



## Public Transport Operations and Supply

### Development of rotating duty rosters

**Dipl.-Ing. Steffen Dutsch**



## Structure of the faculty

Institute of Traffic Planning and Road Traffic

Institute of Traffic Telematics

Institute of Logistics and Aviation

Institute of Rail Vehicle Engineering

Institute of Transport & Economics

Institute of Automotive Engineering Dresden

Institute of Railway Systems and Public Transport

Founded in: 1992  
Professors: 26  
Students: ≈1600

## Structure of the institute

Chair of Transportation Systems Engineering

Chair of Planning and Design of Railway Infrastructure

Chair of Rail and Public Urban Transport

Chair of Railway Signalling and Transport Safety Technology

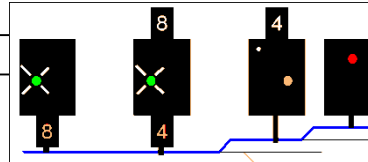
## Structure of the chair

Professional group of railway traffic

Professional group of railway operation

Professional group of public urban transport

Railway Operation Laboratory



## Public Transport Operations and Supply

### Development of rotating duty rosters

#### **1. Introduction**

- 1.1. Task and steps of duty-scheduling
- 1.2. Tasks and problems of duty-rostering
- 1.3. Types of duty rosters

#### **2. Development of rotating duty rosters**

- 2.1. Duty rosters with fixed rhythm
- 2.2. Duty rosters with weekly rhythm

#### **3. General examples**

- 3.1. Shift similarly duty roster
- 3.2. Duty roster with fast rotation

#### **4. Exercise**

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### Development of rotating duty rosters

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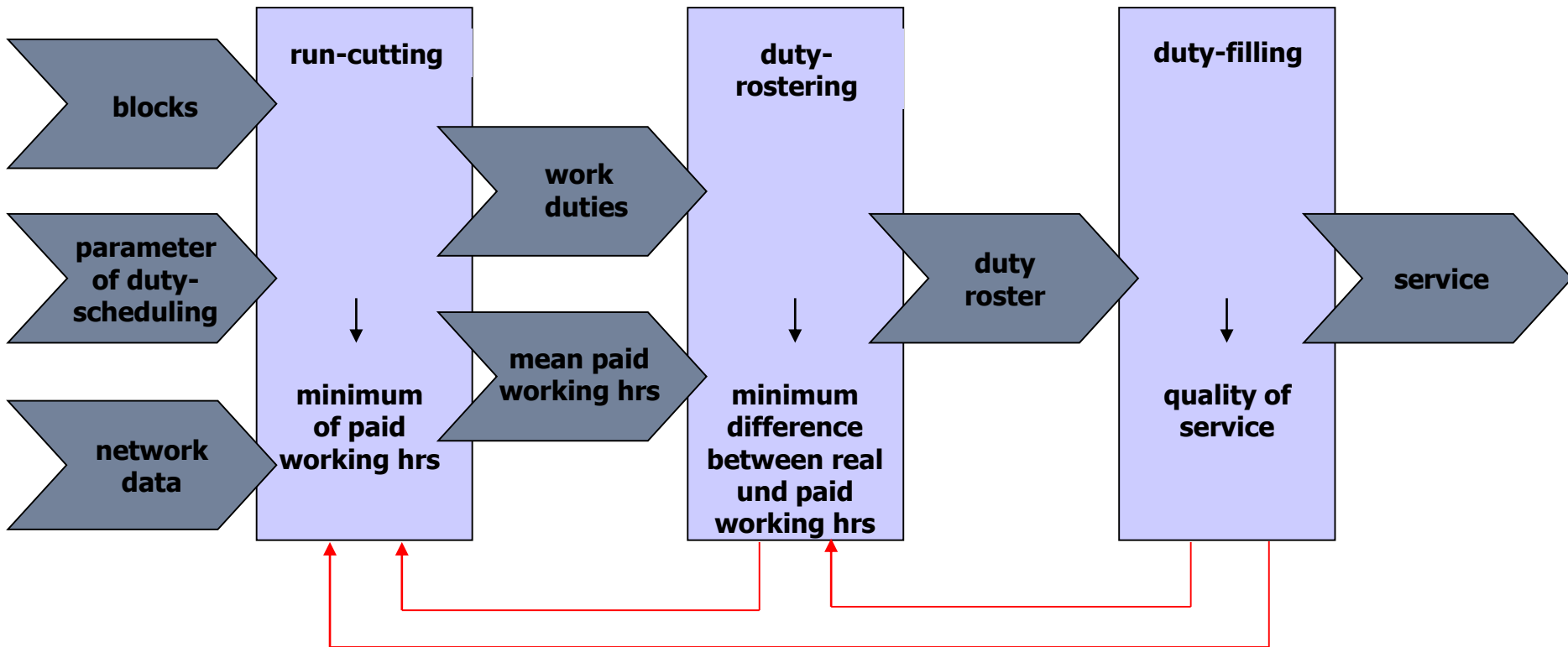
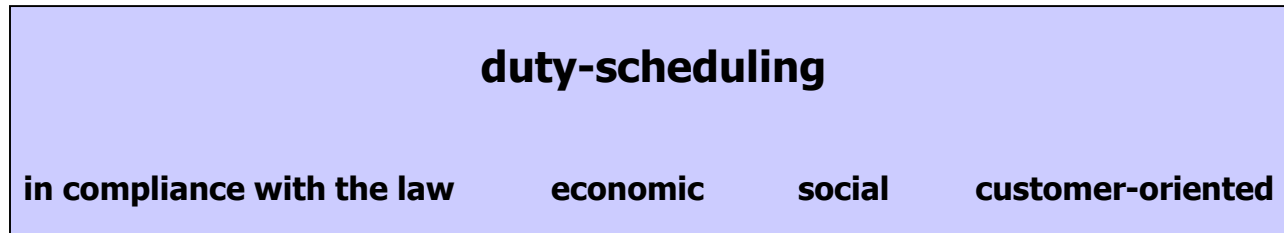
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- 2.1. Duty rosters with fixed rhythm
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#### **4. Exercise**



