BWR Stability and Bifurcation analysis on the basis of system codes and reduced order models

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Introduction

The dynamics of boiling water reactors (BWR) can be described by a system of nonlinear partial differential equations coupled. From the nonlinear dynamics, it is well known that such systems can have unusual and strange behavior which is reflected by the solution manifold of the corresponding equation system [1-5]. Consequently, to understand the nonlinear stability behavior of a BWR, the solution manifold of the differential equation systems must be examined. In particular, with regard to the existence of operational points where stable and unstable power oscillations are observed, stable or unstable fixed points and stable or unstable oscillatory solutions (or turning points/saddle node bifurcations) are of paramount importance [2].

These investigations have reactor safety relevance because power oscillations could induce undesirable hot spots in a BWR [2]. If the amplitudes become large enough, technical limit values (as critical power ratio) could be exceeded and fuel element failure could be expected. In addition to that there exist BWR states where (for example) unstable limit cycles occur (in the neighborhood of subcritical bifurcations where unstable periodic orbits occur) [5]. In this case, small perturbations imposed to the system, lead to a stable behavior. But if critical perturbation amplitude is exceeded, the system behaves unstable. Accordingly, this behavior hides the danger that instabilities can be undetected. Therefore the operational safety could be violated. Hence the methodology of the nonlinear stability analysis of BWR, applied in the current work, will uncover such phenomena.

In the framework of this, integrated BWR (system) codes and simplified BWR models (reduced order models, ROM) are used parallel to reveal the stability characteristic of fixed points and periodic solutions of the nonlinear differential equations describing the stability behavior of a BWR loop [2,3]. This work is a continuation of the previous work at the Paul Scherrer Institute (PSI, Switzerland) and University of Illinois (USA) on this field. The ROM developed at PSI was extended by an external loop and a model which takes into account the effect of subcooled boiling. Furthermore, a new calculation methodology for the feedback reactivity was implemented. Afterwards the modified ROM was coupled with the bifurcation code BIFDD [3,4] which performs so-called semi-analytical bifurcation analysis.

This article presents the motivation and the basic principles of this methodology. In addition to that the influence of the external loop on the stability boundary and the Poincarè-Andronov-Hopf bifurcation will be demonstrated.

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Methodology of nonlinear stability analysis

Under variation of one or more selected system parameters (control parameters) a fixed point (stationary point) can bifurcate to an isolated stable or unstable periodic solution (also termed stable or unstable limit cycle) of the system equations. The investigation of such nonlinear system behavior is the principal objective of nonlinear stability analysis [1-5]. In the following, a briefly characterization of system codes and reduced order models is given.

Complex or integrated system codes are computer programs which include detailed physical models of all nuclear power plant components which are significant for a particular transient analysis [2]. Therefore, such detailed BWR models represent the stability characteristics of a BWR close to the physical reality. In face of this, nonlinear BWR stability analysis with the aid of complex system codes is currently common practice in many laboratories [3]. A particular demand is the integration of a 3D neutron kinetic model for the core, thereby permitting analysis of regional or higher mode stability behavior (as so-called out-of-phase oscillations) [1-3].

A detailed investigation of the complete solution manifold of the nonlinear equations describing BWR stability behaviour by employing system codes needs comprehensive parameter variation studies which require large computational effort, hence system codes are inappropriate to reveal the complete stability characteristics of a BWR. Therefore, reduced order analytical models become necessary [2,3]. The ROM is characterized by a minimum number of system equations which is mainly realized by reducing the geometrical complexity. One demand on the ROM is that the corresponding equation system should present the real stability behaviour of a BWR loop. The main advantage of employing ROM is the coupling with methods of semi-analytical bifurcation analysis. In such a methodology the stability properties of fixed points and periodic solutions are investigated analytically without the need for solving the system of nonlinear differential equations [2-4].

