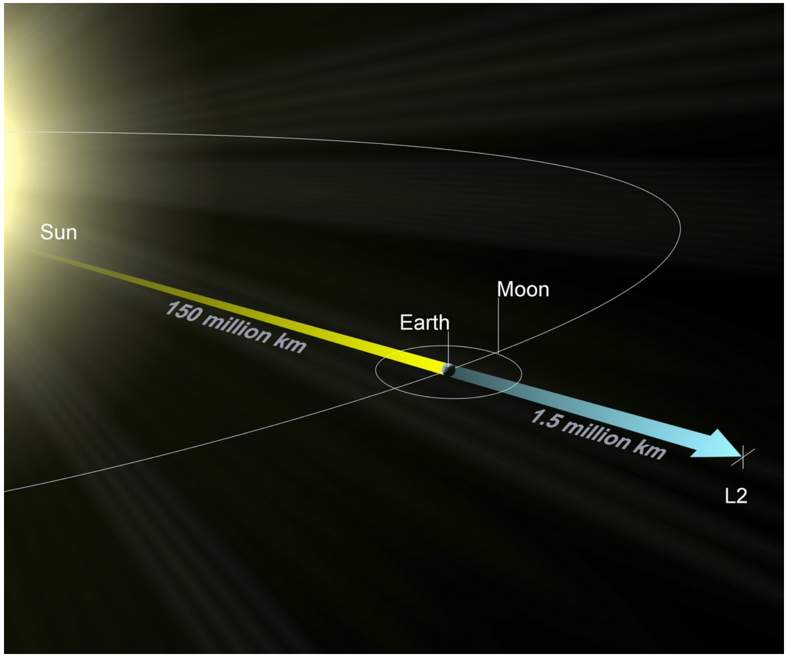
2. 2.500.000 stars with astrometric precision up to $\Delta\varphi$ = 20 mas for stars with apparent magnitude V = 10 mag (Tycho-2 Catalogue, 2000)

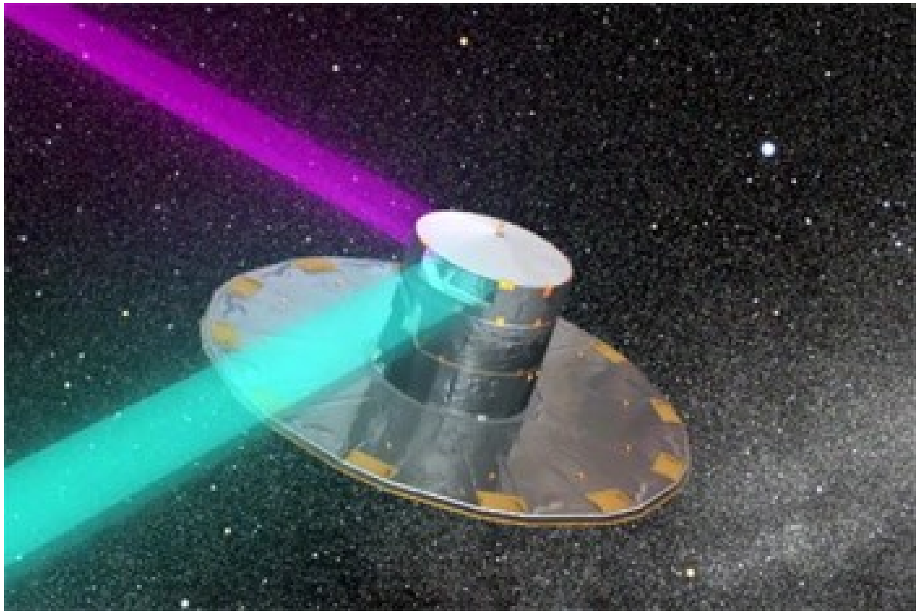
Some examples of apparent magnitude:

apparent magnitude of Wega:	0 mag
apparent magnitude of Polarstar:	2 mag
apparent magnitude of 61 Cyg:	5 mag
apparent magnitude visible to eye:	6 mag

3. Astrometry at micro - arcsecond (μas) level of accuracy

The **Gaia** Space Astrometry Mission (ESA, 2013)