**Task:**

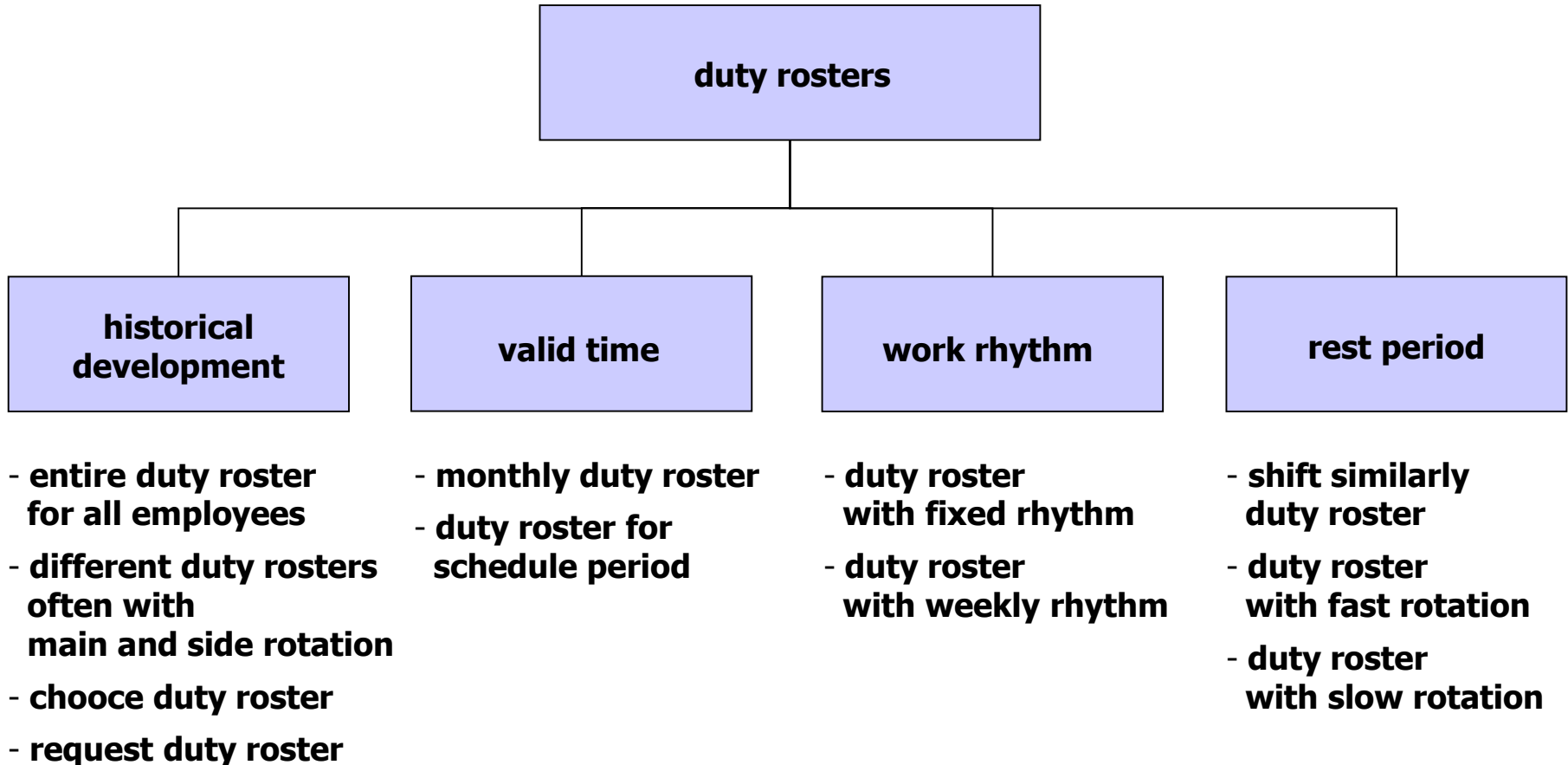
group all duties and days-off into a roster, that is