The main objective of the current work is to combine system code analysis and ROM analysis. The intention is first to identify the stability properties of certain operational points by performing ROM analysis and then to use the system code for a detailed stability investigation in the neighbourhood of these operational points [2]. To this end, plant model data and data characterizing the operational point of a specified BWR plant will be extracted from the system code RAMONA. These data, respectively recalculated in an appropriate manner, are ROM inputs. In the first step of the ROM investigation, semi-analytical bifurcation analysis will be performed. As a result, the stability boundary (SB) and the nature of the Poincarè-Andronov-Hopf bifurcation (PAH-B) are determined. In the second step, for independent confirmation of the results, numerical integrations of the ROM differential equations will be carried out for specified parameter values [2-4].

Influence of the external loop model on the stability behavior

The current reduced order model based on the PSI ROM consists of tree sub-models [3]. These are a neutron kinetic model, a fuel heat conduction model and a thermal-hydraulic two channel model. In the original PSI-ROM, the recirculation loop was replaced by the boundary condition of a constant external pressure drop [1,2]. This is a reasonable assumption in an out-of-phase oscillation mode but not in an in-phase oscillation state [3]. Hence a recirculation loop model was developed and implemented in the ROM.

First investigations of the recirculation loop impact on the stability properties are concentrated on a thermal-hydraulic one-heated-channel model in the homogeneous equilibrium limit. To this end (or to understand this) the ratio A_{ol}, which appears in the momentum balance of the ROM after the recirculation loop is implemented, was varied in small steps. This ratio is defined as $A_{ol} = A_{inlet} / A_{doc}$ in which A_{inlet} is the inlet flow cross section of the heated channel and A_{doc} is the flow cross section of the downcomer. From the physical point of view, if the downcomer cross section will be increased (A_{ol} decreases), inertial effects of the downcomer mass flow are decreasing which lead to a constant external pressure drop. Consequently, if the ratio A_{ol} is zero ($A_{ol} = 0$) inertial effects of the downcomer will vanish. In the opposite, if the ratio A_{ol} will be increased, inertial effects of the downcomer will increase too. Because of practical relevance the ratio A_{ol} will be varied in the interval $A_{ol} \in [0,...,2]$.

