POST-LINEAR METRIC OF A SOLAR SYSTEM BODY

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ABSTRACT.

A precise modeling of light trajectories in the solar system on the sub-micro-arcsecond and nano-arcsecond scale of accuracy requires the metric tensor of solar system bodies in post-linear approximation. The Multipolar Post-Minkowskian formalism represents a framework for determining the metric density in the exterior of a compact source of matter, which can be regarded as massive solar system body. The knowledge of the metric density, frequently been called gothic metric, allows to deduce the metric tensor. Some aspects are considered about how to determine the metric density and the metric tensor from the field equations of gravity.

1. INTRODUCTION

An advancement in astrometric science towards sub-micro-arcsecond and nano-arcsecond level in angular measurements of celestial objects requires considerable progress in the theory of light propagation through the curvilinear space-time of the solar system. In curved space-time the light signals propagate along null-geodesics, governed by the geodesic equation which reads $\ddot{x}^{\alpha}(\lambda)+\Gamma^{\alpha}_{\mu\nu}\,\dot{x}^{\mu}(\lambda)\,\dot{x}^{\nu}(\lambda)=0$, where $x^{\alpha}(\lambda)$ is the four-coordinate of the light signal as function of the affine curve parameter λ, a dot means total derivative with respect to λ, and the Christoffel symbols $\Gamma^{\alpha}_{\mu\nu} = g^{\alpha\beta}\left(g_{\beta\mu\,,\,\nu} + g_{\beta\nu\,,\,\mu} - g_{\mu\nu\,,\,\beta}\right)/2$ are functions of the metric tensor $g_{\alpha\beta}$, and a comma denotes partial derivative with respect to the four-coordinates, e.g. $f_{\,,\,\mu} = \partial f / \partial x^{\mu}$ and $f_{\,,\,\mu\nu} = \partial^2 f / \partial x^{\mu}\,\partial x^{\nu}$, etc. Accordingly, a precise modeling of light trajectories implies a precise knowledge of the metric of solar system bodies. The metric tensor can be series expanded in powers of the gravitational constant G, called post-Minkowskian expansion,

$$g_{\alpha\beta}\left(x\right) = \eta_{\alpha\beta} + \sum_{n=1}^{\infty} G^n\, h^{(\mathrm{nPM})}_{\alpha\beta}\left(x\right) \tag{1}$$

where the first and second term, $h^{(\mathrm{1PM})}_{\alpha\beta}$ and $h^{(\mathrm{2PM})}_{\alpha\beta}$, are the linear and post-linear term of the metric perturbation, which are required for determining the light trajectory on the sub-micro-arcsecond and nano-arcsecond scale of accuracy. The orthogonality relation $g^{\alpha\rho}\,g_{\rho\beta} = \delta^{\alpha}_{\beta}$ enables to switch between the contravariant and covariant components of the metric tensor.

The Multipolar Post-Minkowskian (MPM) formalism represents a perturbative approach for determining the metric density, $\overline{g}^{\alpha\beta}$, in the exterior of a compact source of matter, defined by

$$\overline{g}^{\alpha\beta} = \sqrt{-g}\;\, g^{\alpha\beta} \qquad \text{or} \qquad g^{\alpha\beta} = \sqrt{-\overline{g}}\;\, \overline{g}^{\alpha\beta} \tag{2}$$

where $g = \det\left(g_{\rho\sigma}\right)$ and $\overline{g} = \det\left(\overline{g}_{\rho\sigma}\right)$ is the determinant of the covariant components of the metric tensor and metric density, respectively. The post-Minkowskian expansion of the metric density reads

$$\overline{g}^{\alpha\beta}\left(x\right) = \eta^{\alpha\beta} - \sum_{n=1}^{\infty} G^n\, \overline{h}^{\alpha\beta}_{(\mathrm{nPM})}\left(x\right) \tag{3}$$

where the first and second term, $\overline{h}^{\alpha\beta}_{(\mathrm{1PM})}$ and $\overline{h}^{\alpha\beta}_{(\mathrm{2PM})}$, are the linear and post-linear term of the gothic metric perturbation. The orthogonality relation $\overline{g}^{\alpha\rho}\,\overline{g}_{\rho\beta} = \delta^{\alpha}_{\beta}$ enables to switch between the contravariant and covariant components of the gothic metric.