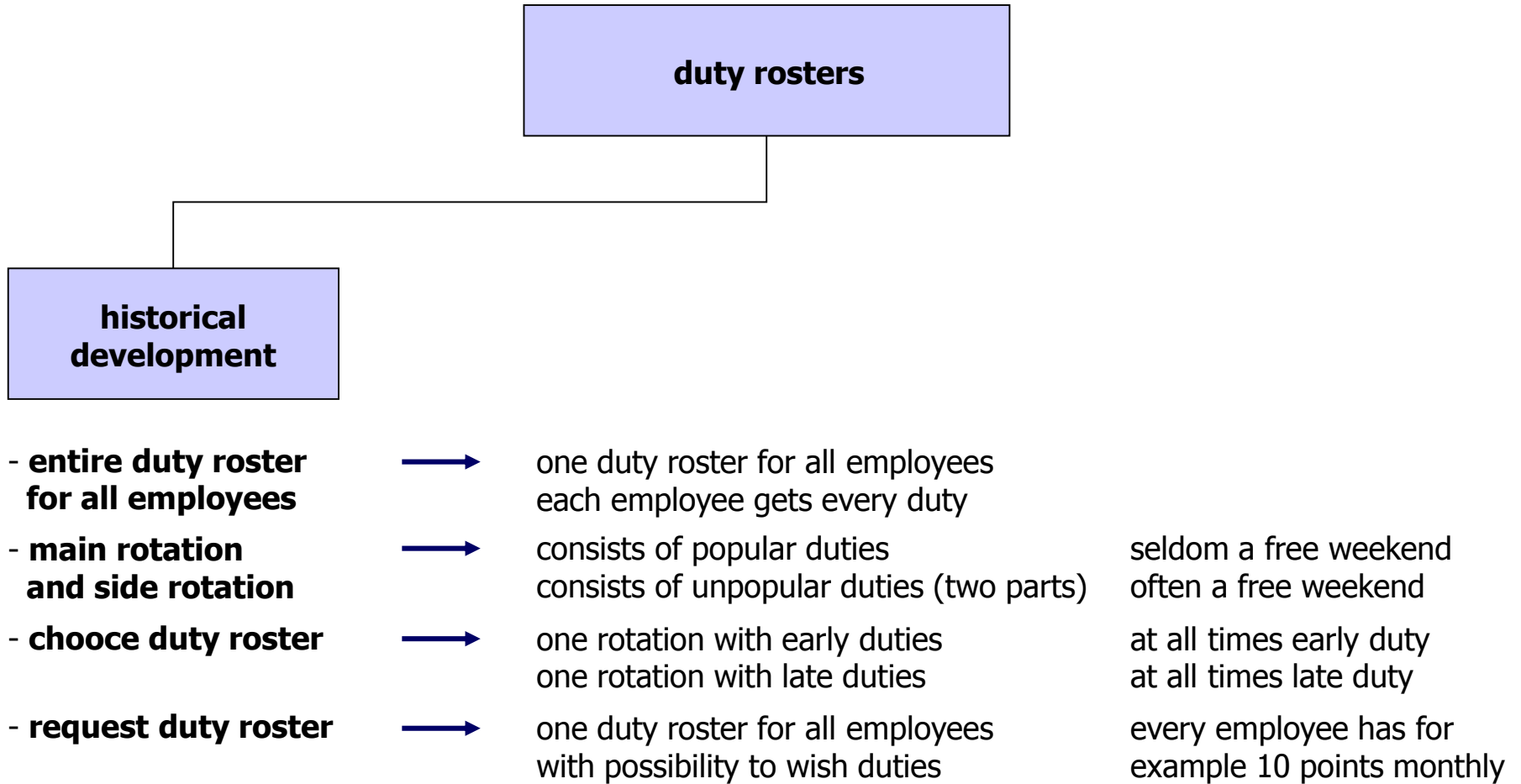
- with exactly one employee per duty each day
- in compliance with the law
- as social as possible

