TECHNISCHE UNIVERSITÄT DRESDEN

Faculty of Traffic Sciences Chair of Traffic Flow Science

On Extracting Minimally Infeasible Periodic Event Networks

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Outline

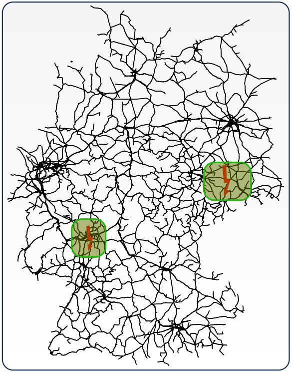
Motivation

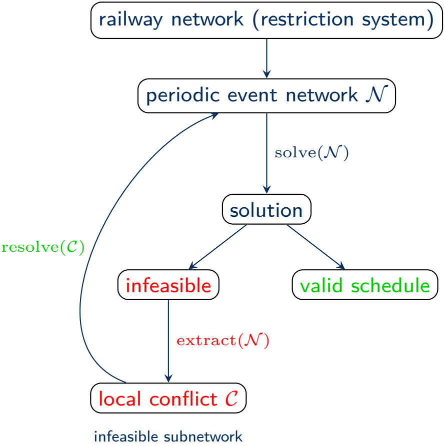
- many industrial problems encoded to SAT
- e. g., periodic event scheduling problem (PESP)
- used for timetabling in railway networks
- SAT solver's answer: formula $\mathcal{F}$ either satisfiable or unsatisfiable
- $\mathcal{F}$ resp. railway network often over-constrained in real-world instances
  $\rightarrow$ minimally unsatisfiable subformula extraction (MUS)

Motivation

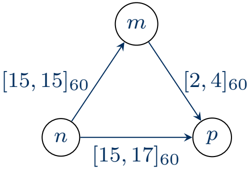
Public Railway Transport Networks

goal: compute valid timetable



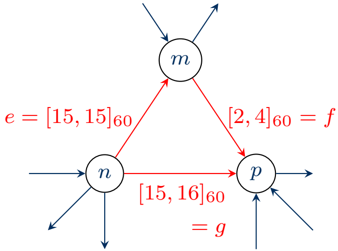


Periodic Event Scheduling Problem (PESP)



- period $T = 60$
- (periodic) events (nodes) n, m, $p \in \mathcal{V}$
  - schedule $\Pi : \mathcal{V} \to [0, T-1] \subseteq \mathbb{N}$
  - potential $\Pi(n)$, $\Pi(m)$, $\Pi(p)$
- constraints resp. activities (edges) $\mathcal{E}$
- e. g., (n, m) with $[15, 15]_{60}$
  - constraint holds iff $\Pi(m) - \Pi(n) \in [15, 15]_{60}$
  - e. g., $\Pi(n) = 2, \Pi(m) = 17$ or $\Pi(n) = 50, \Pi(m) = 5$
- schedule Π is valid iff all constraints hold
- tuple $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$ is called periodic event network

Local Conflict



- infeasible periodic event network $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$

Definition (Local Conflict)

Periodic event network $\mathcal{C} = (\mathcal{V}, \mathcal{Z}, T)$ with $\mathcal{Z} \subseteq \mathcal{E}$ is called **local conflict** iff

(i) $\mathcal{C}$ is infeasible

(ii) $\mathcal{C}$ feasible for any removed constraint in $\mathcal{Z}$

- local conflict extraction: find $\mathcal{C} = (\mathcal{V}, \{e, f, g\}, T)$

Propositional Logic (SAT)

$$\mathcal{F} = \neg p \wedge (p \vee q)$$

- literal: $L = p$ or $L = \neg p$ (variable or its negation), $p \in \mathcal{R}$
- clause c: disjunction of literals
- formula $\mathcal{F}$ in conjunctive normal form (CNF): conjunction of clauses
- interpretation $J : \mathcal{R} \to \{F, T\}$
- $\mathcal{F}$ is satisfiable ($\mathcal{F}^J = T$) iff for all clauses at least one literal $L^J = T$
- e. g., $p^J = F$, $q^J = T$ implies $\mathcal{F}^J = T$
- $\mathcal{F}$ unsatisfiable if no such J exists

Minimally Unsatisfiable Subformula

$$\mathcal{F} = \neg p \wedge (p \vee q) \wedge \neg q \wedge \ldots$$

- split unsatisfiable $\mathcal{F}$ into disjoint sets of clauses $\{R\} \cup \mathcal{G}$ (domain-specific)
  $\rightarrow \mathcal{F} = R \wedge (\bigwedge_{K \in \mathcal{G}} K)$
- e. g., $\mathcal{G} = \{K_1, K_2, \ldots\}$, $R = \emptyset$

