# Gravitational Induction with Weber's Force


M. Tajmar and A. K. T. Assis*
Institute of Aerospace Engineering
Technische Universität Dresden
01062 Dresden, Germany
Email: martin.tajmar@tu-dresden.de

* Institute of Physics 'Gleb Wataghin'
University of Campinas — UNICAMP
13083-859 Campinas, SP, Brazil
Email: assis@ifi.unicamp.br
Homepage: `www.ifi.unicamp.br/~assis`



**Abstract**

According to Faraday's law of induction, when we change the current intensity in a primary electric circuit we can induce a current in a secondary circuit under appropriate conditions. An electric current means charges in motion. Microscopically we can express Faraday's law by saying that when we accelerate charges in the primary circuit, a force is exerted on charges of the secondary circuit which can accelerate them. A similar effect also exists in gravity with accelerated masses but of course with much less intensity. The phenomenon is called frame dragging and can be derived from general relativity theory. Here we present an alternative way to calculate such gravitational induction forces based on Weber's law which only involves simple mathematics and incorporates other fundamental concepts such as Newton's third law and Mach's principle as the origin of inertia. It therefore summarizes all low velocity gravitationally-relevant effects into a single equation.



**Key Words:** Electromagnetic induction. Gravitational induction. Relational mechanics.

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# 1 Introduction

Frame dragging (also called the Lense-Thirring effect) is a well-known phenomenon in general relativity. It affects e.g. the trajectory of satellites in polar orbit or generates precession forces on spinning gyroscopes. This effect has been experimentally verified most notably with the LAGEOS and Gravity-Probe B missions.[1, 2] General relativity involves advanced mathematics which is not easily accessible to undergraduate students or engineers, therefore scientists tried to derive a simple framework to calculate these effects which is called Gravitomagnetism.[3, 4, 5, 6, 7] Forward was one of the first scientists to derive a set of Maxwell-like equations[8] that allow to calculate frame-dragging effects similar to electromagnetic effects in the weak-field and low-speed limit of general relativity.

We will show that the same effects can be derived out of Weber's force law[9, 10, 11] for gravitation that includes all well known mechanical forces such as Newton's third law and the so-called fictive forces (such as the centrifugal or Coriolis force). Weber's law also incorporates Mach's principle (the origin of inertia is due to the gravitational interaction with the distant galaxies) and therefore summarizes gravitational phenomena in a single equation easily accessible to undergraduate students. Even more, as we will show in this paper, Weber's force also includes the frame-dragging predictions of general relativity.

Weber's law therefore seems to be an excellent starting point to teach the concepts of mechanics and even higher order gravitational effects with simple mathematics. As an example, we will show how frame-dragging or gravitational induction works with Weber's law and calculate the difference in the orders of magnitude between electromagnetic and gravitational induction.

# 2 Electromagnetic Induction

According to Faraday's law of induction (1831), the electromotive force $emf$ induced in a closed circuit $C$ is given by

$$emf = \oint_C \frac{\vec{F}}{q} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt} \ . \tag{1}$$

Here $\vec{F}$ represents the force acting on an electric charge $q$ belonging to the circuit, $d\vec{\ell}$ is an oriented infinitesimal length pointing along the circuit in each point, $\Phi_B = \int\int_A \vec{B} \cdot d\vec{a}$ is the flux of the magnetic field $\vec{B}$ through the loop of the circuit having a total area $A$, an infinitesimal area orthogonal to the loop in each point of its surface is represented by $d\vec{a}$ (with the directions of $d\vec{\ell}$ and $d\vec{a}$ connected by the right-hand rule), while $t$ represents the time.

As a specific example we consider a spherical shell of radius $R$ uniformly electrified with a total charge $Q$ spinning around the $z$ axis passing through the center of the sphere with an angular velocity $\vec{\Omega}$ relative to an inertial frame of reference $S$. The center $O$ of the coordinate system will be considered at the center of the shell. This situation produces a vector potential $\vec{A}$ and a magnetic field $\vec{B}$ at a point $\vec{r}$ inside the shell given by, respectively:[12, 13, 14]

$$\vec{A} = \frac{\mu_o Q}{12\pi R}\vec{\Omega} \times \vec{r} \ , \tag{2}$$

and

$$\vec{B} = \frac{\mu_o Q \vec{\Omega}}{6\pi R} \ , \tag{3}$$

where $\mu_o = 4\pi \times 10^{-7} \ kgm/(A^2 s^2)$ is the permeability of free space.

It should be observed that the magnetic field inside this spherical shell is uniform, having the same direction and intensity at all internal points.