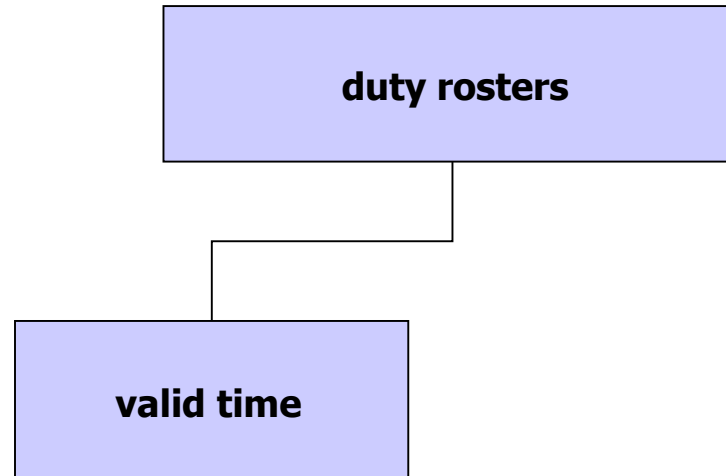
**Problems:**

- Different number of duties at different days (monday to friday, saturday and sunday)
- minimal rest period: 10 hours  
maximal number of duties without days-off: 6 duties  
minimal number of free sundays per year: 15 sundays
- Number of cycledays, sum of duties and days-off should be between 4 and 8 days but never 7 days









- **monthly  
duty  
roster**



regional transport, public urban transport in small networks

without any rhythm

possibility to take bank holidays, special features or wishes into account

- **duty roster  
for schedule  
period**

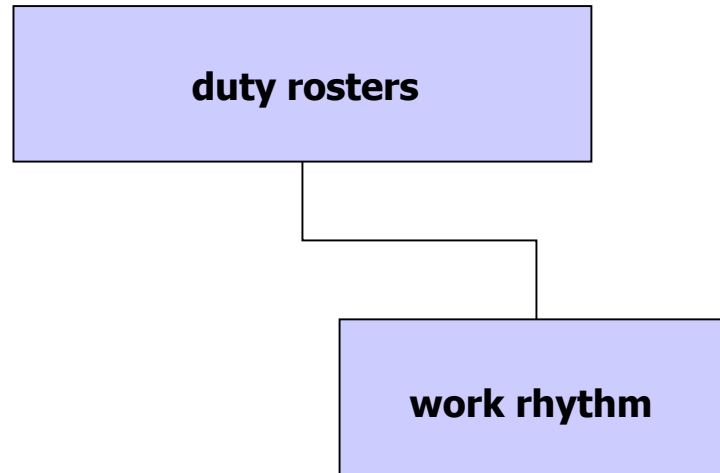


rail transport, public urban transport in large networks

with fixed or weekly rhythm

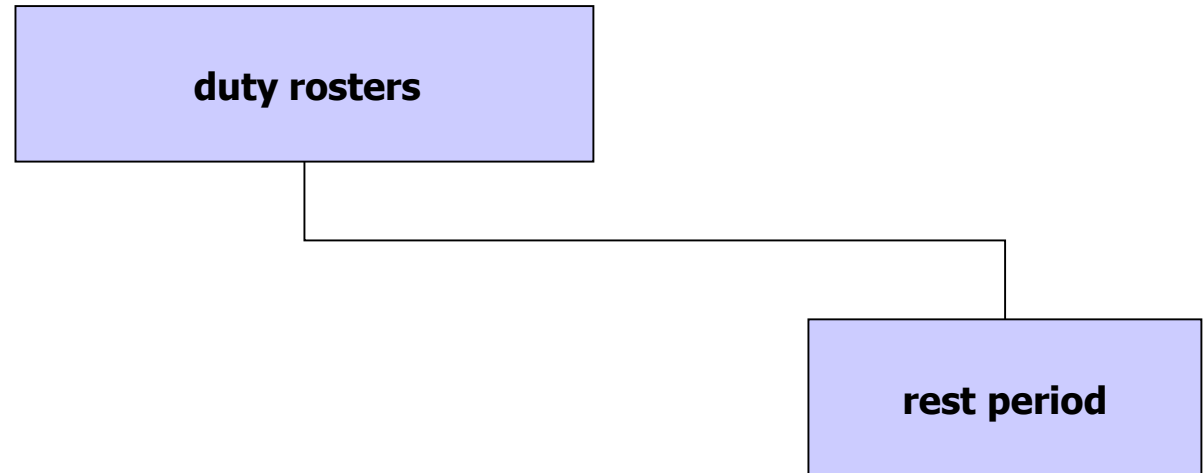
no possibility to take bank holidays, special features or wishes into account;

this possibility exists only in the step duty-filling



- **duty roster with fixed rhythm**
- **duty roster with weekly rhythm**

→ Point 2



second duty begins and ends about 24 hours after the first  
best possibility for families with children

second duty begins and ends less than 24 hours after the first  
popular with young people

second duty begins and ends more than 24 hours after the first  
best possibility for elderly employees <sup>1)</sup>

- ← - **shift similarly  
duty roster**
- ← - **duty roster  
with fast rotation**
- ← - **duty roster  
with slow rotation**

1) reference:  
 Tarifvertrag zur Bewältigung des  
 demografischen Wandels im Nahverkehr.-  
 Vereinigung der kommunalen  
 Arbeitgeberverbände (VKA), 2013

