

## Simplified Mathematical Model of Combustion in Stoker Systems

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### Abstract

For thermal treatment of household waste stoker systems are most frequently used. Since the beginning of the 1980's the focus has been on pollution reduction by development of high performance gas cleaning processes connected onto stoker systems (secondary measures). Now investigations focus on the optimization of the thermal process itself (primary measures). Along with the experimental research with pilot and industrial plants the mathematical modeling of the stoker process is as before an important goal. Due to the variations in waste properties and the difficulties in their determination, simplified assumptions were used along with experimental results from pilot plants to develop a mathematical model that describes the combustion and gasification in a stoker system.

### Key Words

Thermal treatment of waste; stoker firing systems; combustion; gasification; mathematical model; experimental investigations; pilot plant.

### 1. Introduction

Over the last decades, stoker systems have become a standard for thermal treatment processes. Prior to 1940 substantial amount of developmental work was carried out on the combustion of different types of coal for energy conversion. Since then, design alterations on the different stoker systems for thermal treatment of wastes have mainly resulted from empirical findings. However, certain experimental and theoretical investigations have revealed potential for process optimisation through regulation along the grate path of the so-called main variables (Figure 1). Oxygen enrichment, flue gas recycling [e.g. 1] or gasification at the grate [e.g. 2, 3] and release or retention of elements as a function of the waste properties or the process control [4] (level and distribution of the main variables given in Figure 1) are examples of such measures for optimization.

Process adjustment to the various types of waste (e.g. household waste, paper mill remnants, wood scraps, etc.) requires, in addition to information from specific experimental investigations on pilot and industrial plants, model concepts that describe the conversion of solids in stoker furnaces.

A mathematical model not only allows a description of processes which are very difficult to determine experimentally, but also provides solutions to problems arising in the planning of new plants or in optimisation of existing plants. This report presents a model describing interaction among the essential variables for the conversion of carbon in stoker systems (Figure 1). This report is confined to a summary of the partial stages, a more detailed account and the inherent equations can be found in [5, 6].

We will deal with interim results from model calculations and experimental investigations as well as with the development of the model set-up for gasification conditions. Model set-up is necessary particularly in connection with the gasification on the grate and the independent afterburning. The advantage of a separate process control over conventional methods is a considerably higher degree of freedom to optimise the partial tasks (control of the solid residues from the grate, reduction of the CO and CxHy concentrations in the flue gas, etc.). Possible application for such a separate process control is operation of stoker systems with gasification for recovery of valuable materials.

### 2. Model Set-Ups

Material conversion on a grate can be roughly divided into the partial stages: drying, degasification, gasification and residual burnout of the solid (Figure 2). Gasification and residual burnout are of particular

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interest and depending on the composition of waste they can claim up to 90 % of the overall conversion time. The following discussion will firstly consider the carbon conversion during gasification and combustion processes. It is well-known that the gasification and combustion of carbon have highly complex chemical reaction mechanisms. For residues with highly fluctuating material composition it is advantageous to describe different process stages using simplified models which are subsequently combined to so-called summarised kinetic data. For the development of the model presented below, the following assumptions and steps have been made:

- Reactor characteristics can be described by a serial cascade of continuous stirred reactors (CSR).
- One of these stirred reactor elements can be observed moving along the length of the stoker. Depending on the supplied reaction-gas mass-flow and on reaction conditions (temperature, oxygen partial pressure, reactive surface) a carbon conversion results. With the enthalpy of chemical reaction and gas mass-flow, the storage capacity of the bed, and the heat transfer conditions one obtains a corresponding balance temperature for the observed cell.
- An effective reaction coefficient (so-called "summarised kinetic data") can be determined in a batch-stoker research plant:
  - first of all for basic investigations with different model fuels (e.g. lignite with different inert material content) and
  - subsequently for investigations with waste materials.
- Through series-connection of several cells in which there is a solid material exchange caused by grate movements, one obtains, for instance, integral values of the temperature and oxygen concentration.
- Theoretical and experimental results for steady state as well as for nonsteady state conditions in the pilot stoker plant are compared.