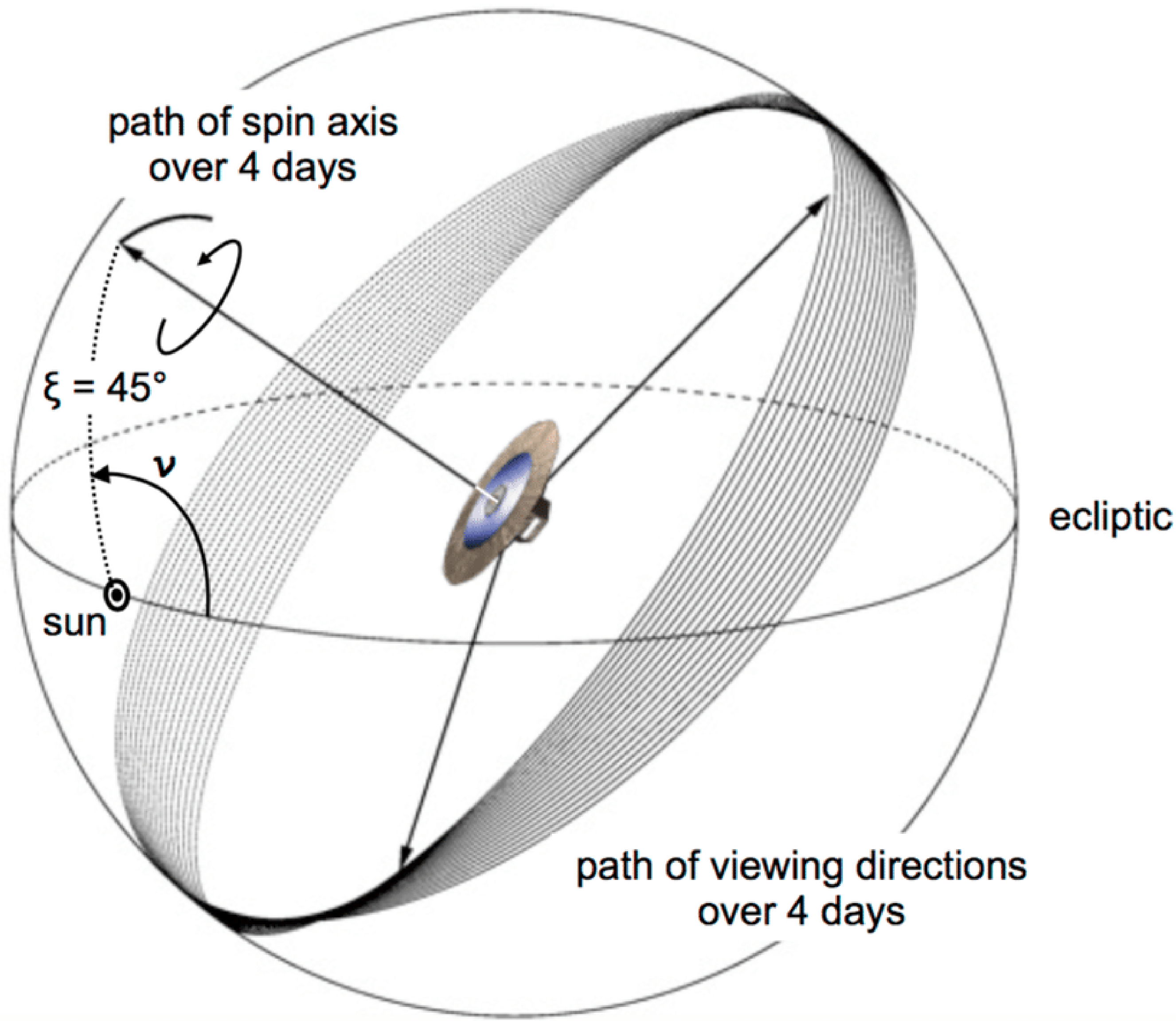




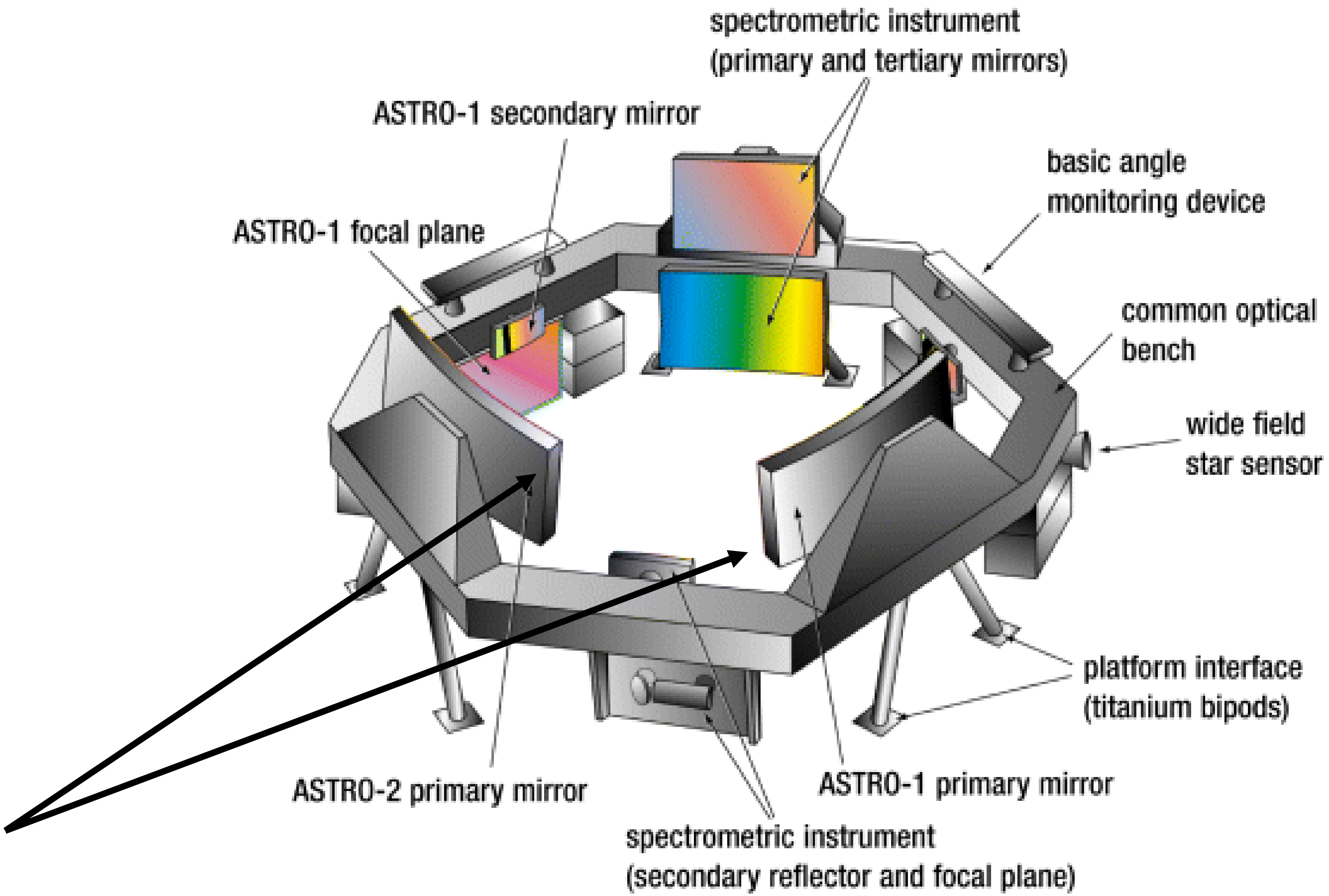
Gaia orbits around L2

- two telescopes on-board
- Gaia rotates slowly around its axis
- scans the entire sky

The scanning law of Gaia spacecraft:



The optical equipment of Gaia spacecraft:



two telescopes (1.4 m x 0.5 m)

The primary aims of the Gaia Mission:

- measurement of positions and velocities of 1 billion stars
- determination of their brightness and temperature
- creation of a three-dimensional map of our galaxy

The additional discoveries to be expected:

- about 7.000 exoplanets
- about 500.000 quasars
- about 1.000.000 Solar System objects

Some recent results of Gaia Space Astrometry Mission

1. 1.700.000.000 stars with astrometric precision up to $\Delta\varphi$ = 30 μas for stars with brightness V = 15 mag (Gaia Data Release 2, 2018)

2. 1.700.000.000 stars with astrometric precision up to $\Delta\varphi$ = 5 μas for stars with brightness V = 10 mag (final Gaia Data Release)

4. Astrometry at sub-micro - arcsecond (sub-μas) level

- it is obvious that a long-term goal of astrometry is sub-μas precision

- the scientific objectives of sub-μas are overwhelming, for instance:

a) detection of Earth-like exoplanets

b) enables direct distance measurements of extra-galactic sources

c) precise mapping of dark matter outside the Milky Way

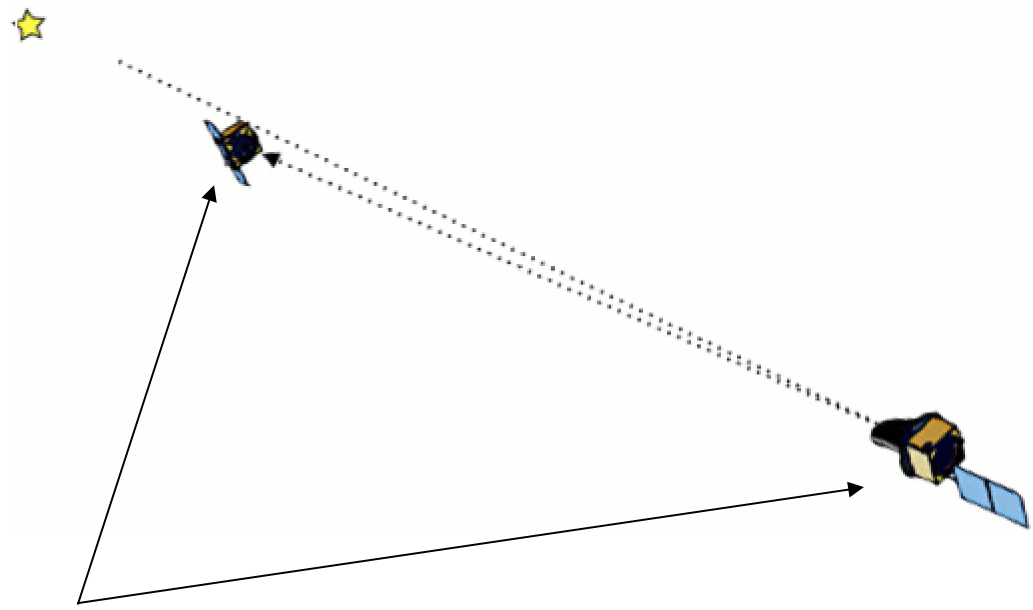
d) would allow for more precise tests of relativity

Space Astrometry Missions proposed to ESA:

1. Gaia-NIR (μas – astrometry)

2. Theia (sub-μas – astrometry)

3. NEAT (nas – astrometry)

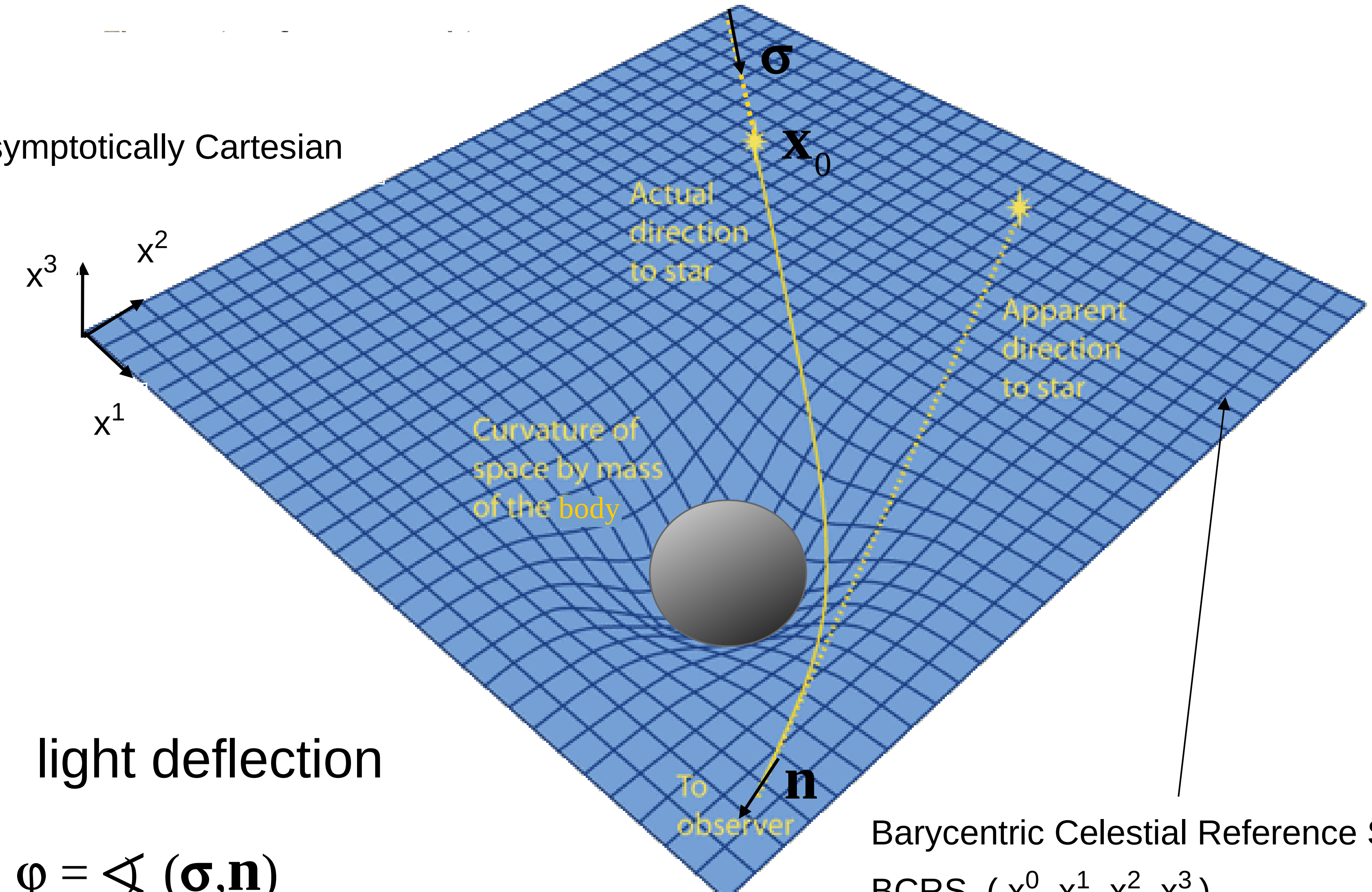


- concept: pair of spacecraft flying in formation at 40 m distance
- aim: detection of Earth-like planets within 50 light-years
- aimed astrometric accuracy: 50 nas

5. Relativistic theory of light propagation

5.1 The effect of light deflection

asymptotically Cartesian



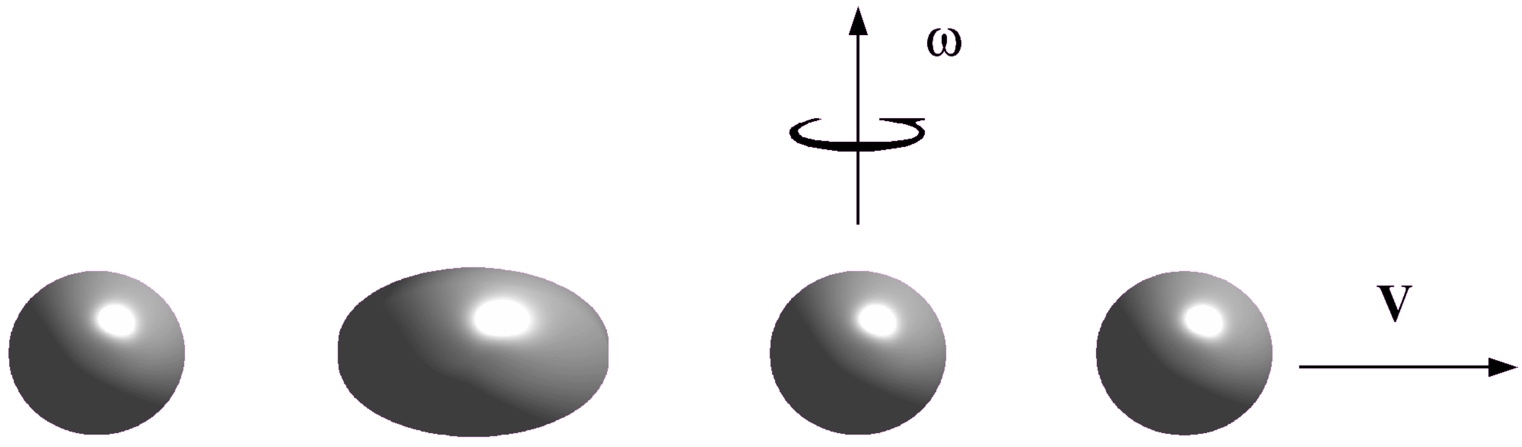
light deflection

$\varphi = \sphericalangle\,(\boldsymbol{\sigma},\mathbf{n})$

Barycentric Celestial Reference System

BCRS (x^0, x^1, x^2, x^3)

Magnitude of light deflection for grazing ray at giant planets



	Monopole	Quadrupole	Rotation	Translation
Jupiter	16.3 mas	240 μas	0.2 μas	0.7 μas
Saturn	5.8 mas	95 μas	0.04 μas	0.2 μas

S.A. Klioner, Sov. Astron. **35** (1991) 523

conclusion: sub – μas astrometry has to account for higher multipoles as well as for the motion of Solar System bodies

5.2 The exact geodesic equation

geodesic equation:

$$\frac{\ddot{x}^i(t)}{c^2} + \Gamma^i_{\alpha\beta}\,\frac{\dot{x}^\alpha(t)}{c}\,\frac{\dot{x}^\beta(t)}{c} - \Gamma^0_{\alpha\beta}\,\frac{\dot{x}^\alpha(t)}{c}\,\frac{\dot{x}^\beta(t)}{c}\,\frac{\dot{x}^i(t)}{c} = 0$$

$x^\alpha(t)$ is the four-coordinate of the light signal

Christoffel symbols:

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2}\,g^{\alpha\beta}\left(\frac{\partial g_{\beta\mu}}{\partial x^\nu} + \frac{\partial g_{\beta\nu}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\beta}\right)$$

What is the metric tensor $g_{\alpha\beta}(t, \boldsymbol{x})$ of the Solar System?