Definition

$\mathcal{M} = R \wedge (\bigwedge_{K \in \mathcal{G}'} K)$ with $\mathcal{G}' \subseteq \mathcal{G}$ is called **high-level minimally unsatisfiable subformula** (HLMUS) iff

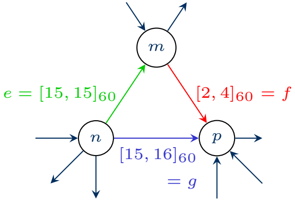
(i) $\mathcal{M}$ is unsatisfiable

(ii) for any $K \in \mathcal{G}'$: $\mathcal{M} \setminus K$ is satisfiable

- HLMUS extraction: find $\mathcal{G}' = \{K_1, K_2\}$

Local Conflict Extraction

Reduce Local Conflict to HLMUS



- let $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$ infeasible periodic event network with $\mathcal{E} = \{e, f, g, \ldots\}$
- encode $\mathcal{N}$ as propositional formula $\mathcal{F}$ in CNF:

$$\begin{aligned}\mathcal{F} = \text{encode}(\mathcal{N}) = {} & \text{encodeNode}(n) \wedge \text{encodeNode}(m) \wedge \text{encodeNode}(p) \wedge \ldots \\ & \wedge \text{encodeEdge}(e) \wedge \text{encodeEdge}(f) \wedge \text{encodeEdge}(g) \wedge \ldots\end{aligned}$$

- split $\mathcal{F}$ in sets of clauses $\{R\} \cup \mathcal{G}$:

$$R = \text{encodeNode}(n) \wedge \text{encodeNode}(m) \wedge \text{encodeNode}(p) \wedge \ldots$$
$$\mathcal{G} = \{\text{encodeEdge}(e), \text{encodeEdge}(f), \text{encodeEdge}(g), \ldots\}$$

Results

- goal: extract local conflicts effectively/efficiently from periodic event network $\mathcal{N}$
- 4 solvers under test
  1. polynomial-based algorithm by Nachtigall, Opitz et al.
  2. `SAText`: comparison solver (SAT-based)
  3. `MoUsSaKa`: by Kottler (HLMUS extractor, destructive algorithm)
  4. `Haifa-MUC`: by Ryvchin et al. (HLMUS extractor, resolution-based)

Results

- periodic event networks $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$: industrial scenarios
- generated by software system TAKT (TU Dresden)
- railway networks: south-east, south-west Germany and intercity network
- data from DB Netz AG
- runtime in seconds (2 days timeout)

$\mathcal{N}$	$\lvert\mathcal{E}\rvert$	polyn.	SAText	MoUsSaKa	Haifa-MUC
1	128	1	2	1	1
2	211	1	5	1	1
3	327	1	9	1	1
4	533	2	12	1	1
5	689	2	7	1	1
6	702	3	82	10	4
7	788	-	102 203	383	out of memory
8	5 571	-	7 269	72	84
9	6 305	-	timeout	timeout	out of memory
10	9 821	-	36	2	5
11	11 082	-	48	2	4
12	14 589	-	23	3	5

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Conclusion

- automatic extraction of local conflicts has decisive importance
- can be encoded into a SAT-based domain (HLMUS)
- instances can be solved efficiently by state-of-the-art solvers
- outlook:
  - better encodings for difficult/complex local conflicts
  - use solvers in parallel

Sources

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Modulo Interval resp. Periodic Extension

- interval $[a, b] := \{x \in \mathbb{Z} \mid a \leq x \leq b\} \subset \mathbb{Z}$, $a, b \in \mathbb{Z}$
- e. g., $[2, 4] = \{2, 3, 4\}$
- interval modulo $T \in \mathbb{N}$:

$$[a, b]_T := \bigcup_{z \in \mathbb{Z}} [a + z \cdot T,\ b + z \cdot T] \subseteq \mathbb{Z}$$

- e. g., $[2, 4]_{10} = \ldots \cup [-8, -6] \cup [2, 4] \cup [12, 14] \cup [22, 24] \cup \ldots$

Complexity Classification

- which complexity class does Local Conflicts (LC) belong to?
- complexity class $coNP = \{\bar{L} \mid L \in NP\}$
- e. g., $\mathrm{UNSAT} = \{\mathcal{F} \mid \mathcal{F}$ unsatisfiable$\}$ is $coNP$-complete
- complexity class $DP = \{L_1 \cap L_2 \mid L_1 \in NP, L_2 \in coNP\}$