### 3. Batch Process Model

One continuous stirred reactor element is selected from the cascade depicted in fig. 2 and is considered from the viewpoint of an "accompanying observer" (Batch process model) (Figure 3). It is assumed that the carbon and inert material (combustible solid) have equivalent diameters in the observed cell. The void volume of the fixed bed remains constant. The mass flow of carbon  $\dot{m}_C(t)$  released by combustion yields a corresponding flow of enthalpy of reaction  $h_{ch}$ . Depending on this energy flow, the supplied reaction-gas mass flow  $\dot{m}_{g,in}$ , the storage capacity of the bed (residual carbon and inert material), and the heat-transfer ratios (supply and loss) one obtains an balance temperature. The energy balance can be summarised as follows:

$$\begin{array}{cccc} \boxed{A} & \boxed{B} & \boxed{C} & \boxed{D} \\ \dot{m}_{g,in} \cdot c_{p,g,in} \cdot (T_{g,in} - T_0) dt + \dot{m}_C(t) \cdot (-\Delta h_{ch}) dt + \dot{m}_C(t) \cdot c_C(T - T_0) dt + \dot{Q}_R dt = & & & (1) \\ \dot{m}_{g,out} \cdot c_{p,g,out} \cdot (T - T_0) dt + m_{C,Re}(t) \cdot c_C dT + m_{In} \cdot c_{In} dT + \dot{Q}_L dt & & \boxed{E} & \boxed{F} & \boxed{G} & \boxed{H} \end{array}$$

Only overstoichiometrical conditions are considered first of all. It is assumed that the heterogeneous gasification reaction (a)  $C + 1/2 O_2 \rightarrow CO$  is followed directly by the homogeneous gas phase reaction (b)  $CO + 1/2 O_2 \rightarrow CO_2$ . With the simplified formulation for the converted-carbon mass flow  $\dot{m}_C(t)$

$$\dot{m}_C(t) = M_C \cdot A_{sp,os}(t) \cdot k_{eff,os}(t) \cdot \frac{p_{O_2}(t)}{R T(t)} \quad (2)$$

where

$$k_{eff,os} = \frac{1}{\beta + \frac{1}{2 \cdot k_{ch,os}}} \quad (3)$$

and

$$k_{ch,os} = k_{max,os} \cdot \exp \left[ -\frac{E_{A,os}}{R \cdot T} \right] \quad (4)$$

one obtains after corresponding transformations of eq. (2) - among other things with the oxygen balance -

$$\dot{m}_C(t) = m_{C,0} \cdot \left( \frac{1}{k_{eff,os}(t,T)} \cdot \frac{d_{p,0} \rho_p}{6} \cdot \left( \frac{m_{C,0}}{m_{C,Re}(t)} \right)^{\frac{2}{3}} \cdot \frac{RT(t)}{M_C P_{tot}} + \frac{M_{O_2}}{M_C} \cdot \frac{\rho_{g,in}}{\rho_0} \cdot \frac{m_{C,0}}{\dot{m}_{g,in}} \right)^{-1} \cdot \frac{P_{O_2,in}}{P_{tot}} \quad (5)$$

As is represented [5], this model set-up yields the expected curves for oxygen concentration, temperature and residual carbon content as a function of time for the cell in question. Figure 4 exemplifies the curve calculated for the carbon conversion as a function of time for four different partial stages. Curve 1 is determined by initially assuming a pre-set fixed bed temperature and neglecting the inert component and the heat-transfer conditions (terms D, G, and H in eq. 1). A further stage (curve 2) takes the inert component (term G) into consideration. The curve illustrates that at the end of the reaction residual carbon content is considerably lower. This is due to the capacity of the inert material to store sufficient energy during the main combustion phase to thus reducing the risk of "blow out" [5]. If the heat-transfer (curve 3 in Figure 4) is now taken into account, the result, after an initial "heating stage", is as expected: a curve similar to those obtained in the previous cases. Finally curve 4 exemplifies air staging as a function of time. By assuming a transport speed of solid in a stoker system the solid conversion over a period of time (batch process) can be used for the solid conversion along the reaction path of the stoker plant.