5.3 The metric tensor of the Solar system

expansion of metric tensor for weak gravitational fields:

$$\frac{m_A}{P_A} \ll 1$$

m_A ... Schwarzschild radius

P_A ... radius of body

$$g_{\alpha\beta}(t, \boldsymbol{x}) = \eta_{\alpha\beta} + h_{\alpha\beta}(t, \boldsymbol{x}) + \mathcal{O}\left(G^2\right)$$

where $\eta_{\alpha\beta} = \mathrm{diag}\,(-1, +1, +1, +1)$ is the flat metric

linearized field equations:

$$\Box\, h_{\alpha\beta}(t, \boldsymbol{x}) = -\frac{16\,\pi\,G}{c^4}\left(T_{\alpha\beta}(t, \boldsymbol{x}) - \frac{1}{2}\,\eta_{\alpha\beta}\,T(t, \boldsymbol{x})\right)$$

solution of linearized field equations:

$$h_{\alpha\beta}(t, \boldsymbol{x}) = \frac{4\,G}{c^4}\int d^3x'\,\frac{T_{\alpha\beta}(t', \boldsymbol{x}') - \dfrac{1}{2}\,\eta_{\alpha\beta}\,T(t', \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}$$

where $t' = t - \frac{|\boldsymbol{x} - \boldsymbol{x}'|}{c}$ is the retarded time expansion of metric tensor for weak gravitational fields and slow-motion of bodies:

$\frac{v_A}{c} \ll 1$ where v_A ... orbital velocity of body

post-Newtonian expansion (1.5PN approximation) of metric tensor:

$$g_{\alpha\beta}(t, \boldsymbol{x}) = \eta_{\alpha\beta} + h^{(2)}_{\alpha\beta}(t, \boldsymbol{x}) + h^{(3)}_{\alpha\beta}(t, \boldsymbol{x}) + \mathcal{O}\left(c^{-4}\right)$$

$h^{(2)}_{\alpha\beta}(t, \boldsymbol{x}) = \mathcal{O}\left(c^{-2}\right)$ and $h^{(3)}_{\alpha\beta}(t, \boldsymbol{x}) = \mathcal{O}\left(c^{-3}\right)$

The multipole-expansion of metric tensor of a system of N bodies:

$$h_{00}^{(2)}(t,\boldsymbol{x}) = \frac{2\,G}{c^2}\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}M_{\langle L\rangle}^{A}(t)\;\partial_{\langle L\rangle}\frac{1}{r_A(t)}$$

$$h_{ij}^{(2)}(t,\boldsymbol{x}) = h_{00}^{(2)}(t,\boldsymbol{x})\;\delta_{ij}$$

$$h_{0i}^{(3)}(t,\boldsymbol{x}) = \frac{4\,G}{c^3}\sum_{l=1}^{\infty}\frac{(-1)^l}{l!}\;\dot{M}_{\langle iL-1\rangle}^{A}(t)\;\partial_{\langle L-1\rangle}\frac{1}{r_A(t)}$$

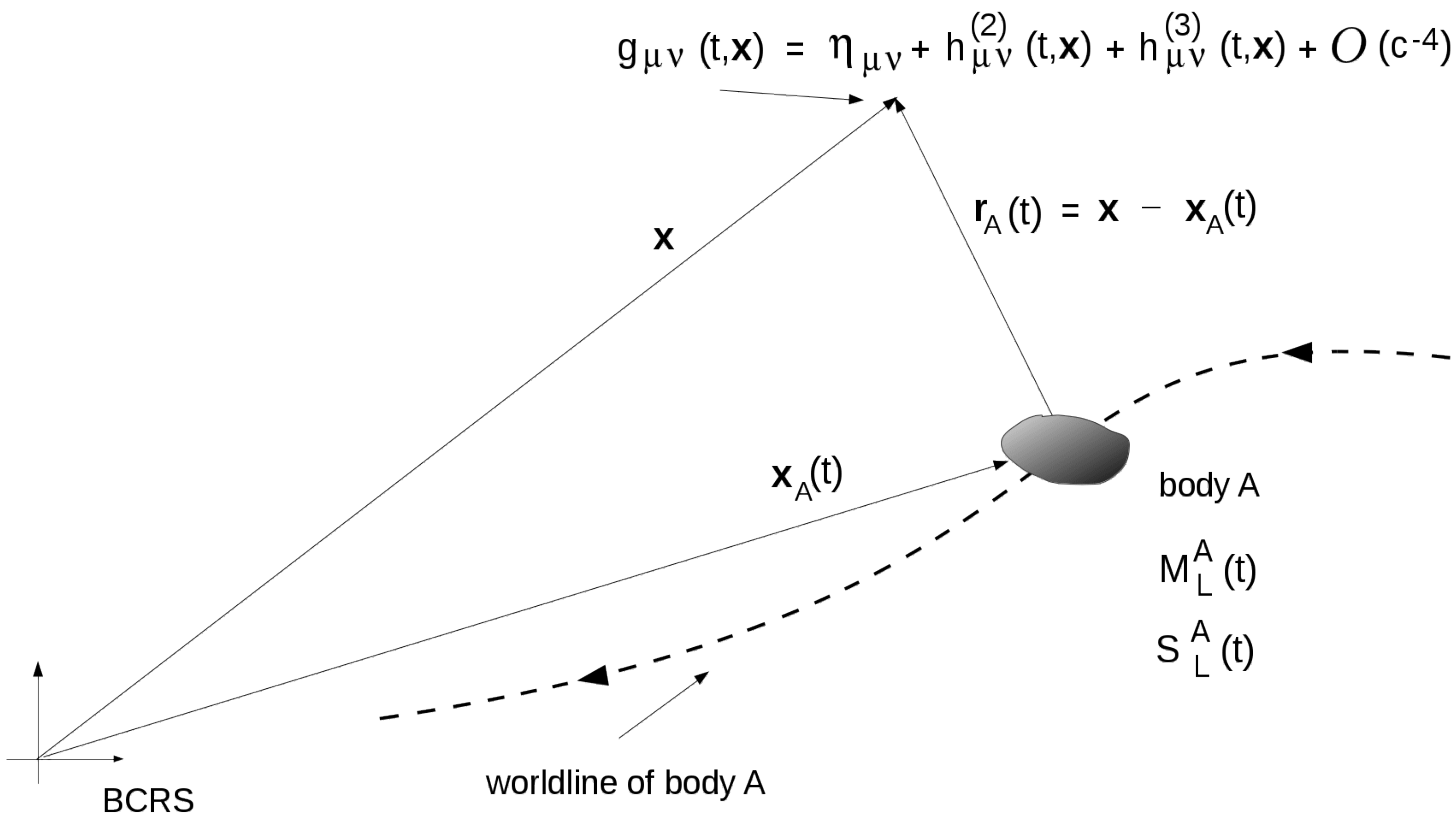
$$+\frac{4\,G}{c^3}\sum_{l=1}^{\infty}\frac{(-1)^l\;l}{(l+1)!}\,\epsilon_{iab}\,S_{\langle bL-1\rangle}^{A}(t)\;\partial_{\langle aL-1\rangle}\frac{1}{r_A(t)}$$

$$-\frac{4\,G}{c^3}\,v_A^i(t)\sum_{l=0}^{\infty}\frac{(-1)^l}{l!}M_{\langle L\rangle}^{A}(t)\;\partial_{\langle L\rangle}\frac{1}{r_A(t)}$$

Thorne 1980, Blanchet/Damour 1986, Damour/Iyer 1990, Damour/Soffel/Xu (DSX) 1991

STF (symmetric tracefree) differential operator: $\partial_{\langle L\rangle} = \underset{i_1\ldots i_l}{\mathrm{STF}}\;\frac{\partial}{\partial x^{i_1}}\cdots\frac{\partial}{\partial x^{i_l}}$

metric tensor describes a system of N arbitrarily moving bodies, having arbitrary shape, inner structure, oscillations, rotations:



multipoles are integrals over stress-energy tensor $T_A^{\mu\nu}$ of the body

- mass-multipoles (shape, inner structure, oscillations):

$$M^A_{\langle L \rangle}(T_A) = \underset{L}{\mathrm{STF}} \int d^3 X_A \; X^A_L \; \frac{T_A^{00}(T_A, \boldsymbol{X}_A)}{c^2} + \mathcal{O}\left(c^{-2}\right)$$