1) reference:  
 collective agreement to overcome  
 demographic change in public transport.-  
 Union of Employers' Associations, 2013  
 enacts January, 1 2014

## Transportation systems (M. Sc.)

### Development of rotating duty rosters

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- 4. Exercise**

## 4 working days and 2 days-off

weekday	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7			
cycleday	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	1	2	3	4	5	6	7	
staff no. 1	■	■					■	■	■				■	■					■	■							■	■			■	■						
staff no. 2		■	■						■	■				■	■					■	■							■	■			■	■					
staff no. 3			■	■						■	■				■	■					■	■							■	■			■	■				
staff no. 4				■	■						■	■				■	■					■	■							■	■			■	■			
staff no. 5					■	■						■	■				■	■					■	■							■	■			■	■		
staff no. 6	■					■	■						■	■					■	■							■	■			■	■						

## 4 working days and 1 day-off

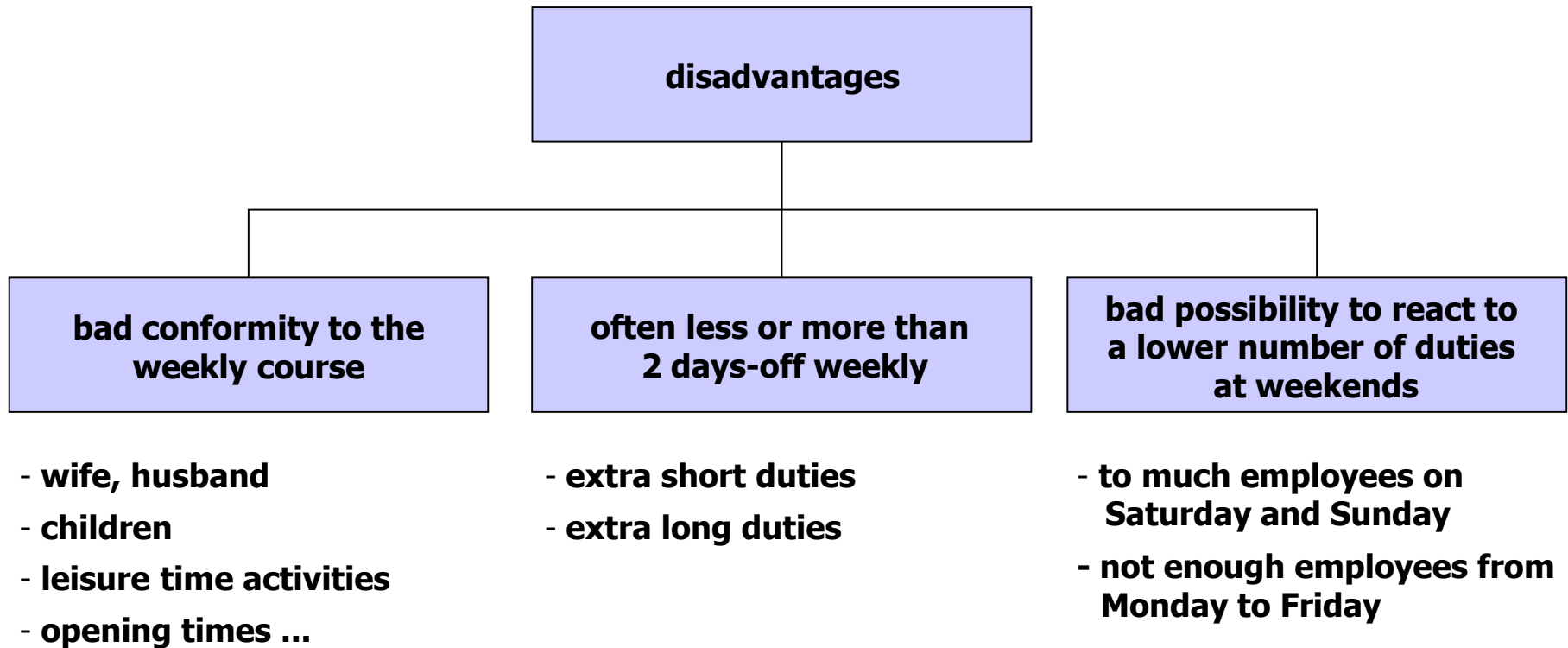
weekday	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su						
cycleday	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5				
staff no. 1	■					■						■				■				■							■			■				
staff no. 2		■					■					■					■			■							■			■				
staff no. 3			■						■				■					■			■							■			■			
staff no. 4				■						■				■						■								■			■			
staff no. 5					■						■							■			■							■			■			

weekday	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su						
cycleday	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5				
staff no. 1	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■				
staff no. 2	■					■						■				■				■							■			■				
staff no. 3		■					■					■					■			■							■			■				
staff no. 4			■						■				■					■			■							■			■			
staff no. 5				■						■				■						■								■			■			