Figure 5 depicts exemplary calculated and experimental data determined from a batch reactor with lignite [6]. Within the scope of the simplifications made, it can be seen, that the measured results agree well with the calculated results.

#### 4. Model for a Continuous Stoker Process

A continuous model implies the series connection of several cells. If each cell initially moves along the grate path as a function of time  $\Delta t$  (Figure 6) and no material transfer between the cells is permitted, this corresponds for each cell to the air staging conditions (see curve 4 in Figure 4).

For a closer examination of the ratios for a continuous grate it is necessary to record the forced-draft zones of several cells connected in series and to take the mechanical material conversion between the cells, caused by grate movement, into consideration. It is assumed that a specific amount of the material contained in the element  $i$  is, governed by the operated grate, discontinuously either fed or pushed back at a point of time  $t^*$ . The mass balance at the point of time  $t^*$  is expressed for the element  $i$  of the continuous stirred reactor as

$$m_{C,0}(t^*, i) = m_{C,Re}(t^*, i) + m_{C,fw}(t^*, i-1) + m_{C,bw}(t^*, i+1) - m_{C,fw}(t^*, i) - m_{C,bw}(t^*, i) \quad (6)$$

The balance for the inert substance can be stated in the same manner. The energy balance is derived using the enthalpies for the substituted materials. Furthermore, formation of a new equivalent diameter is required due to the material conversion. The applicable data for the element  $i$  of the continuous stirred reactor at time  $t^*$  correspond to the initial data for the cell model. By applying the modified equations (1) to (4) it is now possible to determine the solid conversion separately for each cell. Figure 7 exemplifies the curves for the oxygen concentrations as a function of time for a grate with 5 forced-draft zones and 2 CSR elements per zone: a total of 10 CSR elements. A pre-set temperature and a grate initially coated with fuel are assumed. A steady state is set for each CSR element in the example given after an initial run-up phase. A combination of the individual CSR elements yields an integral oxygen concentration or balance temperature for the overall flue-gas flow as well as residual carbon content at the end of the grate.

The experimental results of Figures 8 to 10 from a pilot plant [2] having a thermal capacity of ca. 0.5 MW and the respective model calculations show an irregular change in the air staging along the grate path for three examples. Depending on the initial ratios, an irregular increase in the combustion air or even a redistribution of air, for example, can result in either a "kindling" (reduction in oxygen concentration and increase in temperature) or a "blow out". Figures 8 to 10 also show that the present model is capable of describing these nonsteady combustion processes correctly.

## 5. Enlargement of the Model for Gasification

An expansion of the model for gasification requires first of all modification of the Batch model. The heterogeneous reaction (c)  $C + CO_2 \rightarrow CO$  (Boudouard reaction) is here also taken into account. It is possible to envisage that the overall converted carbon mass flow  $\dot{m}_C(t)$  can be separated into partial flows in accordance with Equation (7)

$$\dot{m}_C(t) = \dot{m}_C(t)_{a,b} + \dot{m}_C(t)_a + \dot{m}_C(t)_c \quad (7)$$

The first term on the right-hand side in equation (7) determines the partial flow which is converted to carbon dioxide in accordance with the heterogeneous reaction (a) with subsequent gas phase reaction (b). Furthermore, the second term integrates the partial flow, which reacts according to the heterogeneous reaction (a) only to carbon monoxide. The partial flow which is converted by the Boudouard reaction is subsequently taken into consideration by the third term. The element B in the energy balance given in Equation (1) can now be completed in accordance with the partial reactions and the inherent reaction enthalpies.