Consider now a metal ring of smaller radius $r_o$ and larger radius $r$, with $r_o \ll r < R$, centered at $O$ and located in the $xy$ plane orthogonal to this magnetic field. The magnetic flux through the area $A = \pi r^2$ of the ring is given by $\Phi_B = \pi r^2 B = \mu_o Q r^2 \Omega / 6R$. According to equation (1), the $emf$ induced in the ring is given by:

$$emf = -\pi r^2 \frac{dB}{dt} = -\frac{\mu_o Q r^2}{6R}\frac{d\Omega}{dt} \ . \tag{4}$$

According to Ohm's law, neglecting the self-inductance of the ring and considering the time interval in which the angular acceleration $d\Omega/dt$ remains constant, the current $I$ induced in the ring having a resistance $R_o$ is given by $I = emf/R_o$.

A similar effect is known to exist in general relativity, although the calculations leading to this prediction are reasonably complex.[8, 15, 16] In this work we present an alternative formulation of gravitational theory which also predicts a similar effect, namely, Weber's force.

# 3 Gravitational Induction with Weber's Force

In the sequence of the paper we consider how it might be possible to produce a similar effect with gravitation. We will be dealing only with neutral bodies here, so that the electromagnetic forces can be neglected.

Consider a spherical shell of mass $M$ and radius $R$ which is spinning around an axis passing through the center of the shell with an angular velocity $\vec{\Omega}$ relative to an inertial frame of reference $S$. An internal test body of mass $m$ is located at a position vector $\vec{r}$ relative to the center $O$ of the shell, moving with velocity $\vec{v}$ and acceleration $\vec{a}$ relative to $S$. According to Weber's law applied to gravitation, the force exerted by this spinning spherical shell and acting on the test body is given by:[9, 11]

$$\vec{F} = -\frac{2GmM}{Rc^2}\left[\vec{a} + \vec{\Omega} \times \left(\vec{\Omega} \times \vec{r}\right) + 2\vec{v} \times \vec{\Omega} + \vec{r} \times \frac{d\vec{\Omega}}{dt}\right] \ , \tag{5}$$

where $G = 6.67 \times 10^{-11}\ m^3/kgs^2$ is the constant of universal gravitation and $c = 2.998 \times 10^8\ m/s$ is the velocity of light in vacuum. The second term in the square bracket yields in non-inertial frames of reference the centrifugal force, while the third term yields the Coriolis' force, yielding a mathematical implementation of Mach's principle.[9, 10, 11]

In what follows we will consider that the test body of mass $m$ belongs to a ring of radius $r$ centered at the center $O$ of the spherical shell and located in the $xy$ plane, orthogonal to the direction $z$ of the rotation of the spherical shell, $\vec{\Omega} = |\vec{\Omega}|\hat{z} = \Omega\hat{z}$. The ring will be considered initially at rest and the position vector of one of its elements can be represented in polar coordinates by $(r,\ \varphi)$, where $\varphi$ is the azimuthal angle, figure 1.

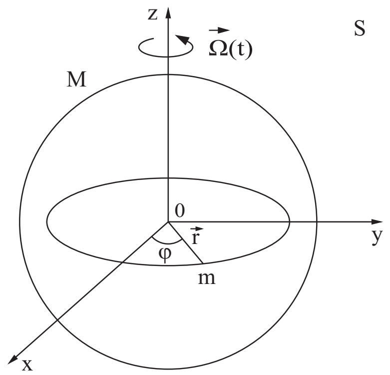


Figure 1: Spherical shell spinning around the $z$ axis with an internal ring at the equatorial plane.

We will be interested only in the azimuthal motion of the ring and how it is influenced by a variable rate of rotation of the surrounding spherical shell. Therefore the position, velocity and azimuthal acceleration of the test particle can be represented by, respectively: $\vec{r} = r\hat{r}$, $\vec{v} = r\dot{\varphi}\hat{\varphi}$ and $\vec{a} = r\ddot{\varphi}\hat{\varphi}$, where $\hat{\varphi}$ is the unit azimuthal vector at the location of the particle. The component of the force $\vec{F}$ proportional to $\vec{\Omega} \times (\vec{\Omega} \times \vec{r})$ points along the radial $\hat{r}$ direction, the same happening with the component proportional to $2\vec{v} \times \vec{\Omega}$, so that they will not be considered in the sequence.

Combining equation (5) with Newton's second law of motion, $\vec{F} = m\vec{a}$, yields the following equation describing the motion of the test particle in the azimuthal direction:

$$-m_W r\ddot{\varphi} + m_W r\frac{d\Omega}{dt} = mr\ddot{\varphi}\ , \tag{6}$$

where $m_W = 2GmM/Rc^2$ is what we call Weber's mass for this geometry.