- spin-multipoles (rotations, inner circulations, convections):

$$S^A_{\langle L \rangle}(T_A) = \underset{L}{\mathrm{STF}} \int d^3 X_A \; \epsilon_{abc_l} \; X^A_{aL-1} \; \frac{T_A^{0b}(T_A, \boldsymbol{X}_A)}{c} + \mathcal{O}\left(c^{-2}\right)$$

these so-called local multipoles are defined in the local coordinate system of the body

5.4 The geodesic equation in 1.5PN approximation

$$\frac{\ddot{x}^i(t)}{c^2} = \frac{1}{2}\,h^{(2)}_{00,i} - h^{(2)}_{00,j}\,\frac{\dot{x}^i(t)}{c}\,\frac{\dot{x}^j(t)}{c} - h^{(2)}_{ij,k}\,\frac{\dot{x}^j(t)}{c}\,\frac{\dot{x}^k(t)}{c}$$

$$+\frac{1}{2}\,h^{(2)}_{jk,i}\,\frac{\dot{x}^j(t)}{c}\,\frac{\dot{x}^k(t)}{c} - \frac{1}{2}\,h^{(2)}_{00,0}\,\frac{\dot{x}^i(t)}{c} - h^{(2)}_{ij,0}\,\frac{\dot{x}^j(t)}{c}$$

$$+\frac{1}{2}\,h^{(2)}_{jk,0}\,\frac{\dot{x}^i(t)}{c}\,\frac{\dot{x}^j(t)}{c}\,\frac{\dot{x}^k(t)}{c} - h^{(3)}_{0i,j}\,\frac{\dot{x}^j(t)}{c} + h^{(3)}_{0j,i}\,\frac{\dot{x}^j(t)}{c}$$

$$-h^{(3)}_{0j,k}\,\frac{\dot{x}^i(t)}{c}\,\frac{\dot{x}^j(t)}{c}\,\frac{\dot{x}^k(t)}{c} + \mathcal{O}\left(c^{-4}\right)$$

where $$h^{(n)}_{\alpha\beta,\mu} = \left.\frac{\partial h^{(n)}_{\alpha\beta}(t,\boldsymbol{x})}{\partial x^\mu}\right|_{\boldsymbol{x}=\boldsymbol{x}(t)}, \quad n = 2,3$$

6. Integration of geodesic equation in 1.5PN approximation

unique solution of geodesic equation requires two conditions

- first condition defines direction of the photon at past null-infinity

$$\boldsymbol{\sigma} = \lim_{t \to -\infty} \frac{\dot{\boldsymbol{x}}(t)}{c}$$

- second condition defines coordinate of the photon at the moment of emission

$$\boldsymbol{x}_0 = \boldsymbol{x}(t_0)$$

first integration of geodesic equation yields the velocity of the light signal:

$$\frac{\dot{\boldsymbol{x}}\left(t\right)}{c} = \int\limits_{-\infty}^{t} dct' \, \frac{\ddot{\boldsymbol{x}}\left(t'\right)}{c^2} = \boldsymbol{\sigma} + \frac{\Delta\dot{\boldsymbol{x}}\left(t\right)}{c}$$

second integration of geodesic equation yields the trajectory of the light signal:

$$\boldsymbol{x}\left(t\right) = \int\limits_{t_0}^{t} dct' \, \frac{\dot{\boldsymbol{x}}\left(t'\right)}{c} = \boldsymbol{x}_0 + c\left(t - t_0\right)\boldsymbol{\sigma} + \Delta\boldsymbol{x}\left(t, t_0\right)$$

geodesic equation is solved by iteration

1. solution of first iteration is just the unperturbed light ray:

$$\boldsymbol{x}\left(t\right) = \underbrace{\boldsymbol{x}_0 + c\left(t - t_0\right)\,\boldsymbol{\sigma}}_{\boldsymbol{x}_{\rm N}(t)} + \mathcal{O}\left(c^{-2}\right)$$

2. solution of second iteration is the light ray in 1PN approximation:

$$\boldsymbol{x}\left(t\right) = \boldsymbol{x}_0 + c\left(t - t_0\right)\,\boldsymbol{\sigma} + \Delta\boldsymbol{x}_{\rm 1PN}\left(t\right) + \mathcal{O}\left(c^{-3}\right)$$

3. solution of third iteration is the light ray in 1.5PN approximation:

$$\boldsymbol{x}\left(t\right) = \boldsymbol{x}_0 + c\left(t - t_0\right)\,\boldsymbol{\sigma} + \Delta\boldsymbol{x}_{\rm 1PN}\left(t\right) + \Delta\boldsymbol{x}_{\rm 1.5PN}\left(t\right) + \mathcal{O}\left(c^{-4}\right)$$

one is confronted with a serious problem when integrating the geodesic equation

Let us consider an example

By inserting the multipole-expansion of the metric tensor in the geodesic equation one encounters the following kind of integrals:

$$\Delta \dot{\boldsymbol{x}}^{A}_{1\mathrm{PN}}(t) \sim \frac{G}{c} \sum_{l=0}^{\infty} \frac{(-1)^{l}}{l!} \int_{-\infty}^{t} dct' \; M_{L}^{A}(t') \; \partial_{L} \frac{\boldsymbol{r}_{A}(t')}{(r_{A}(t'))^{3}} \Bigg|_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{N}}(t')}$$

where $\boldsymbol{r}_{A}(t') = \boldsymbol{x} - \boldsymbol{x}_{A}(t')$

The differentiation ∂_L leads to involved terms, e.g.:

$$\partial_{L} \frac{1}{(r_{A}(t'))^{3}} = (-1)^{l} \; \frac{3\,(2l-1)!!}{(r_{A}(t'))^{2l+3}} \; \underset{i_1 \ldots i_l}{\mathrm{STF}} \; r_{A}^{i_1}(t') \ldots r_{A}^{i_l}(t') \Bigg|_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{N}}(t')}$$

Problem: Such procedure leads to terrible integrals,

because the differentiation has to be performed before integration

The following terrible integral is only a piece of the example under consideration:

$$\Delta \dot{\boldsymbol{x}}^{A}_{\rm 1PN}\left(t\right) \sim \frac{G}{c} \sum_{l=0}^{\infty} \frac{3\,(2l-1)!!}{l!} \mathop{\rm STF}_{i_1 \dots i_l} \int\limits_{-\infty}^{t} dct'\; M^{A}_{i_1 \dots i_l}\left(t'\right)$$

$$\times \left[\boldsymbol{x}_0 + c\left(t' - t_0\right)\boldsymbol{\sigma} - \boldsymbol{x}_A\left(t'\right)\right]$$

$$\times \frac{\left[x_0^{i_1} + c\left(t' - t_0\right)\sigma^{i_1} - x_A^{i_1}\left(t'\right)\right] \dots \left[x_0^{i_l} + c\left(t' - t_0\right)\sigma^{i_l} - x_A^{i_l}\left(t'\right)\right]}{\left|\boldsymbol{x}_0 + c\left(t' - t_0\right)\boldsymbol{\sigma} - \boldsymbol{x}_A\left(t'\right)\right|^{2l+3}}$$

Problem is caused by the fact that one has

1. first of all to differentiate with respect to the field point $\boldsymbol{x}$
2. to insert the unperturbed light ray $\boldsymbol{x} = \boldsymbol{x}_0 + c\left(t' - t_0\right)\boldsymbol{\sigma}$
3. afterwards to perform the integration

solution of this problem found by S. Kopeikin, J. Math. Phys. **38** (1997) 2587
for bodies at rest with full multipole-structure:

- Introduction of new variables:

new time-variable: $$c\,\tau = \boldsymbol{\sigma} \cdot \boldsymbol{x}_{\rm N}\,(t)$$

new spatial-variable: $$\xi^i = \Big(\underbrace{\delta_{ij} - \sigma_i\,\sigma_j}_{P_{ij}}\Big)\,x^j_{\rm N}\,(t)$$

new auxiliary constant: $$t^* = t_0 - \frac{\boldsymbol{\sigma} \cdot \boldsymbol{x}_0}{c}$$

- Inverse transformation yields:

coordinate time: $$t = \tau + t^*$$

unperturbed light ray: $$\boldsymbol{x}_{\rm N} = \boldsymbol{\xi} + c\,\tau\,\boldsymbol{\sigma}$$

- Time – derivative in terms of the new variables

$$\left[\frac{\partial F\,(t,\boldsymbol{x})}{\partial ct}\right]_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{N}}(t)} \;=\; \frac{\partial}{\partial ct^{*}}F\left(\tau + t^{*}\,,\,\boldsymbol{\xi} + c\,\tau\,\boldsymbol{\sigma}\right)$$

- Spatial derivative in terms of the new variables

$$\left[\frac{\partial F\,(t,\boldsymbol{x})}{\partial x^{i}}\right]_{\boldsymbol{x}=\boldsymbol{x}_{\mathrm{N}}(t)} \;=\; \left(P_{ij}\,\frac{\partial}{\partial \xi^{j}} + \sigma^{i}\,\frac{\partial}{\partial c\tau} - \sigma^{i}\,\frac{\partial}{\partial ct^{*}}\right)F\left(\tau + t^{*}\,,\,\boldsymbol{\xi} + c\,\tau\,\boldsymbol{\sigma}\right)$$

- Solution of the problem: integration in terms of new variables

$$\int dc\tau' \, \frac{\partial}{\partial c\tau'} F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right) = F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right) + C\left(\boldsymbol{\xi}\right)$$

in such cases the integration can be performed immediately

$$\int dc\tau' \, \frac{\partial}{\partial ct^*} F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right) = \frac{\partial}{\partial ct^*} \int dc\tau' \, F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right)$$

$$\int dc\tau' \, \frac{\partial}{\partial \xi^i} F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right) = \frac{\partial}{\partial \xi^i} \int dc\tau' \, F\left(\tau' + t^*\,,\,\boldsymbol{\xi}\right)$$

in such cases the differentiation is performed after integration

- the STF differential operation in terms of the old variables

$$\partial_{\langle L\rangle} = \underset{i_1\ldots i_l}{\mathrm{STF}} \frac{\partial}{\partial x^{i_1}} \cdots \frac{\partial}{\partial x^{i_l}}$$

- Trinomial formula

$$(a+b+c)^l = \sum_{p=0}^{l} \binom{l}{p} a^{l-p} \sum_{q=0}^{p} \binom{p}{q} b^{p-q}\, c^q$$

- the STF differential operation in terms of the new variables

$$\partial_{\langle L\rangle} = \underset{i_1\ldots i_l}{\mathrm{STF}} \sum_{p=0}^{l} \binom{l}{p} \sum_{q=0}^{p} \binom{p}{q} \sigma^{i_1} \ldots \sigma^{i_p} P_{i_{p+1}j_{p+1}} \ldots P_{i_1 j_1} \frac{\partial}{\partial \xi^{j_{p+1}}} \cdots \frac{\partial}{\partial \xi^{j_1}} \left(\frac{\partial}{\partial c\tau}\right)^{p-q} \left(\frac{\partial}{\partial ct^*}\right)^{q}$$

The example under consideration leads now to the following expression:

$$\Delta \dot{\boldsymbol{x}}^{A}_{\rm 1PN}\left(\tau + t^{*}\right) \sim \frac{G}{c} \sum_{l=0}^{\infty} \frac{\left(-1\right)^{l}}{l!} \sum_{p=0}^{l} \frac{l!}{(l-p)!\;p!} \sum_{q=0}^{p} \left(-1\right)^{q} \frac{p!}{(p-q)!\;q!}$$

$$\times\; \sigma^{i_1} \ldots \sigma^{i_p}\; P^{i_{p+1}\,j_{p+1}} \ldots P^{i_l\,j_l}\; \frac{\partial}{\partial \xi^{j_{p+1}}} \ldots \frac{\partial}{\partial \xi^{j_l}} \left(\frac{\partial}{\partial c\, t^{*}}\right)^{q}$$

$$\times \int\limits_{-\infty}^{\tau} \left(\frac{\partial}{\partial c\,\tau'}\right)^{p-q} \frac{\boldsymbol{r}^{\rm N}_{A}\left(\tau' + t^{*}\right)}{\left(r^{\rm N}_{A}\left(\tau' + t^{*}\right)\right)^{3}}\; M^{A}_{L}\left(\tau' + t^{*}\right)\, dc\tau'$$

where $\boldsymbol{r}^{\rm N}_{A}\left(\tau' + t^{*}\right) = \boldsymbol{\xi} + c\,\tau'\,\boldsymbol{\sigma} - \boldsymbol{x}_{A}\left(\tau' + t^{*}\right)$

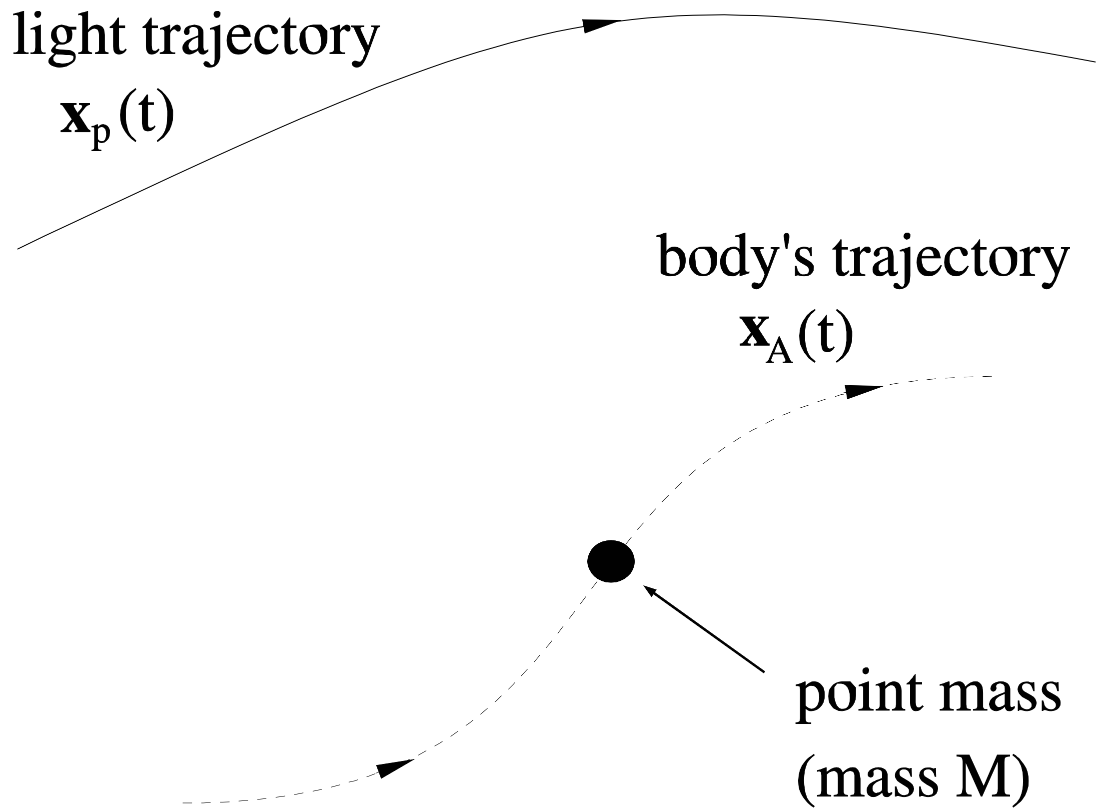
The Integral is of considerably simpler structure now