The MPM formalism determines the metric density in the exterior of a massive body, having arbitrary shape, inner structure, oscillations, and rotational motions. Due to Eq. (2) the knowledge of the metric density allows to deduce the metric tensor. In what follows, some aspects are considered about how to obtain the metric density and metric tensor from the field equations.

2. THE FIELD EQUATIONS OF GRAVITY

The field equations relate the metric tensor $g_{\mu\nu}$ to the stress-energy tensor of matter $T_{\mu\nu}$,

$$R_{\mu\nu} - \frac{1}{2}\, g_{\mu\nu}\, R = \frac{8\,\pi\, G}{c^4}\, T_{\mu\nu} \tag{4}$$

where $R_{\mu\nu} = \Gamma^{\rho}_{\mu\nu\,,\,\rho} - \Gamma^{\rho}_{\mu\rho\,,\,\nu} + \Gamma^{\rho}_{\sigma\rho}\,\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\rho}_{\sigma\nu}\,\Gamma^{\sigma}_{\mu\rho}$ is the Ricci tensor and $R = g^{\mu\nu}\, R_{\mu\nu}$ is the Ricci scalar. The field equations constitute a set of ten coupled non-linear partial differential equations for the ten components of the metric tensor $g_{\mu\nu}$ of space-time, which in differential geometry is modeled by a semi-Riemannian manifold $\mathcal{M}$. The contracted Bianchi identities imply that only six of these field equations (4) are independent, which determine the ten components of the metric tensor up to a passive coordinate transformation (keep points of manifold fixed and change coordinates) from the old $\{y\}$ to the new coordinate system $\{y'\}$,

$$y^{\mu} \rightarrow y'^{\,\mu}\,. \tag{5}$$

The field equations (4) are invariant under these (infinitely many) coordinate transformations, known as passive general covariance of the field equations. That means, if the set $(\mathcal{M}, \mathbf{g})$ is a solution of the field equations, then the set $(\mathcal{M}, \mathbf{g}')$ is also a solution of the same field equations, where $g'_{\alpha\beta} = A^{\mu}_{\alpha}\, A^{\nu}_{\beta}\, g_{\mu\nu}$ is the metric tensor in these new coordinates with A^{μ}_{α} being the Jacobian matrix $A^{\mu}_{\alpha} = \partial y^{\mu} / \partial y'^{\alpha}$ of the passive coordinate transformation. These sets are physically equivalent and describe the same physical system. The metric tensors have different components in different coordinate systems, $g'_{\alpha\beta} \neq g_{\mu\nu}$, but as geometrical objects (Eq. (2.23) in *Hawking,Ellis* (1974)) they are equal, $\mathbf{g}' = \mathbf{g}$, because they attribute the same distance to the same pair of points $\mathcal{P}$ and $\mathcal{Q}$ of the manifold: $d_{\mathbf{g}'}\,(\mathcal{P}, \mathcal{Q}) = d_{\mathbf{g}}\,(\mathcal{P}, \mathcal{Q})$ (infinitesimal distance of these pairs is assumed). For later purposes it is useful to consider an active coordinate transformation (keep coordinates fixed and change points of manifold),

