Workshop on
Stochastic Processes and Random Trees

Program and abstracts

October 11–12, 2018
TU Dresden, Germany

Program

Thursday, October 11

09:00–09:45	Rudolf Grübel: From algorithms to arbotons
09:45–10:30	Zakhar Kabluchko: Edgeworth expansions for profiles of random trees
10:30–11:00	coffee break
11:00–11:45	Jan Swart: Recursive tree processes and the mean-field limit of stochastic flows
11:45–12:30	Anita Winter: Aldous Markov chain on cladograms
12:30–14:00	lunch break
14:00–15:30	open problems session
15:30–16:00	coffee break
16:00–16:45	Cornelia Pokalyuk: A host-parasite model for the human cytomegalovirus
16:45–17:30	Amaury Lambert: Nested coalescents and coagulation-transport equations

Friday, October 12

09:00–09:45	Nina Gantert: Large deviations for the maximum of a branching random walk
09:45–10:30	Jan Nagel: Random walk on a barely supercritical branching random walk
10:30–11:00	Coffee break
11:00–11:45	Martin Byrenheid: On the maximum degree in preferential attachment trees
11:45–12:30	Nicolas Broutin: Limits of inhomogeneous random graphs
12:30–14:00	Lunch break
14:00-14:45	Steffen Dereich: Quasi-processes for branching Markov chains
14:45–15:30	Peter Mörters: Branching with reinforcement
15:30–16:00	Coffee break

Abstracts

Trees: From algorithms to arbotons

Rudolf Grübel

Leibniz Universität Hannover

We briefly review the description of large graphs and permutations by graphons and permutons respectively and then discuss these in the context of Markov chain boundary theory. We then turn to binary trees and show that two different analogous concepts (for arbotons, if one so wishes) result, depending on the underlying algorithm. First, trees that arise from the binary search tree algorithm lead to the set of diffuse probability measures on the set of infinite 0-1 sequences. Second, with Rémy's algorithm for generating uniform random binary trees, there are several representations of the boundary, one of them reminiscent of graphons.

I will concentrate on ideas. Details can be complicated and are (mostly) given in the papers listed below.

1. S. N. Evans, R. Grübel, A. Wakolbinger (2012) Trickle-down processes and their boundaries. *Electronic Journal of Probability* 17, 1-58

2. S. N. Evans, R. Grübel, A. Wakolbinger (2017) Doob-Martin boundary of Rémy's tree growth chain. *Annals of Probability* 45, 225-277

3. J. Gerstenberg (2018) Exchangeable interval hypergraphs and limits of ordered discrete structures. *arXiv*:1802.09015

4. R. Grübel (2015) Persisting randomness in randomly growing discrete structures: graphs and search trees. *Discrete Mathematics & Theoretical Computer Science* 18(1), 23pp.

Edgeworth expansions for profiles of random trees

Zakhar Kabluchko

WWU Münster

Binary search trees are random trees that appear, for example, in the analysis of the Quicksort algorithm. The profile of the binary search tree with n vertices is a function $k \mapsto L_n(k)$ that counts the number of vertices at distance k from the root. We prove asymptotic expansions for $L_n(k)$ as $n \to \infty$ (where k may be a function of n) by using the embedding of the binary search trees into continuous-time branching random walks. This talk is based on a joint work with A. Marynych and H. Sulzbach.

Recursive tree processes and the mean-field limit of stochastic flows

Jan Swart

The Czech Academy of Sciences

Interacting particle systems can often be constructed from a graphical representation, by applying local maps at the times of associated Poisson processes. This leads to a natural coupling of systems started in different initial states. We consider interacting particle systems on the complete graph in the mean-field limit, i.e., as the number of vertices tends to infinity. We are not only interested in the mean-field limit of a single process, but mainly in how several coupled processes behave in the limit. This naturally leads to the study of Recursive Tree Processes (RTPs), first introduced by Aldous and Bandyopadyay, which are a sort of Markov chains in which time has a tree-like structure and flows in the direction of the root, and the state of each vertex is a random function of its descendants. If the state at the root is measurable with respect to the sigma field generated by the random functions attached to all vertices, then the RTP is said to be endogenous. We illustrate the abstract theory on an example of a particle system with cooperative branching. This yields an interesting new example of a recursive tree process that is not endogenous. Joint work with Tibor Mach and Anja Sturm.