Referring to [7] the summation reaction coefficients are expressed by taking the partial flows into account and by making a corresponding transformation:

$$k_{eff,os} = \beta \cdot \left[ \frac{2}{(2+1/Da_a)} + \frac{2 \cdot e^{-0,23 \cdot Ha}}{(2+1/Da_a - e^{-0,23 \cdot Ha})(2+1/Da_a)} + \frac{2 - 2 \cdot e^{-0,23 \cdot Ha}}{2+1/Da_c - e^{-0,23 \cdot Ha}} \right] \quad (8)$$

The partial pressures of the gas components oxygen, carbon monoxide, carbon dioxide and nitrogen are derived from the mass balances. An example is given by the relation for oxygen  $p_{O_2}$ :

$$p_{O_2} = \frac{p_{O_2,in} \cdot \frac{M_{O_2}}{M_C} \cdot \frac{\rho_{g,in,N}}{\rho_{O_2,N}} \cdot \frac{1}{\dot{m}_{g,in}} \cdot (\dot{m}_C(t)_{a,b} + \frac{1}{2} \cdot \dot{m}_C(t)_a)}{1 + \frac{M_{O_2}}{M_C} \cdot \frac{\rho_{g,in,N}}{\rho_{O_2,N}} \cdot \frac{1}{\dot{m}_{g,in}} \cdot (\frac{1}{2} \cdot \dot{m}_C(t)_a + \dot{m}_C(t)_c)} \cdot p_{tot} \quad (9)$$

This equation, as also the relations for the coefficients, can be used for the case of an infinitely fast homogeneous gas phase reaction (b) and, with neglectable Boudouard reaction (c), with the known combustion equations. Figure 11 depicts an example calculated using the model formulation expanded to include gasification. A more detailed discussion on the conversion of solids by gasification is beyond the scope of this contribution.

However investigations on gasification on a grate and the separate independent afterburning in an combustion chamber system appear highly promising, particularly in connection with the application to the thermal treatment of wastes. Although this mode of operation is not as yet in general use for stoker systems, investigations on a pilot plant show that it has the following advantages over conventional modes of operation:

- a considerable reduction in the flue gas mass flows,
- generation of fuel gas so that the afterburning process can be optimised independently and run without the use of additional fuel (e.g. primary measures for the reduction of  $CO$ ,  $C_xH_y$ , soot,  $NO_x$  etc.).
- control of solid residues properties and separation of the valuable substances after the stoker process.

This type of separate process control enables considerable improvements to be made using stoker systems so that with corresponding modification of process controls they will remain competitive in the future.

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## Notation

### Latin characters

(large)

*A* area  
*Da* Damköhler number  
*E* energy  
*Ha* Hatta number  
*M* molar mass  
*Q* heat  
*R* universal gas constant  
*T* temperature

(small)

*c* specific heat capacity  
*d* diameter  
*h* specific enthalpy  
*k* reaction coefficient  
*m* mass  
*t* time

### Greek characters

(small)

$\beta$  mass transfer coefficient  
 $\vartheta$  temperature  
 $\rho$  density  
 $\psi$  concentration

### Indices

(overline)

$\dot{\phantom{x}}$  flow rate

(subscript)

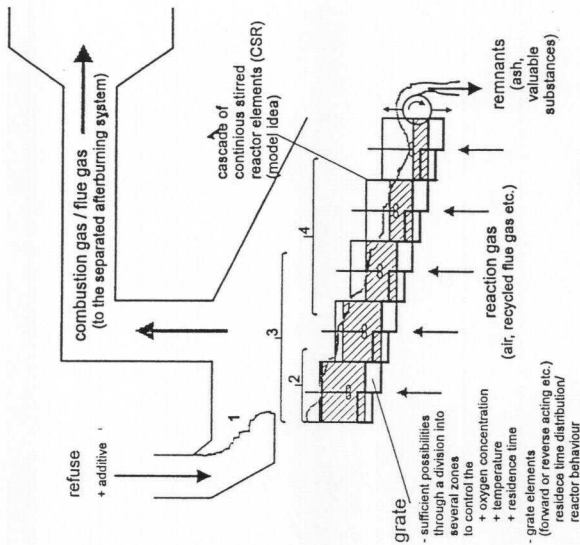
0 start  
*A* activation  
*bw* back  
*fw* forward