$$\Psi : \mathcal{M} \rightarrow \mathcal{M} \tag{6}$$

which is a C^{∞} differentiable mapping of each point of the manifold reversibly unique to another image point of the same manifold, $\mathcal{P} \rightarrow \Psi\,(\mathcal{P})$. Hence, the coordinates are changed $y^{\mu}\,(\mathcal{P}) \rightarrow y'^{\,\mu}\,(\mathcal{P})$. The field equations (4) are invariant under these (infinitely many) diffeomorphisms, known as active general covariance of the field equations. That means, if the set $(\mathcal{M}, \mathbf{g})$ is a solution of the field equations, then the set $(\mathcal{M}, \mathbf{g}')$ is also a solution of the same field equations, where $\mathbf{g}' = \Psi^{*}\mathbf{g}$ is the pullback of the metric tensor, $g'_{\alpha\beta} = A^{\mu}_{\alpha}\, A^{\nu}_{\beta}\, g_{\mu\nu}$, with A^{μ}_{α} being the Jacobian matrix $A^{\mu}_{\alpha} = \partial y^{\mu} / \partial y'^{\alpha}$ of the active coordinate transformation. These sets are physically equivalent and describe the same physical system (Section 7.1 in *Hawking,Ellis* (1974); for the associated problem of Leibniz Equivalence see *Earman,Norton* (1987) and *Lusanna,Pauri* (2006)). These metric tensors attribute the same distance of a pair of points of the manifold and their images, $d_{\mathbf{g}'}\,(\mathcal{P}, \mathcal{Q}) = d_{\mathbf{g}}\,(\Psi\,(\mathcal{P})\,, \Psi\,(\mathcal{Q}))$ (infinitesimal distance of these pairs and their images is assumed). But these metric tensors are not equal, $\mathbf{g}' \neq \mathbf{g}$, because they attribute different distances to the same pair of points of the manifold: $d_{\mathbf{g}'}\,(\mathcal{P}, \mathcal{Q}) \neq d_{\mathbf{g}}\,(\mathcal{P}, \mathcal{Q})$ (e.g. *Gaul,Rovelli*, (2000)). However, if a Killing vector field exists on $\mathcal{M}$ and the diffeomorphism Ψ proceeds along the congruence of that Killing vector field, then the metric and pullback metric are equal, $\mathbf{g}' = \mathbf{g}$, and the diffeomorphism is an isometry (Section 2.6 in *Hawking,Ellis* (1974)).

3. LANDAU-LIFSCHITZ FORMULATION OF GRAVITY

The theory of gravity has a geometrical interpretation in physical curvilinear space-time and a field-theoretical interpretation in auxiliary flat space-time (e.g. text below Eq. (11) in *Gupta* (1954) or Section 8.4 in *Feynman* (1995) or part 5 in Box 17.2 in *Misner, Thorne, Wheeler* (1973)); for an excellent historical overview we refer to *Brian Pitts,Schieve* (2018). So one distinguishes between a physical manifold $\mathcal{M}$ covered by curvilinear coordinates y^μ and endowed with metric $g_{\mu\nu}(y)$, a flat background manifold $\mathcal{M}_0$ covered by curvilinear coordinates x^α and endowed with metric $g^0_{\alpha\beta}(x)$, and a diffeomorphism

$$\Phi : \mathcal{M}_0 \rightarrow \mathcal{M} \tag{7}$$

which is a C^∞ differentiable mapping of each point $q \in \mathcal{M}_0$ of the flat background manifold $\mathcal{M}_0$ reversibly unique to another point $p \in \mathcal{M}$ of the physical manifold $\mathcal{M}$ (hence $\dim \mathcal{M}_0 = \dim \mathcal{M}$); it is not relevant whether (7) exists everywhere or only on finite domains $\Phi : \mathcal{V} \subseteq \mathcal{M}_0 \rightarrow \mathcal{U} \subseteq \mathcal{M}$.

The field equations (4) are not invariant under (7), because the manifolds $\mathcal{M}$ and $\mathcal{M}_0$ are different with respect to their geometrical properties: the curvature tensor of $\mathcal{M}$ expressed in terms of $g_{\mu\nu}(y)$ is non-zero, $R^\mu_{\alpha\nu\beta}(y) \neq 0$, in any coordinate system $\{y\}$ which maps the physical manifold, while the curvature tensor of $\mathcal{M}_0$ expressed in terms of $g^0_{\alpha\beta}(x)$ vanishes, $R^\mu_{\alpha\nu\beta}(x) = 0$, in any coordinate system $\{x\}$ which maps the flat background manifold. In particular, the metric tensor $\mathbf{g}^0$ of $\mathcal{M}_0$ (e.g. in Cartesian coordinates $\mathbf{g}^0$ is given by $\eta_{\alpha\beta} = \mathrm{diag}\,(-1,+1,+1,+1)$) and the metric tensor $\mathbf{g}$ of $\mathcal{M}$ can never be related by a pullback: $\mathbf{g}^0 \neq \Phi^*\mathbf{g}$. In this context it is noticed that Eqs. (A.11) - (A.13) in *Carroll* (2013) consider a map with $\dim \mathcal{M}_0 \neq \dim \mathcal{M}$, but not a diffeomorphism where necessarily $\dim \mathcal{M}_0 = \dim \mathcal{M}$.