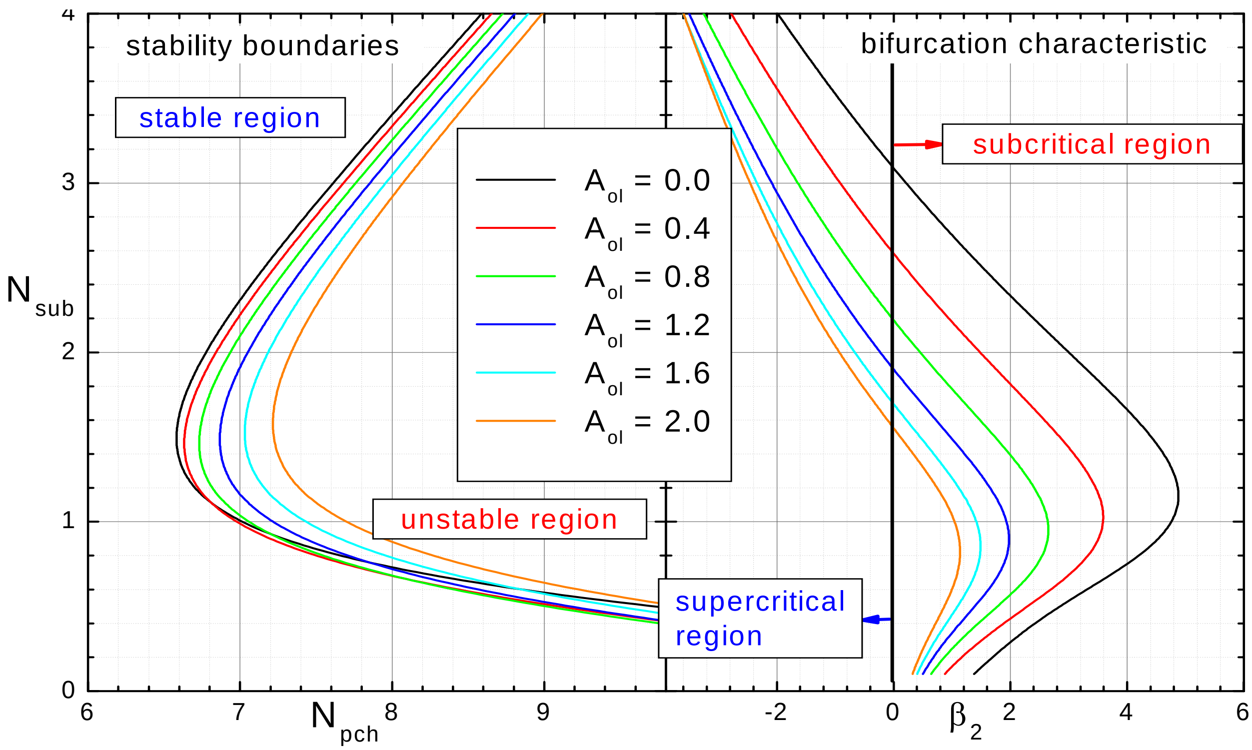

Figure 1: Stability boundaries in the N_{sub} - N_{pch} -parameter space and the corresponding bifurcation characteristic for different ratios A_{ol} with $A_{ol} \in [0,...,2]$

Figure 1 shows stability boundaries in the N_{sub} - N_{pch} -parameter space and the corresponding bifurcation characteristics for different A_{ol} values calculated by employing semi-analytical bifurcation analysis. The stability boundary[1] separates stable fixed points from the unstable one. The parameter β_2 results from the Floquet theory [2-4] and determines the bifurcation characteristic. If $\beta_2 < 0$ the bifurcation is called supercritical (the periodic solution is stable which corresponds to stable limit cycles) and if $\beta_2 > 0$ the bifurcation is called subcritical (the periodic solution is unstable which corresponds to unstable limit cycles). The stability boundaries in figure 1 shift to the right hand site and the number of subcritical fixed points decreases for increasing A_{ol} values. From the

[1] in terms of non linear dynamics this means, that there exist at least a pair of complex conjugated eigenvalues with zero real parts

stability point of view, the number of stable fixed points increases. According to this, the system becomes more stable.

After the semi-analytical bifurcation analysis was performed the results were verified by using the numerical integration of the system of equations for chosen points in the N_{sub}-N_{pch}-parameter space. The results (not presented in the current paper) confirm the predictions of the semi-analytical bifurcation analysis.

A similar study was performed to analyze the influence of the downcomer friction on the stability behavior. The investigation shows that the system is not very sensitive to the downcomer friction variation. The numerical integration confirms this prediction.

Comment: The subcooling number N_{sub} represents the core inlet subcooling and appears as a boundary condition in the single phase energy equation. The phase change number (also called Zuber number) scales the phase change due to the heat addition into the coolant of the heated channel. These dimensionless numbers are defined as

$$N_{sub} = \frac{(h^*_{sat} - h^*_{inlet})}{\Delta h^*_{fg}} \cdot \frac{\Delta\rho^*}{\rho^*_g} \qquad N_{pch} = \frac{q''\xi^*_h l^* \Delta\rho^*}{A^*_{inlet}\Delta h^*_{fg}\rho^*_g v^*_0}$$

where h^*_{sat} is the saturation enthalpy, h^*_{inlet} inlet enthalpy, $\Delta\rho^* = \rho^*_f - \rho^*_g$ liquid-vapor density, $\Delta h^*_{fg} = h^*_g - h^*_f$ liquid-vapor enthalpy, v^*_0 reference velocity (steady state channel inlet velocity), l^* length of the channel, q'' wall heat flux and ξ^*_h is the heated perimeter. The N_{sub}-N_{pch}-space shows the thermodynamic state within the heated channel. In face of this the N_{sub}-N_{pch}-parameter space is often used in the literature as stability map [2-4].

Conclusions

The analyzed results confirm that the recirculation loop model is an essential element in the BWR-ROM. The dominant term in the momentum balance is the inertial term. On the other hand, the downcomer friction has a very small impact on the stability behavior. Consequently, it can be neglected in further investigations. Notice, the numerical integration confirms the results of the semi-analytical bifurcation analysis.

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