### C

*ch* carbon  
*eff* chemical effective  
*g* gaseous  
*In* inert  
*in* input  
*l* loss  
*O<sub>2</sub>* oxygen  
*os* outer surface  
*out* out  
*p* pressure, particle  
*R* radiation  
*Re* remainder  
*sp* spherical shape  
*tot* total

waste materials	physical properties	gaseous, liquid, pasty, solid (dusty or coarse)	
	chemical properties	composition (problematic substances)	
oxygen supply	thermolysis	gasification	combustion
reaction gas	$\lambda = 0$	$\lambda < 1$	$\lambda \geq 1$
temperature	air, oxygen, (nitrogen), carbon dioxide, steam, recycled flue gas, etc.		} drying, degasification, melting, evaporation, sublimation
pressure	low pressure	atmospheric pressure	
reactor behavior	$p \ll 0,1 \text{ MPa}$	$p \approx 0,1 \text{ MPa}$	$p \gg 0,1 \text{ MPa}$
method of material input	continuous stirred reactor (CSR)	real reactor	plug flow reactor (PFR)
residence time	$Pe = 0$	$0 < Pe < \infty$	$Pe \rightarrow \infty$
	short	long	very long
	several s	several min up to hours	several hours up to days
additional substances	additive (e.g. bonding of pollutants, controlling the melting behavior) additional fuel bed (e.g. fluidized bed, solid bed, circulating bed, binding matrix, ash feedback)		

Fig 1. Main influential parameters for thermal treatment.



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Fig. 2. Combustion steps of a stoker firing process and model idea

### Model idea / energy balance

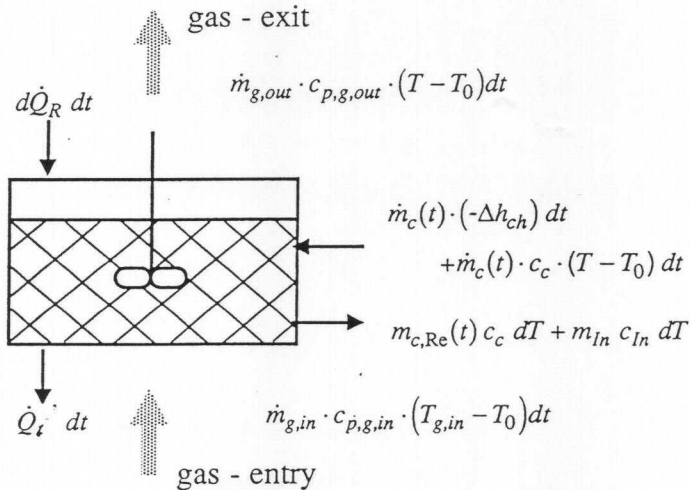


Fig. 3. Energy balance of a continuous stirred reactor element (CSR)