However, the diffeomorphism (7) is an active coordinate transformation, which makes it possible to pullback the metric tensor $\mathbf{g}$ of the physical manifold $\mathcal{M}$ (given by $g_{\mu\nu}(y)$) to the metric tensor $\Phi^*\mathbf{g}$ which propagates as tensorial field (given by $g_{\alpha\beta}(x)$) on the flat background $\mathcal{M}_0$

$$g_{\alpha\beta}(x) = \frac{\partial y^\mu}{\partial x^\alpha}\,\frac{\partial y^\nu}{\partial x^\beta}\,g_{\mu\nu}(y)\;. \tag{8}$$

In the same way, the Ricci tensor and energy-momentum tensor on $\mathcal{M}$ are pulled back on $\mathcal{M}_0$. By means of these relations the field equations of gravity (4) on the physical manifold $\mathcal{M}$ can be pulled back to field equations on the flat background manifold $\mathcal{M}_0$. Then, the sets $(\mathcal{M}, \mathbf{g})$ and $(\mathcal{M}_0, \Phi^*\,\mathbf{g})$ are physically equivalent, iff the metric tensor $\mathbf{g}$ on the physical manifold $\mathcal{M}$ is determined by the field equations (4), while the pulled-back metric tensor $\Phi^*\,\mathbf{g}$ on the flat background manifold $\mathcal{M}_0$ (i.e. $g_{\alpha\beta} = \Phi^{*\,\mu\nu}_{\;\alpha\beta}\,g_{\mu\nu}$ in Eq. (8)) is determined by the pulled-back field equations on $\mathcal{M}_0$ (cf. Section 7 in *Hawking,Ellis* (1974), especially text below Eq. (7.51) in *Hawking,Ellis* (1974), as well as text below Eq. (7.10) in *Carroll* (2013)).

In the Landau-Lifschitz formulation one makes a detour and does not consider the metric tensor $g_{\mu\nu}(y)$ but the metric density $\overline{g}^{\mu\nu}(y)$, which is pulled back from the physical manifold to the flat background manifold. A detailed mathematical representation of the Landau-Lifschitz formulation is given by Sections 1 and 2 in *Petrov,Kopeikin,Lompay,Tekin* (2017) as well as by Section 7 in *Hawking,Ellis* (1974). These field equations take the following form (cf. Eqs. (20.20) - (20.22) in *Misner, Thorne, Wheeler* (1973), Eq. (6.6) in *Poisson, Will* (2014)),

$$H^{\alpha\rho\beta\sigma}{}_{,\,\rho\sigma}(x) = \frac{16\,\pi\,G}{c^4}\,(-g(x))\,\Big(T^{\alpha\beta}(x) + t^{\alpha\beta}_{\mathrm{LL}}(x)\Big)\,. \tag{9}$$

The l.h.s. is the Landau-Lifschitz superpotential, $H^{\alpha\rho\beta\sigma} = \overline{g}^{\alpha\beta}\,\overline{g}^{\rho\sigma} - \overline{g}^{\alpha\sigma}\,\overline{g}^{\beta\rho}$, while the r.h.s. is the Landau-Lifschitz complex, where $t^{\alpha\beta}_{\mathrm{LL}}$ is the Landau-Lifschitz pseudotensor which represents, roughly to speak, the energy-momentum distribution of the gravitational fields. The field equations (9) are manifestly Lorentz-covariant and constitute a set of ten coupled non-linear partial differential equations for the ten components of the metric density $\overline{g}^{\alpha\beta}$. Because of the identity $H^{\alpha\rho\beta\sigma}{}_{,\rho\sigma\beta} = 0$ (implying energy-momentum conservation, cf. Eqs. (6.7) - (6.8) in *Poisson, Will* (2014)) only six equations are independent, which determine the ten components of the metric density up to a passive transformation of coordinates which map the flat background manifold.