Mo	Tu	We	Th	Fr	Sa	Su
				■		
■		■			■	
			■			
	■					■







**duty roster with weekly rhythm**



**consists of frame basic groups with two days-off weekly**

Number of week	su	mo	tu	we	th	fr	sa
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B 1 (1 week, 1 cycle basis group, 2 days-off)

1.	day-off						day-off
----	---------	--	--	--	--	--	---------

B 3 (3 weeks, 4 cycle basic groups, 6 days off)

1.	(day-off)				day-off	day-off	
2.				day-off	↓		
3.		day-off	day-off				(day-off)

(day-off)

extra day-off, either  
Saturday or Sunday  
day-off, with the possibility  
to put it into next week

day-off



B 4 (4 weeks, 5 cycle basic groups, 8 days-off)

1.	day-off					day-off	(day-off)
2.				day-off	day-off		
3.			day-off	↓			
4.	(day-off)	day-off					day-off

B 6 (5) (6 weeks, 5 cycle basic groups, 12 days-off)

1.	day-off	day-off					
2.		day-off	day-off				
3.			day-off	day-off			
4.				day-off	day-off		
5.					day-off	day-off	
6.						day-off	day-off

B 6 (7) (6 weeks, 7 cycle basic groups, 12 days off)

1.	<day-off>				<day-off>	day-off	<day-off>
2.				<day-off>	day-off	<day-off>	
3.			<day-off>	day-off	<day-off>		
4.		<day-off>	day-off	<day-off>			
5.	<day-off>	day-off	<day-off>				<day-off>
6.	day-off	<day-off>				<day-off>	day-off

<day-off>

extra day-off, take only  
5 from there

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number of week	shift	su	mo	tu	we	th	fr	sa
$B_e$								
1.	e	day-off	e	e	e	e	e	day-off
$B_l$								
1.	l	day-off	l	l	l	l	l	day-off
$B_{e/l}$								
1.	e	day-off	e	e	e	e	day-off	l
2.	l/e	l	l	l	day-off	day-off	e	e
3.	l	e	day-off	day-off	l	l	l	l
$B_{l/e}$								
1.	l	l	l	l	l	day-off	day-off	e
2.	l/e	e	e	e	day-off	l	l	l
3.	e	l	day-off	day-off	e	e	e	day-off

e early shift  
l late shift

it goes:

$$\begin{aligned}
 n_{B_e} &= n_{emf} - k \\
 n_{B_l} &= n_{lmf} - k \\
 n_{B_{e/l}} &= n_{lsa} - k \\
 n_{B_{l/e}} &= n_{lsu} - k \\
 k &= 1/5(n_{esa} + n_{lsa} + n_{esu} + n_{lsu})
 \end{aligned}$$

boundary conditions:

$$\begin{aligned}
 &n_{emf}; n_{lmf}; n_{esa}; n_{lsa}; n_{esu}; n_{lsu} \geq 0 \text{ (a whole number)} \\
 &n_{emf}; n_{lmf} \geq k \\
 &n_{esa}; n_{esu} = k \\
 &k \leq n_{lsa}; n_{lsu} \leq 2k \\
 &n_{esa} + n_{lsa} + n_{esu} + n_{lsu} \equiv 0 \pmod{5}
 \end{aligned}$$

- k [ - ] a whole constant
- $n_B$  [ - ] number of basic groups
- $n_{mf}$  [ - ] number of duties on every monday to friday
- $n_{sa}$  [ - ] number of duties on saturday
- $n_{su}$  [ - ] number of duties on sunday and public holiday

number of week	su	mo	tu	we	th	fr	sa
B 1							
1.	day-off						day-off
B 6 (5)							
1.	day-off	day-off					
2.		day-off	day-off				
3.			day-off	day-off			
4.				day-off	day-off		
5.					day-off	day-off	
6.						day-off	day-off

it goes:

$$n_{B1} = n_{mf} - 4 n_{B6(5)}$$

$$n_{B6(5)} = 1/5 n_{sa} = 1/5 n_{su}$$

boundary conditions:

$$n_{mf} ; n_{sa} ; n_{su} \geq 0 \text{ (a whole number)}$$

$$n_{mf} \geq 4/5 n_{sa}$$

$$n_{mf} \geq 4/5 n_{su}$$

$$n_{sa} ; n_{su} \equiv 0 \pmod{5}$$

- $n_B$  [ - ] number of basic groups
- $n_{mf}$  [ - ] number of duties on every monday to friday
- $n_{sa}$  [ - ] number of duties on saturday
- $n_{su}$  [ - ] number of duties on sunday and public holiday