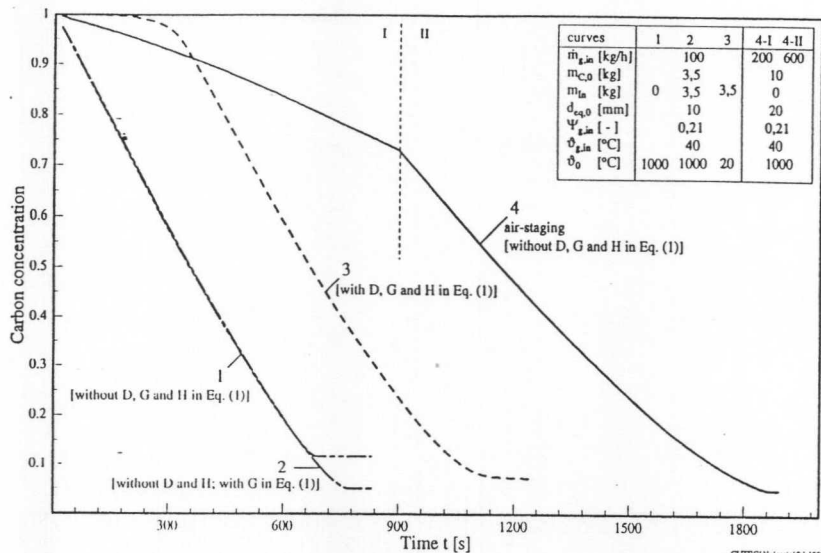


Fig. 4. Calculated carbon concentration over a period of time for a given bed

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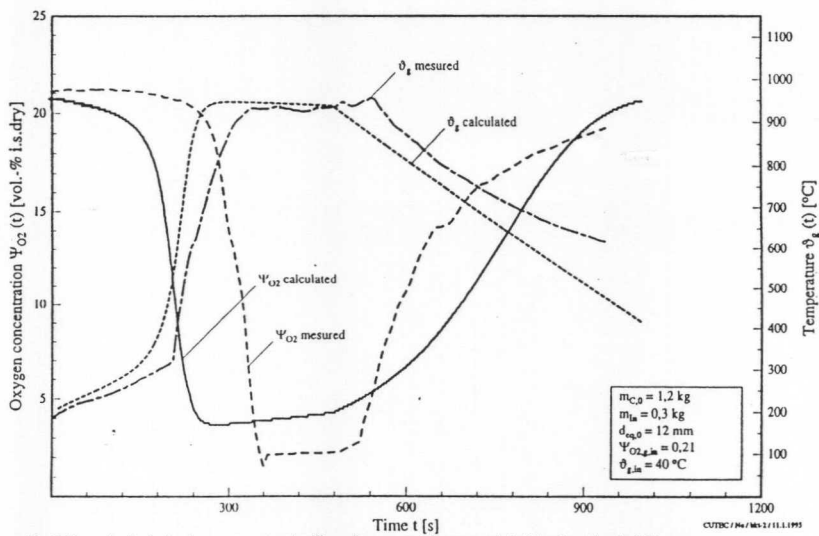
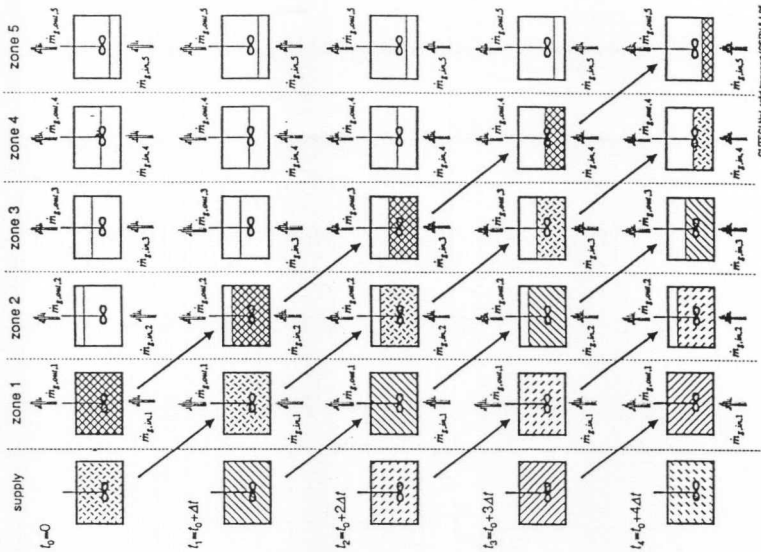


Fig. 5. Measured and calculated oxygen concentration  $\Psi_{O_2}$  and temperature  $\vartheta_a$  over a period of time for a given bed [6]