Thus far, no specific choice of the coordinates of the flat background manifold has been imposed. For practical calculations in celestial mechanics, in the theory of light propagation, or in the theory of gravitational waves, it is, however, very useful to choose harmonic coordinates to cover the flat background space-time $\mathcal{M}_0$, which are introduced by the gauge condition

$$\overline{g}^{\alpha\beta}{}_{,\beta}(x) = 0 \qquad \Longrightarrow \qquad \Box_g\, x^\alpha = 0 \tag{10}$$

where the relation on the r.h.s. follows from the relation on the l.h.s. where $\Box_g$ is the covariant d'Alembert operator which in harmonic coordinates reads $\Box_g = g^{\rho\sigma}\,\nabla_\rho\,\nabla_\sigma$ and ∇_ρ denotes covariant derivative with respect to x^ρ. Harmonic coordinates are small deformations of the Minkowski coordinates, therefore it is useful to decompose the pulled-back metric density into the flat Minkowskian metric plus a small perturbation,

$$\overline{g}^{\alpha\beta}(x) = \eta^{\alpha\beta} - \overline{h}^{\alpha\beta}(x) \tag{11}$$

so that the gothic metric perturbation $\overline{h}^{\alpha\beta}$ propagates as dynamical field on the flat background space-time $\mathcal{M}_0$ (Section 7.1 in *Carroll* (2013) and Section 6.2 in *Poisson, Will* (2014)). By inserting (10) and (11) into (9) one obtains the Landau-Lifschitz field equations (also known as reduced field equations of gravity) in the following form (Eq. (5.2b) in *Thorne* (1980))

$$\Box\,\overline{h}^{\alpha\beta}(x) = -\frac{16\,\pi\,G}{c^4}\left(\tau^{\alpha\beta}(x) + t^{\alpha\beta}(x)\right) \tag{12}$$

where $\Box = \eta^{\rho\sigma}\,\partial_\rho\partial_\sigma$ is the flat d'Alembert operator in terms of harmonic coordinates in the flat background space-time $\mathcal{M}_0$. The terms on the r.h.s. in (12) are given by

$$\tau^{\alpha\beta} = (-g)\;T^{\alpha\beta} \quad \text{and} \quad t^{\alpha\beta} = (-g)\;t^{\alpha\beta}_{\rm LL} + \frac{c^4}{16\,\pi\,G}\left(\overline{h}^{\alpha\rho}{}_{,\sigma}\,\overline{h}^{\beta\sigma}{}_{,\rho} - \overline{h}^{\alpha\beta}{}_{,\rho\sigma}\,\overline{h}^{\rho\sigma}\right). \tag{13}$$

The ten coupled non-linear partial differential equations (12) are exact field equations of gravity in the Landau-Lifschitz formulation in harmonic coordinates. Because of the gauge condition $\overline{h}^{\alpha\beta}{}_{,\beta} = 0$, which follows from (10) and (11), only six equations are independent of each other.

The harmonic gauge (10) does not uniquely select one harmonic coordinate system but a class of infinitely many harmonic systems, because it allows for a residual gauge transformation between two arbitrary harmonic reference systems $\{x\}$ and $\{x'\}$,

$$x'^{\,\alpha} = x^\alpha + \varphi^\alpha(x) \tag{14}$$

if the gauge vector φ^α satisfies the homogeneous Laplace-Beltrami equation $\Box_g\,\varphi^\alpha = 0$; Eq. (14) has been elucidated by Fig. 1 in *Zschocke* (2019). The field equations (12) are invariant under the residual gauge transformation (14), which permits extensive simplifications of the form of the metric density. Moreover, the calculations of the MPM formalism are considerably simplified by assuming that $\{x\}$ are just Minkowskian (i.e. straight harmonic) coordinates, while $\{x'\}$ are considered as curvilinear harmonic coordinates.