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**monday – friday:**

duty no.	train	begin and end working time	working time
1	1/3.	3.56 – 12.06	8:10
2	1/6.	4.14 – 12.51	8:37
3	1/7.	4.34 – 13.06	8:32
4	1/8.	4.49 – 13.04	8:15
5	11/1. + 1/7.	4.56 – 7.59 + 12.56 – 16.49	6:56
6	11/2. + 11/3.	5.11 – 8.14 + 15.11 – 18.14	6:06
7	11/3. + 1/6.	5.26 – 7.44 + 12.41 – 16.51	6:28
8	1/3.	11.56 – 19.49	7:53
9	11/1.	14.41 – 18.29	3:48
10	1/8.	12.54 – 19.04	6:10
11	11/2. + 1/3.	14.56 – 18.44 + 19.39 – 0.11	9:15
12	1/7.	16.39 – 1.18	8:39
13	1/6.	16.41 – 0.48	8:07
14	1/8.	18.54 – 4.21	9:27

**saturday, sunday:**

duty no.	train	begin and end working time	working time
21	1/3.	3.56 – 12.06	8:10
22	1/6.	4.14 – 12.51	8:37
23	1/7. + 1/8.	4.34 – 8.49 + 8.49 – 13.04	8:30
24	1/7.	8.39 – 17.06	8:27
25	1/3.	11.56 – 17.49	5:53
26	1/6.	12.41 – 18.34	5:53
27	1/8.	12.54 – 19.04	6:10
28	1/7.	16.56 – 1.18	8:22
29	1/3.	17.39 – 0.11	6:32
30	1/6.	18.24 – 0.48	6:24
31	1/8.	18.54 – 4.21	9:27

**You have the duties number 1 to 14 and 21 to 31. Create a duty roster with weekly rhythm. The first duty after days-off has to start as late as possible, the last one before days-off has to end as early as possible. The rest period between two duties may never be smaller than 10 hours.**



**monday – friday:**

duty no.	train	begin and end working time	working time
1	1/3.	3.56 – 12.06	8:10
1.1	on-call duty	3.56 – 12.06	8:10
1.2	spare duty	(3.56 – 13.06)	7:37
2	1/6.	4.14 – 12.51	8:37
3	1/7.	4.34 – 13.06	8:32
4	1/8.	4.49 – 13.04	8:15
5	11/1. + 1/7.	4.56 – 7.59 + 12.56 – 16.49	6:56
6	11/2. + 11/3.	5.11 – 8.14 + 15.11 – 18.14	6:06
7	11/3. + 1/6.	5.26 – 7.44 + 12.41 – 16.51	6:28
8	1/3.	11.56 – 19.49	7:53
8.1	on-call duty	11.56 – 19.15	7:19
8.2	spare duty	(11.56 – 4.21)	7:37
9	11/1.	14.41 – 18.29	3:48
10	1/8.	12.54 – 19.04	6:10
11	11/2. + 1/3.	14.56 – 18.44 + 19.39 – 0.11	9:15
12	1/7.	16.39 – 1.18	8:39
13	1/6.	16.41 – 0.48	8:07
14	1/8.	18.54 – 4.21	9:27

**saturday, sunday:**

duty no.	train	begin and end working time	working time
21	1/3.	3.56 – 12.06	8:10
21.1	on-call duty	3.56 – 13.04	9:08
21.2	spare duty	(3.56 – 17.06)	7:37
22	1/6.	4.14 – 12.51	8:37
23	1/7. + 1/8.	4.34 – 8.49 + 8.49 – 13.04	8:30
24	1/7.	8.39 – 17.06	8:27
25	1/3.	11.56 – 17.49	5:53
25.1	on-call duty	11.56 – 19.24	7:28
25.2	spare duty	(11.56 – 4.21)	7:37
26	1/6.	12.41 – 18.34	5:53
27	1/8.	12.54 – 19.04	6:10
28	1/7.	16.56 – 1.18	8:22
29	1/3.	17.39 – 0.11	6:32
30	1/6.	18.24 – 0.48	6:24
31	1/8.	18.54 – 4.21	9:27

number of week	su	mo	tu	we	th	fr	sa
1.	day-off						day-off
2.	day-off	day-off					
3.		day-off	day-off				
4.			day-off	day-off			
5.				day-off	day-off		
6.					day-off	day-off	
7.						day-off	day-off
8.	day-off						day-off
9.	day-off						day-off
10.	day-off	day-off					
11.		day-off	day-off				
12.			day-off	day-off			
13.				day-off	day-off		
14.					day-off	day-off	
15.						day-off	day-off
16.	day-off						day-off
17.	day-off						day-off
18.	day-off	day-off					
19.		day-off	day-off				
20.			day-off	day-off			
21.				day-off	day-off		
22.					day-off	day-off	
23.						day-off	day-off
24.	day-off						day-off