By considering a spherical shell with $R = 1\ m$ and $M = 100\ kg$ it is easy to see that $m \gg m_W$. We are then led to:

$$\ddot{\varphi} = \frac{m_W}{m}\frac{d\Omega}{dt} = \frac{2GM}{Rc^2}\frac{d\Omega}{dt} \ . \tag{7}$$

This equation represents a gravitational induction. That is, whenever the rate of rotation of the surrounding spherical shell changes as a function of time, there will be an azimuthal force acting on each element of the internal ring. If it is free to rotate around its axis and friction can be neglected, the ring will undergo an induced azimuthal acceleration given by equation (7).

An interesting question which might be asked related to equation (7) is the following gedanken experiment: Is the angular acceleration field propagating inside the shell core? The answer to this question according to Weber's law applied to gravitation would require calculations which are beyond the scope of this paper. In any event, it is relevant to remember here the deduction by Kirchhoff and Weber, in the period 1849-1864, of the telegraphy equation based on Weber's electrodynamics.[17, 18, 19, 20, 21, 22, 23, 24] When the resistance of the wire was negligible, the telegraphy equation simplifies to the wave equation. The wave equation obtained by Kirchhoff and Weber then predicted the propagation of signals along the wire with light velocity. As Kirchhoff put it,[18, 21] the velocity of propagation of an electric wave "is independent of the cross-section, of the conductivity of the wire, also, finally, of the density of the electricity: its value is 41950 German miles in a second, hence very nearly equal to the velocity of light *in vacuo*." These works of Weber and Kirchhoff, based on Weber's electrodynamics, were obtained prior to Maxwell's electromagnetic theory of light in the period 1864-1873.[25, 26, 27] One of the authors (AKTA) discussed these papers of Weber and Kirchhoff in more details in other works.[28, 29, 30, 31, 32, 33, 34] The calculations of Weber and Kirchhoff utilized the interaction of two types of charges, namely, positive and negative ones. As we have only one kind of mass, it is not straightforward to extend their calculations to the case of purely gravitational interactions, as is the case considered in the present paper. But this is an interesting topic to be analyzed in the future.

In order to know if the induced angular accelerations predicted by equation (7) would be relevant to the case of Earth internal geophysics, the Earth should not be considered as a rigid body. After all, in this case there would be no effects, according to Weber's law applied to gravitation, which might be due to a relative acceleration between interacting masses. In any event, it would be valuable to highlight this possibility in the case of differential rotations of the Earth shell (as the rotating sphere) and the Earth core (as the probing mass) rotating at different angular velocities relative to an inertial frame of reference. Additional research might be relevant in this case.

The effect described in this paper represented by equation (7) is analogous to an electromagnetic induction with three main differences:

- The first one is that the tangential acceleration of the ring will happen in the same direction as the tangential acceleration of the surrounding shell. In electromagnetism, on the other hand, the induced current will always flow in a direction such that the flux it produces tends to cancel the change of the external flux, according to Lenz's law.

- The second difference is that the induced angular acceleration $\ddot{\varphi}$ will have always the same value for all test masses, no matter the value of $m$. This is similar to the linear acceleration of free fall in the gravitational field of the Earth. The free fall acceleration does not depend on the mass of the test body. This effect also demonstrates the validity of the equivalence principle.

- The third difference happens in the order of magnitude of the effect. Electromagnetic induction has been known to exist since 1831 with Faraday's experiments. It has a reasonably strong effect which is easily detected in the laboratory. By calculating the dimensionless magnitude $2GM/Rc^2$ of equation (7), on the other hand, we obtain with $R = 1\ m$ and $M = 100\ kg$ the extremely small value of $2GM/Rc^2 \approx 1.5 \times 10^{-25}$. Therefore, by changing the rate of rotation of the surrounding spherical shell such that $d\Omega/dt = 1\ rad/s^2$ we obtain $\ddot{\varphi} = 1.5 \times 10^{-25}\ rad/s^2$. Such an extremely small angular acceleration cannot be measured in the laboratory. Moreover, it would not even be produced in practice, due to the inevitable presence of friction. In any event, as stressed before, this effect might be relevant in the case of the differential rotation of the Earth. If the spinning spherical shell is considered as the Earth mantle, then $2GM/Rc^2 \approx \times 10^{-11}$. The test body might be considered the Earth's core. Therefore, this effect might be relevant for the Earth's internal geophysics, as stated before.

Weber's law complies with the principle of action and reaction. Therefore the ring will exert an opposite torque in the spherical shell which will try to decrease its changing rate of rotation. This effect is analogous to the mutual inductance of classical electromagnetic theory.


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