Theorem

LC is DP-complete

Complexity Classification

Lemma

$\text{PESP} = \{\mathcal{N} \mid \mathcal{N} \text{ is feasible}\}$ is NP-complete

Lemma

$\overline{\text{PESP}} = \{\mathcal{N} \mid \mathcal{N} \text{ is infeasible}\}$ is $coNP$-complete

Proof. (LC is DP-complete)

to show:

1. LC $\in DP$
2. LC is DP-hard

1.:

- find L_1, L_2, such that $LC = L_1 \cap L_2$ and $L_1 \in NP, L_2 \in coNP$.
- let $\mathcal{N} = (\mathcal{V}, \mathcal{Z}, T)$ be a PEN, $\mathcal{Z} = S \cup \bigcup_{E \in \mathcal{D}} E$
- $L_1 = \{\mathcal{N} \mid \mathcal{N} \text{ is infeasible}\} \in coNP$
- $L_2 = \{\mathcal{N} \mid \forall E \in \mathcal{D} : (\mathcal{V}, \mathcal{Z} \setminus E, T) \text{ is feasible}\} \in NP$, because $|\mathcal{D}| < \infty$

MUS zu LC

Definition (MUS)

$\mathcal{F}$ is **minimally unsatisfiable subformula** iff $\mathcal{F}$ is unsatisfiable and for any clause $c \in \mathcal{F}$: $\mathcal{F} \setminus \{c\}$ is unsatisfiable.

Lemma

MUS is DP-hard

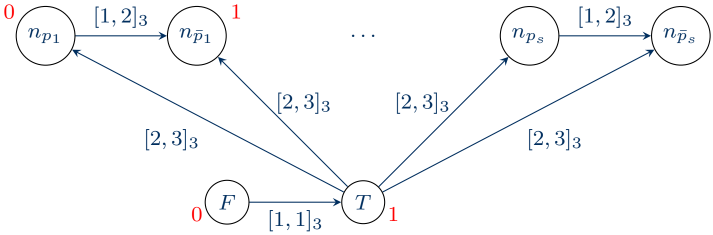
- encode propositional variables as events
- encode disjunction $(p \vee q)$ as periodic event network
- connect them, in order to encode $c \in \mathcal{F}$

Complexity Classification

Encode Propositional Variables

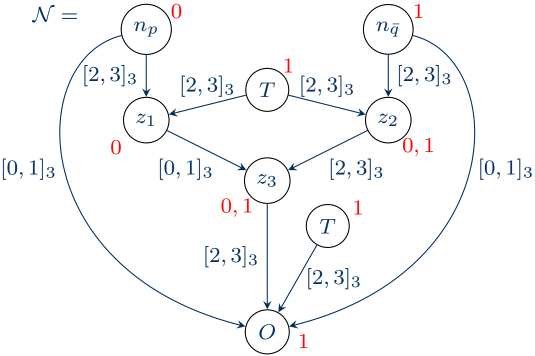
- let p_i $(i \in \{1, \ldots, s\})$ be propositional variables
- periodic event network $\mathcal{N} = (\mathcal{V}, \mathcal{E}, 3)$
- WLOG $\Pi(F) = 0$, $\Pi(T) = 1$
- encode literals p and $\neg p$
- $\Pi(n_p), \Pi(n_{\bar{p}}) \in \{0, 1\}$ semantically means $p^J, (\neg p)^J \in \{T, F\}$

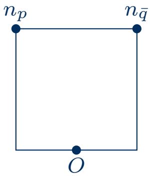
$\mathcal{N} =$



Disjunction

- encode disjunction, e. g., $(p \lor \neg q)$
- encode literals p, $\neg q$ as before
- $(p \lor \neg q)^J$ equivalent to $\Pi(O)$

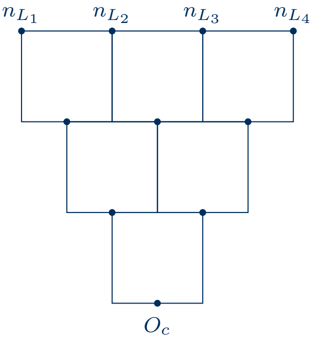




Complexity Classification

Clause

- encode clause as layered encoded disjunctions
- e. g., $(p \vee q \vee r) \equiv (p \vee q) \vee (q \vee r)$
- e. g., network $\mathcal{N}$ for each $c = (L_1 \vee \ldots \vee L_4)$
- assign n_{L_i} $(i \in \{1, \ldots, 4\})$ corresponding to J
- then, $\Pi(O_c)$ is equivalent to c^J