number of week	su	mo	tu	we	th	fr	sa
1.	day-off						day-off
2.	day-off	day-off					
3.		day-off	day-off				
4.			day-off	day-off			
5.				day-off	day-off		
6.					day-off	day-off	
7.						day-off	day-off
8.	day-off						day-off
9.	day-off						day-off
10.	day-off	day-off					
11.		day-off	day-off				
12.			day-off	day-off			
13.				day-off	day-off		
14.					day-off	day-off	
15.						day-off	day-off
16.	day-off						day-off
17.	day-off						day-off
18.	day-off	day-off					
19.		day-off	day-off				
20.			day-off	day-off			
21.				day-off	day-off		
22.					day-off	day-off	
23.						day-off	day-off
24.	day-off						day-off

number of week	su	mo	tu	we	th	fr	sa
1.	day-off						day-off
2.	day-off	day-off					
3.	day-off	day-off	day-off				
4.			day-off	day-off			
5.				day-off	day-off		
6.					day-off	day-off	
7.						day-off	day-off
8.							day-off
9.	day-off						day-off
10.	day-off	day-off					
11.	day-off	day-off	day-off				
12.			day-off	day-off			
13.				day-off	day-off		
14.					day-off	day-off	
15.						day-off	day-off
16.							day-off
17.	day-off						day-off
18.	day-off	day-off					
19.	day-off	day-off	day-off				
20.			day-off	day-off			
21.				day-off	day-off		
22.					day-off	day-off	
23.						day-off	day-off
24.							day-off

**3 groups with ever 6 duties Monday to Friday and ever 5 duties Saturday and Sunday:**

I			II			III			IV			V			VI		
18.54	<b>14</b>	4.21	14.56	<b>11</b>	0.11	11.56	<b>8.1</b>	19.15	5.26	<b>7</b>	16.51	4.34	<b>3</b>	13.06	4.14	<b>2</b>	12.51
18.54	<b>31</b>	4.21	16.56	<b>28</b>	1.18	12.54	<b>27</b>	19.04	8.39	<b>24</b>	17.06	3.56	<b>21.1</b>	13.04			
16.41	<b>13</b>	0.48	(11.56)	<b>8.2</b>	(4.21)	14.41	<b>9</b>	18.29	4.56	<b>5</b>	16.49	(3.56)	<b>1.2</b>	(13.06)	3.56	<b>1</b>	12.06
18.24	<b>30</b>	0.48	(11.56)	<b>25.2</b>	(4.21)	12.41	<b>26</b>	18.34	4.34	<b>23</b>	13.04	3.56	<b>21</b>	12.06			
16.39	<b>12</b>	1.18	11.56	<b>8</b>	19.49	12.54	<b>10</b>	19.04	5.11	<b>6</b>	18.14	4.49	<b>4</b>	13.04	3.56	<b>1.1</b>	12.06
17.39	<b>29</b>	0.11	11.56	<b>25.1</b>	19.24	11.56	<b>25</b>	17.49	(3.56)	<b>21.2</b>	(17.06)	4.14	<b>22</b>	12.51			

number of week	su	mo	tu	we	th	fr	sa
1.	day-off	14	11	8.1	7	3	day-off
2.	day-off	day-off	13	8.2	9	5	21
3.	day-off	day-off	day-off	12	8	10	21.2
4.	22	1.1	day-off	day-off	14	11	27
5.	24	3	2	day-off	day-off	13	25.2
6.	26	5	1.2	1	day-off	day-off	29
7.	25.1	10	6	4	1.1	day-off	day-off
8.	31	11	8.1	7	3	2	day-off
9.	day-off	13	8.2	9	5	1.2	day-off
10.	day-off	day-off	12	8	10	6	22
11.	day-off	day-off	day-off	14	11	8.1	24
12.	21.1	2	day-off	day-off	13	8.2	26
13.	23	1.2	1	day-off	day-off	12	25.1
14.	25	6	4	1.1	day-off	day-off	31
15.	28	8.1	7	3	2	day-off	day-off
16.	30	8.2	9	5	1.2	1	day-off
17.	day-off	12	8	10	6	4	day-off
18.	day-off	day-off	14	11	8.1	7	21.1
19.	day-off	day-off	day-off	13	8.2	9	23
20.	21	1	day-off	day-off	12	8	25
21.	21.2	4	1.1	day-off	day-off	14	28
22.	27	7	3	2	day-off	day-off	30
23.	25.2	9	5	1.2	1	day-off	day-off
24.	29	8	10	6	4	1.1	day-off

Dipl.-Ing. Steffen Dutsch  
Technische Universität Dresden  
Fakultät Verkehrswissenschaften „Friedrich List“  
Institut für Bahnsysteme und Öffentlichen Verkehr  
Professur für Bahnverkehr, öffentlicher Stadt- und Regionalverkehr

Telefon: (0351) 463 36528  
Telefax: (0351) 463 36529  
E-Mail : [Steffen.Dutsch@tu-dresden.de](mailto:Steffen.Dutsch@tu-dresden.de)  
Internet: [www.tu-dresden.de/vkivb/p-srv/p\\_srv.htm](http://www.tu-dresden.de/vkivb/p-srv/p_srv.htm)