4. THE MULTIPOLAR POST-MINKOWSKIAN FORMALISM

The MPM approach has originally been introduced in *Thorne* (1980), while considerable extensions and important advancements have later been worked out in *Blanchet,Damour* (1986) and in a series of subsequent investigations. The MPM formalism is based on the post-Minkowski expansion of the field equations (12),

$$\overline{h}^{\alpha\beta} = \sum_{n=1}^{\infty} G^n \, \overline{h}^{\alpha\beta}_{(\mathrm{nPM})} \quad \text{and} \quad \tau^{\alpha\beta} = T^{\alpha\beta} + \sum_{n=1}^{\infty} G^n \, \tau^{\alpha\beta}_{(\mathrm{nPM})} \quad \text{and} \quad t^{\alpha\beta} = \sum_{n=1}^{\infty} G^n \, t^{\alpha\beta}_{(\mathrm{nPM})} \,. \tag{15}$$

Inserting (15) into (12) yields a hierarchy of field equations,

$$\Box \, \overline{h}^{\alpha\beta}_{(\mathrm{1PM})} (x) = - \frac{16\,\pi}{c^4} \, T^{\alpha\beta} (x) \,, \tag{16}$$

$$\Box \, \overline{h}^{\alpha\beta}_{(\mathrm{2PM})} (x) = - \frac{16\,\pi}{c^4} \left(\tau^{\alpha\beta}_{(\mathrm{1PM})} (x) + t^{\alpha\beta}_{(\mathrm{1PM})} (x) \right) , \tag{17}$$

$$\vdots$$

$$\Box \, \overline{h}^{\alpha\beta}_{(\mathrm{nPM})} (x) = - \frac{16\,\pi}{c^4} \left(\tau^{\alpha\beta}_{(\mathrm{(n-1)PM})} (x) + t^{\alpha\beta}_{(\mathrm{(n-1)PM})} (x) \right) . \tag{18}$$

Each of the field equations (16) $\cdots$ (18) represents an equation in flat space-time. The MPM formalism is an approach for solving that hierarchy of field equations iteratively, starting with the first iteration (16), where $T^{\alpha\beta}$ is the energy-momentum tensor of matter in the approximation of special relativity. The general solution of the gothic metric in linear-order $\overline{h}^{\alpha\beta}_{(\mathrm{1PM})}$ (*Thorne* (1980), *Blanchet,Damour* (1986), *Damour,Iyer* (1991)) is inserted into the second iteration (17) which yields the gothic metric in post-linear order, $\overline{h}^{\alpha\beta}_{(\mathrm{2PM})}$, and so on. Using this iterative approach, it has been demonstrated in *Blanchet,Damour* (1986) that the general solution of these field equations depends on six source-multipoles, $I_L, J_L, W_L, X_L, Y_L, Z_L$, which are integrals over the energy-momentum tensor of the compact source of matter (cf. Eqs. (5.15) - (5.20) in *Blanchet* (1998)). Furthermore, using the residual gauge freedom (14), it has been demonstrated in *Blanchet,Damour* (1986) that the general solution of (16) $\cdots$ (18) can be written as follows,

$$\overline{g}^{\alpha\beta} \left[I_L, J_L, W_L, X_L, Y_L, Z_L \right] \;=\; \eta^{\alpha\beta} - \sum_{n=1}^{\infty} G^n \, \overline{h}^{\alpha\beta\,\mathrm{can}}_{(\mathrm{nPM})} \left[M_L, S_L \right] + \text{ gauge terms} \tag{19}$$

which is valid in the exterior of the body. The canonical piece, $\overline{h}^{\alpha\beta\,\mathrm{can}}_{(\mathrm{nPM})}$, depends on two multipoles: mass-type multipole M_L (accounts for shape, inner structure, and oscillations of the body) and current-type multipole S_L (accounts for rotational motions and inner currents of the body), which are related to the source-multipoles via non-linear equations (Eqs. (6.1a) and (6.1b) in *Blanchet* (1998)). All those terms in the metric density which depend on the gauge vector φ^α are called gauge terms and represent unphysical degrees of freedom because they have no impact on physical observables which are, by definition, coordinate-independent scalars (*Bergmann* (1961)).