Problem is solved because the differentiation is performed after integration

just a comment: the specific case p - q = 0 is not a problem at all

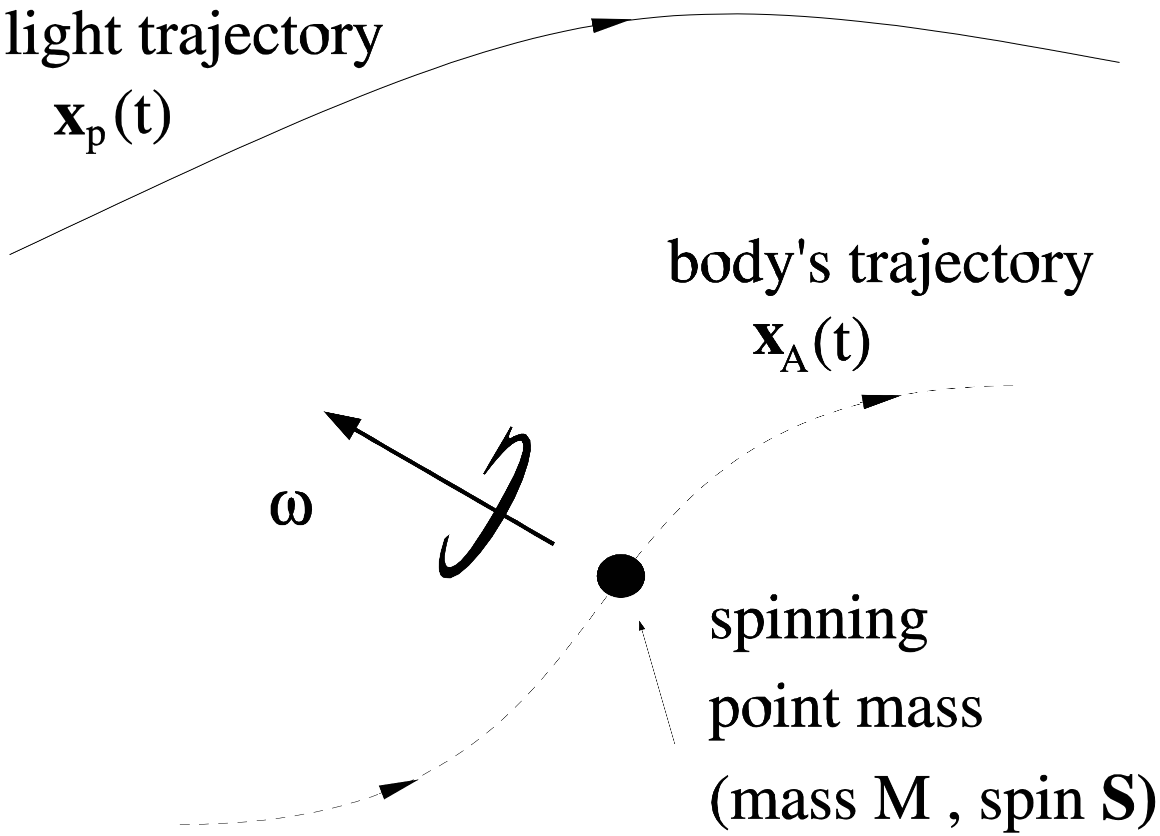
7. The state-of-the-art before our investigation

Light propagation in field of a moving monopole



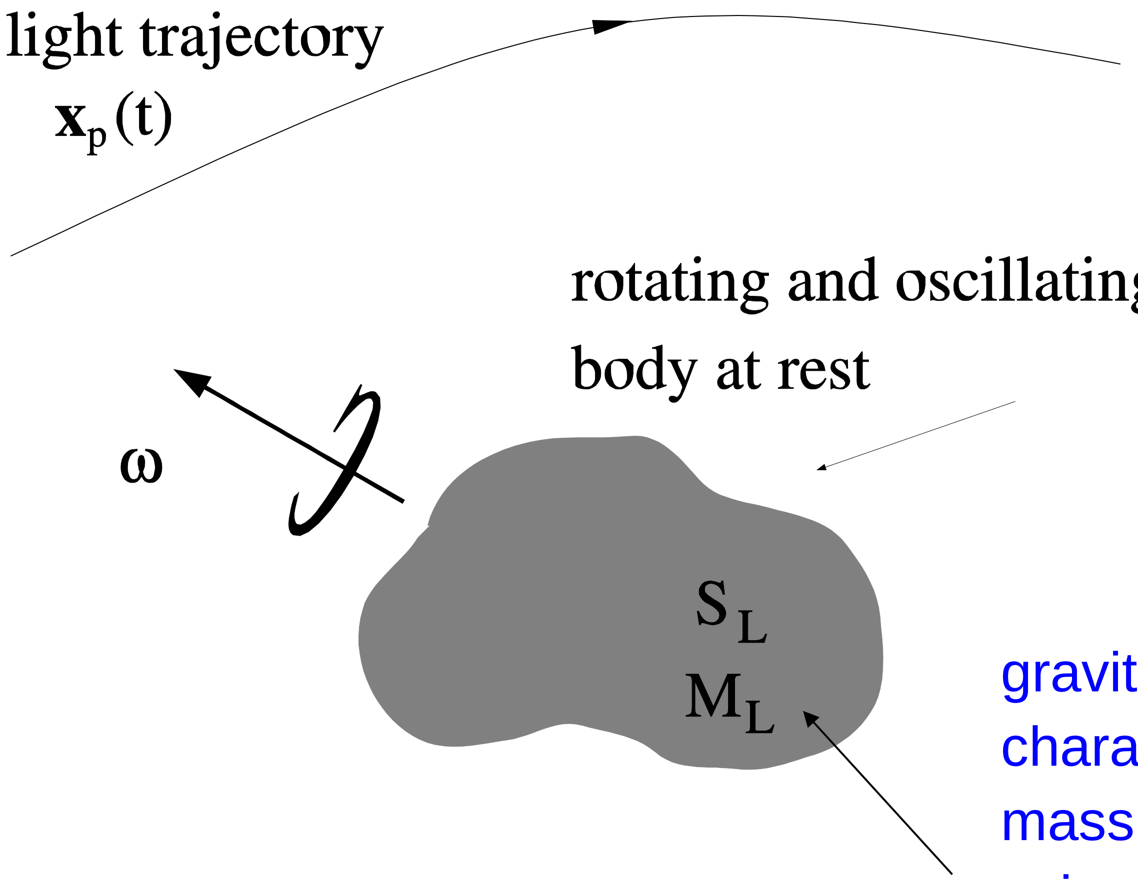
exact analytical post - Minkowskian solution for light trajectory found by
S. Kopeikin, G. Schäfer, Phys. Rev. **D 60** (1999) 124002

Light propagation in field of a <u>moving and rotating monopole</u>



exact analytical post - Minkowskian solution for light trajectory found by
S. Kopeikin, B. Mashhoon, Phys. Rev. **D 65** (2002) 064025