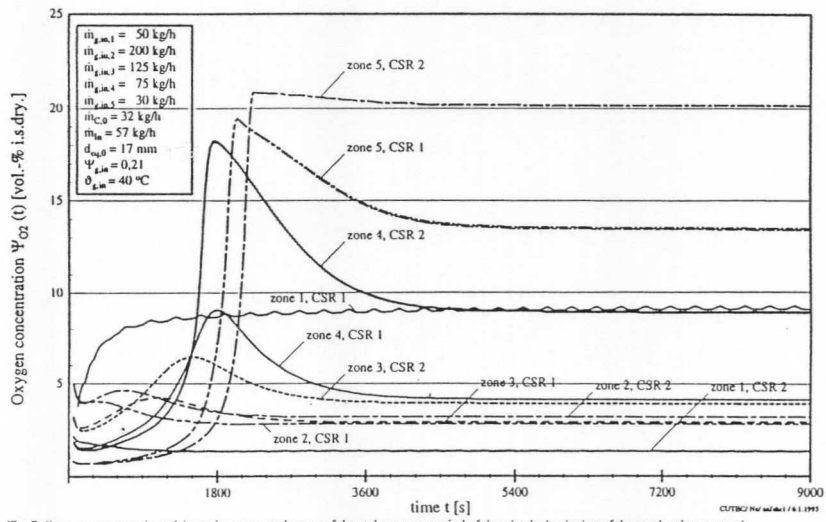
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Fig. 6. Model idea for the description of the solid transport in a grate system



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Fig. 7. Oxygen concentration of the series-connected zones of the stoker over a period of time (at the beginning of the combustion process)

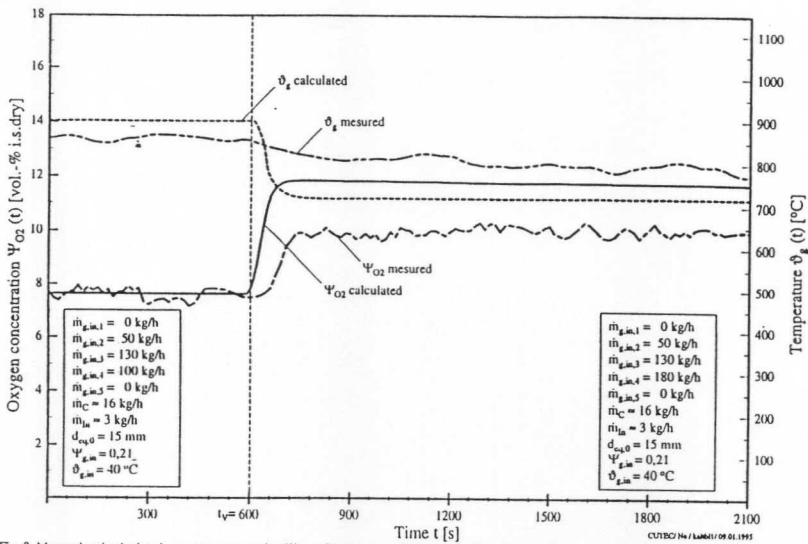


Fig. 8. Measured and calculated oxygen concentration  $\Psi_{O_2}$  and temperature  $\vartheta_g$  (increase of combustion air flow rate in the burn-out zone)