The MPM formalism has been developed for understanding the generation of gravitational waves by an isolated source of matter, like binary black holes. Gravitational waves decouple from the source in the intermediate zone and they do finally propagate with the speed of light into the far wave-zone of the gravitational system. In the far wave-zone the gravitational fields have two degrees of freedom, where the transverse traceless (TT) gauge of the metric tensor becomes relevant because the TT terms in the metric tensor carry the physical information (*Blanchet,Kopeikin,Schäfer* (2001)). In the far wave-zone, the TT projection of the metric density equals the TT projection of the metric tensor (cf. Eq. (7.119) in *Carroll* (2013)),

$$\overline{h}^{\mathrm{TT}}_{\alpha\beta} = h^{\mathrm{TT}}_{\alpha\beta} \qquad \text{in the far} - \text{zone} \,. \tag{20}$$

That is why there is no need to determine the metric tensor in the far wave-zone of the system. The gothic metric perturbation in TT gauge in terms of radiative moments U_L and V_L, which are time-derivatives of source multipoles, is given by Eq. (64) in *Blanchet,Kopeikin,Schäfer* (2001).

5. THE METRIC TENSOR

For determining light trajectories in the near-zone of the solar system one needs the metric tensor of solar system bodies. While in principle one might use the TT gauge, one should, however, not expect much simplification, because such a nice relation like (20) does not exist,

$$\overline{h}_{\alpha\beta}^{\rm TT} \neq h_{\alpha\beta}^{\rm TT} \qquad \text{in the near} - \text{zone} \;. \tag{21}$$

Thus, relativistic astrometry necessarily requires the determination of the metric tensor in the near-zone of the gravitational system. The metric density and the metric tensor contain the same physical information about the gravitational system, because they are related to each other reversibly unique by Eqs. (2). Using these relations, it has been shown in *Zschocke* (2019) that the general form of the metric tensor in the exterior of a solar system body is given by

$$g_{\alpha\beta}\left[I_L, J_L, W_L, X_L, Y_L, Z_L\right] \;\;=\;\; \eta_{\alpha\beta} + \sum_{n=1}^{\infty} G^n \, h_{\alpha\beta\,{\rm can}}^{({\rm nPM})}\left[M_L, S_L\right] + \;\text{gauge terms} \tag{22}$$

where the canonical piece, $h_{\alpha\beta\,{\rm can}}^{({\rm nPM})}$, depends only on two multipoles M_L and S_L. The linear term and the post-linear term of the metric perturbation, $h_{\alpha\beta\,{\rm can}}^{({\rm 1PM})}$ and $h_{\alpha\beta\,{\rm can}}^{({\rm 2PM})}$, respectively, are explicitly given by Eqs. (109) - (111) and (115) - (117) in *Zschocke* (2019). The gauge terms depend on the gauge vector φ^{α} and have no impact on physical observables.

7. CONCLUSION

Future astrometry at the sub-micro-arcsecond and nano-arcsecond level of accuracy in astrometric measurements requires considerable progress in modeling the trajectory of light signals through the curved space-time of the solar system. Such a precise determination of light trajectories implies the knowledge of the metric tensor $g_{\alpha\beta}$ of solar system bodies in the post-linear approximation. The Multipolar Post-Minkowskian formalism represents a framework for determining the metric density $\overline{g}^{\alpha\beta}$ in the exterior of a massive body having arbitrary shape and inner structure, oscillations and rotational motions. The knowledge of the metric density allows to deduce the metric tensor $g_{\alpha\beta}$. Some aspects of that approach have been considered which are relevant for future investigations in the theory of light propagation and relativistic astrometry.

8. REFERENCES

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