Light propagation in field of an extended body at rest

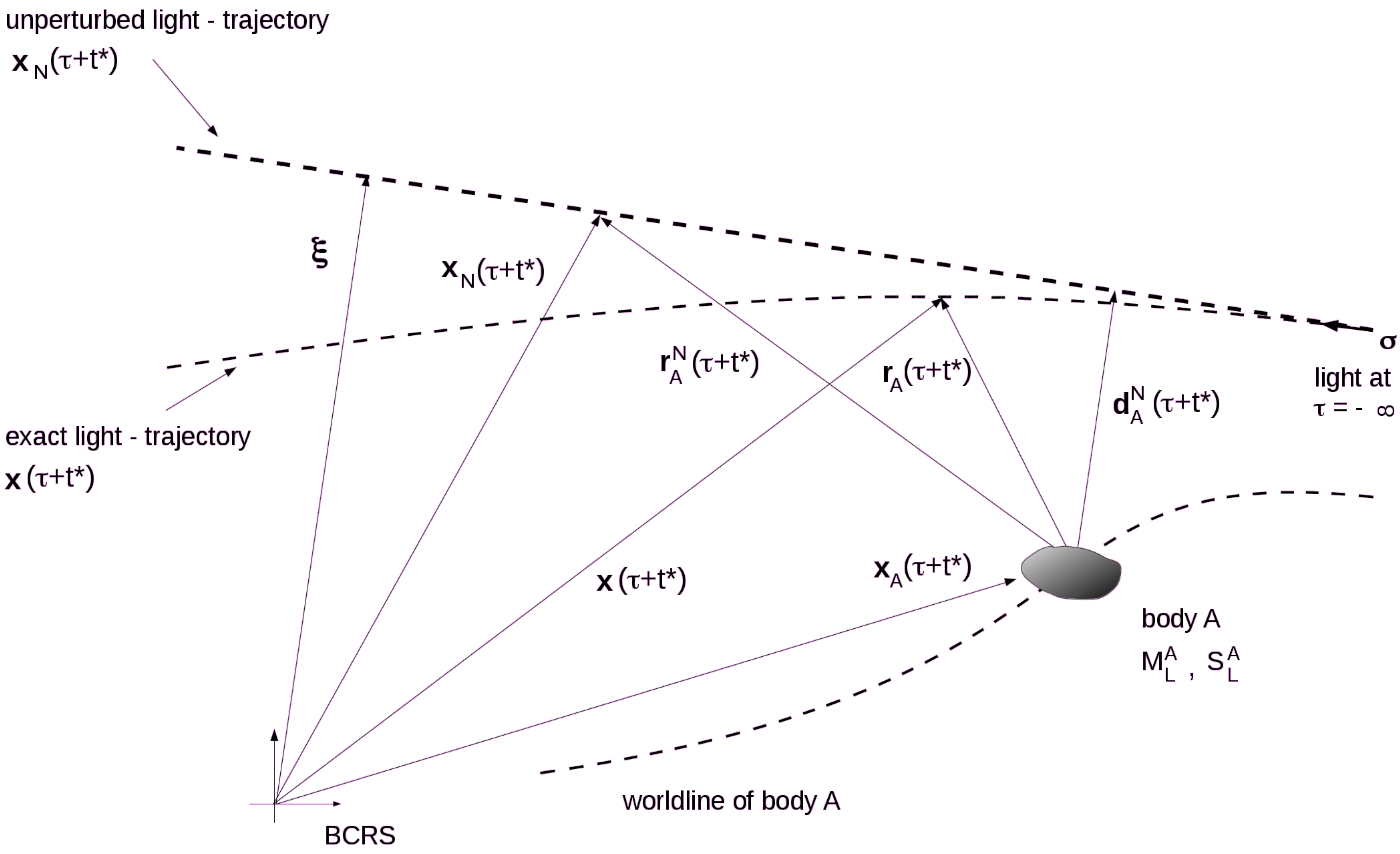


gravitational field characterized by mass moments and spin moments

exact analytical post - Minkowskian solution for light trajectory in terms of global multipoles found by

S. Kopeikin, P. Korobkov, A. Polnarev, Class. Quantum Grav. **23** (2006) 4299

8. Light-trajectory in the field of N moving bodies with M_L, S_L



unknown world-line $\mathbf{x}_A(\tau+t^*)$ of body necessitates to integrate the geodesic equation by parts

1PN solution of light-trajectory in the field of N moving bodies: M_L

$$\Delta \boldsymbol{x}_{1\mathrm{PN}}\left(\tau + t^{*}\right) = -\frac{2\,G}{c^{2}} \sum_{l=0}^{\infty} \frac{\left(-1\right)^{l}}{l!}\; M^{A}_{\langle L \rangle}\left(\tau + t^{*}\right) \partial_{\langle L \rangle}\; \frac{\boldsymbol{d}_{A}\left(\tau + t^{*}\right)}{r^{\mathrm{N}}_{A}\left(\tau + t^{*}\right) - \boldsymbol{\sigma} \cdot \boldsymbol{r}^{\mathrm{N}}_{A}\left(\tau + t^{*}\right)}$$

$$+\frac{2\,G}{c^{2}}\,\boldsymbol{\sigma} \sum_{l=0}^{\infty} \frac{\left(-1\right)^{l}}{l!}\; M^{A}_{\langle L \rangle}\left(\tau + t^{*}\right) \partial_{\langle L \rangle}\; \ln\left[r^{\mathrm{N}}_{A}\left(\tau + t^{*}\right) - \boldsymbol{\sigma} \cdot \boldsymbol{r}^{\mathrm{N}}_{A}\left(\tau + t^{*}\right)\right]$$

S. Zschocke, Physical Review D **92** (2015) 063015

1.5PN solution of light-trajectory in the field of N moving bodies: M_L, S_L

$$\Delta x^{i\,\mathrm{M}}_{1.5\mathrm{PN}}\left(\tau + t^{*}\right) = f^{i}\left(M_L^A\,\frac{v_A}{c}\,,\;\dot{M}_L^A\right)$$

$$\Delta x^{i\,\mathrm{S}}_{1.5\mathrm{PN}}\left(\tau + t^{*}\right) = +\frac{4\,G}{c^3}\sum_{l=1}^{\infty}\frac{(-1)^l\,l}{(l+1)!}\,\epsilon_{iab}\,S^A_{\langle bL-1\rangle}\left(\tau + t^{*}\right)$$

$$\times\partial_{\langle aL-1\rangle}\;\ln\left[r_A^{\mathrm{N}}\left(\tau + t^{*}\right) - \boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\mathrm{N}}\left(\tau + t^{*}\right)\right]$$

$$-\frac{4\,G}{c^3}\,\sigma^j\sum_{l=1}^{\infty}\frac{(-1)^l\,l}{(l+1)!}\,\epsilon_{jab}\,S^A_{\langle bL-1\rangle}\left(\tau + t^{*}\right)$$

$$\times\partial_{\langle aL-1\rangle}\;\frac{d_A^i(\tau+t^{*})}{r_A^{\mathrm{N}}(\tau+t^{*})-\boldsymbol{\sigma}\cdot\boldsymbol{r}_A^{\mathrm{N}}(\tau+t^{*})}$$

S. Zschocke, Physical Review D **93** (2016) 103010

Magnitude of light-deflection for grazing rays at giant planets

$$\varphi = \left| \boldsymbol{\sigma} \times \frac{\Delta \dot{\boldsymbol{x}}(t)}{c} \right|$$

Term	Jupiter [μas]	Saturn [μas]
$\varphi_{M_0^A}$	16300	5800
$\varphi_{M_2^A}$	240	95
$\varphi_{M_4^A}$	9.6	5.46
$\varphi_{M_6^A}$	0.56	0.50
$\varphi_{M_8^A}$	0.04	0.06
$\varphi_{M_{10}^A}$	0.003	0.01
$\varphi_{S_1^A}$	0.2	0.04
$\varphi_{S_3^A}$	0.015	0.006

9. Summary and Outlook

- todays astrometry has reached a precision at micro-arcsecond level (Gaia)
- future astrometry at sub-micro-arcsecond level (e.g. Theia, NEAT) needs a considerably improved theory of light propagation
- the present status in the theory of light propagation is the following:
  - metric of arbitrarily shaped, rotating and oscillating bodies in arbitrary motion is known in the (post-Newtonian) DSX scheme
  - fully analytical model for the light propagation in the Solar System has been obtained in 1PN and 1.5PN approximation
- for sub-micro-arcsecond astrometry many subtle problems have to be solved
  - without details: many work needs to be done in 1PN and 1.5PN approximation
  - light trajectory needs to be known in 2PN approximation (enhanced terms)
  - light trajectory needs also to be known in post-Minkowskian scheme (far-zone)