Complexity Classification

MUS to LC

Lemma

MUS is DP-hard

Lemma

MUS $\leq_p$ LC

Lemma

LC $\in DP$

Theorem

LC is DP-complete

Instance Names

- periodic event networks $\mathcal{N} = (\mathcal{V}, \mathcal{E}, T)$: industrial scenarios
- generated by software system TAKT (TU Dresden)
- railway networks: south-east, south-west Germany and intercity network
- data from DB Netz AG
- runtime in seconds (2 days timeout)

$\mathcal{N}$	instance
1	p_1
2	p_2
3	p_3
4	p_4
5	p_5
6	p_6
7	k
8	fern
9	b
10	dp_sta_1
11	dp_sta_2
12	dp_sta_3

Order Encoding (Finite Ordered Domains)

$$F = p_{x,6} \wedge \neg p_{x,2} \wedge (\neg p_{x,2} \vee p_{x,3}) \\ \wedge (\neg p_{x,3} \vee p_{x,4}) \wedge (\neg p_{x,4} \vee p_{x,5}) \wedge (\neg p_{x,5} \vee p_{x,6})$$

- encoding of $x \in I$ with $I = [l, u] \subset \mathbb{Z}$
- e. g., $x \in [3, 6]$
- propositional variable $p_{x,i}$ ($i \in [l, u]$) with meaning: $x \leq i$
- $\Rightarrow$ two cases:
  1. $p_{x,i} = T$, then $x \leq i$
  2. $p_{x,i} = F$, then $x \not\leq i$, hence $x \geq i + 1$
- since $x \in [3, 6]$, it must hold:
  - $x \leq 6$, implies $p_{x,6}$ must be T
  - $x \geq 3 \Leftrightarrow x \not\leq 2$, hence $\neg p_{x,2}$ must be T
- $\forall i \in \{3, \ldots, 6\} : (x \leq i - 1) \Rightarrow (x \leq i)$
- that is $p_{x,i-1} \rightarrow p_{x,i}$
- which is semantically equivalent to $(\neg p_{x,i-1} \vee p_{x,i})$ $(\forall i \in \{3, \ldots, 6\})$

Encoding PESP as SAT

Order Encoding (Extract Value of x)

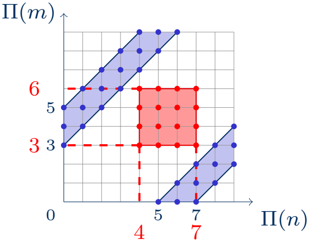
- SAT solver solves SAT instance and returns assignment for $p_{x,i}$ ($\forall i \in \{2, \ldots, 6\}$), if satisfiable
- e. g.,

$p_{x,2}$	$p_{x,3}$	$p_{x,4}$	$p_{x,5}$	$p_{x,6}$
F	F	F	T	T

- hence, $x \leq 6$ and $x \leq 5$
- but $x \not\leq 4$ and $x \not\leq 3$, ...
- $\Rightarrow x \geq 5$
- $x \leq 5$ and $x \geq 5$ implies $x = 5$
- for each variable assignment the exact value for x can be extracted

Encoding PESP as SAT

Constraint



- let be $T = 10$, $e = (n, m)$ with constraint $[3, 5]_{10}$
- exclude not feasible regions wrt. $[3, 5]_{10}$
- e. g., no pair $(\Pi(n), \Pi(m)) : \Pi(n) \leq 7, \Pi(n) \geq 4, \Pi(m) \leq 6, \Pi(m) \geq 3$
- iff $\neg((\Pi(n) \leq 7) \wedge (\Pi(n) \geq 4) \wedge (\Pi(m) \leq 6) \wedge (\Pi(m) \geq 3))$
- iff $\neg((\Pi(n) \leq 7) \wedge \neg(\Pi(n) \leq 3) \wedge (\Pi(m) \leq 6) \wedge \neg(\Pi(m) \leq 2))$
- iff $\neg(p_{n,7} \wedge \neg p_{n,3} \wedge p_{m,6} \wedge \neg p_{m,2})$
- iff $(\neg p_{n,7} \vee p_{n,3} \vee \neg p_{m,6} \vee p_{m,2})$
- connect $[\neg p_{n,7}, p_{n,3}, \neg p_{m,6}, p_{m,2}]$ to formula conjunctively