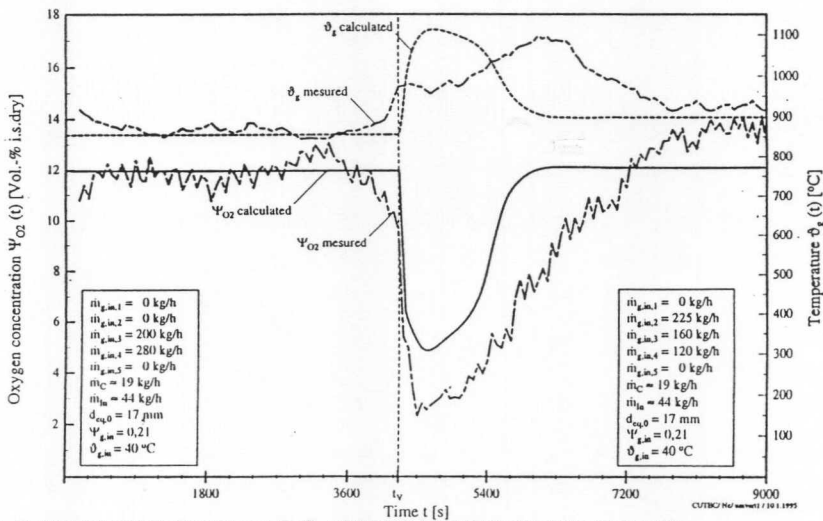


Fig. 9. Measured and calculated oxygen concentration  $\Psi_{O_2}$  and temperature  $\vartheta_g$  (redistribution of combustion air flow rate) [6]

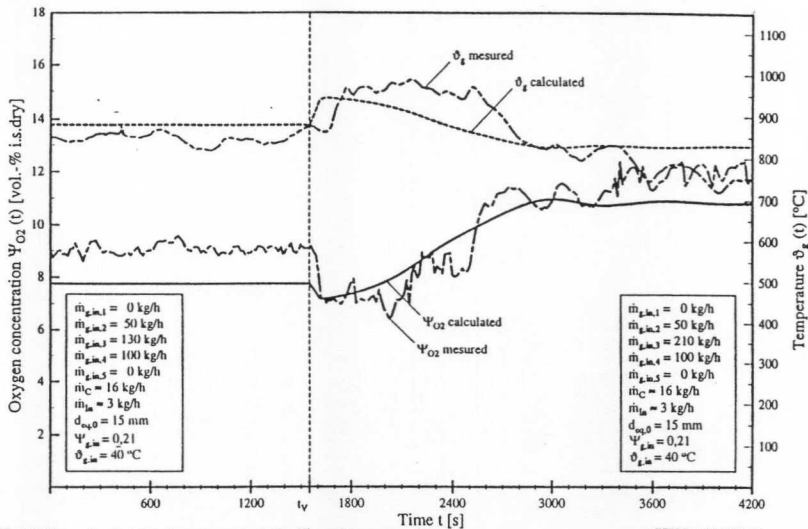


Fig. 10. Measured and calculated oxygen concentration  $\Psi_{O_2}$  and temperature  $\phi_s$  (increase of the combustion air mass flow rate in the main combustion zone)

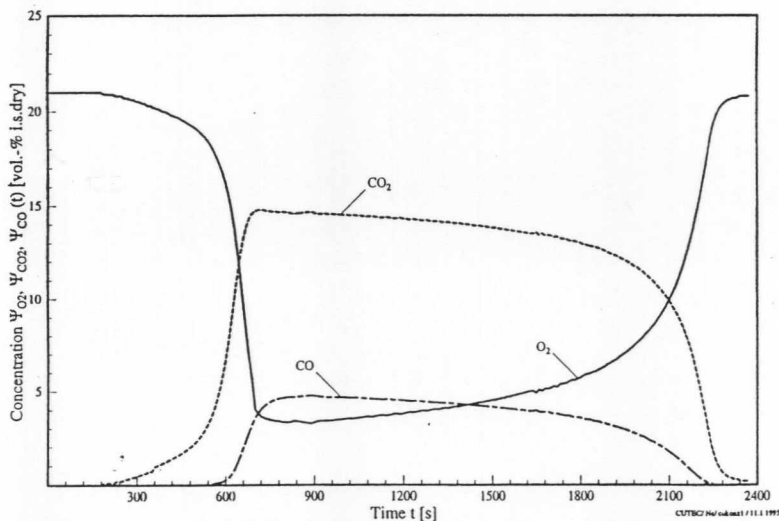


Fig. 11. Calculated oxygen  $\Psi_{O_2}$ , carbon dioxide  $\Psi_{CO_2}$  and carbon monoxide  $\Psi_{CO}$  concentration over a period of time for a given bed (example gasification batch process)