Aldous Markov chain on cladograms

Anita Winter
Universität Duisburg-Essen

In [1], Aldous investigates a symmetric Markov chain on cladograms and gives bounds on its mixing and relaxation times. The latter bound was sharpened in [2]. In this talk we encode cladograms as binary, algebraic measure trees and show that this Markov chain on cladograms with fixed number of leaves converges in distribution as the number of leaves goes to infinity. We give a rigorous construction of the limit, whose existence was conjectured by Aldous, as the solution of a well-posed martingale problem. We show that the Aldous diffusion is a Feller process with continuous paths, and the algebraic measure Brownian CRT is its unique invariant distribution. Furthermore, we consider the vector of the masses of the three subtrees connected to a sampled branch point. In the Brownian CRT, its annealed law is known to be the Dirichlet distribution. Here we give an explicit expression for the infinitesimal evolution of its quenched law under the Aldous diffusion.

[1] David Aldous, *Mixing Time for a Markov Chain on Cladograms*, Combinatorics, Probability and Computing, 2000.

[2] Jason Schweinsberg, *An $O(n^2)$ bound for the relaxation time of a Markov chain on cladograms*, Random Structures Algorithms, 2001.

[3] Wolfgang Loehr, *Leonid Mytnik and Anita Winter*, arXiv:1805.12057.

A host-parasite model for the human cytomegalovirus

Cornelia Pokalyuk
Goethe Universität Frankfurt

Inspired by DNA data of the human cytomegalovirus, we study the dynamics and the maintenance of diversity in a hierarchical model of a parasite population distributed over their hosts.

We assume that each host is infected by a large number of parasites that come in two different types. Parasites are capable to persist in their host till the host dies. Whenever a host dies it is replaced by a host that is infected by a single parasite type. The resulting decrease in diversity is counteracted by reinfections between hosts that add the missing type to a so far purely infected host. Within hosts balancing selection is assumed, which maintains diversity.

A graphical representation for the random genealogy allows us to determine the limit law for the hosts' type dynamics as the number of hosts becomes large, and identify the deterministic dynamical system that governs the host type frequencies.

Using the example of HCMV we provide evidence for the biological relevance of the model.

The talk is based on joint work with Anton Wakolbinger.

Nested coalescents and coagulation-transport equations

Amaury Lambert
Collège de France

A nested coalescent describes the dynamics of particles embedded in larger components where in a Markovian way, components may coalesce together as well as particles embedded in the same component. Such a process generates a tree within a tree, modeling for example the genealogy of individuals inside the phylogeny of species. As a first step, we characterize the nested coalescents viewed as processes with values in the partitions of $\mathbb{N}$, nested and exchangeable. Then we study the small-time behavior of the nested Kingman coalescent, which involves a Smoluchowski-type PDE with transport, whose initial condition is a 'Dirac mass in infinity'. Joint work with Emmanuel Schertzer.

Large deviations for the maximum of a branching random walk

Nina Gantert
Technische Universität München

We consider real-valued branching random walks and prove a large deviation result for the position of the rightmost particle. The position of the rightmost particle is the maximum of a collection of a random number of dependent random walks. We characterise the rate function as the solution of a variational problem. We consider the same random number of independent random walks, and show that the maximum of the branching random walk is dominated by the maximum of the independent random walks. For the maximum of independent random walks, we derive a large deviation principle as well. It turns out that the rate functions for upper large deviations coincide, but in general the rate functions for lower large deviations do not. As time permits, we also give some results about branching random walks in random environment.

The talks is based on joint work with Thomas Höfelsauer.

Random walk on a barely supercritical branching random walk

Jan Nagel

Technische Universiteit Eindhoven

The motivating question behind this project is how a random walk behaves on a barely supercritical percolation cluster, that is, an infinite percolation cluster when the percolation probability is close to the critical value. As a more tractable model, we approximate the percolation cluster by the embedding of a Galton-Watson tree into the lattice. When the random walk runs on the tree, the embedded process is a random walk on a branching random walk. Now we can consider a barely supercritical branching process conditioned on survival, with survival probability approaching zero. In this setting the tree structure allows a fine analysis of the random walk and we can prove a scaling limit for the embedded process under a nonstandard scaling. The talk is based on a joint work with Remco van der Hofstad and Tim Hulshof.

On the Maximum Degree Node in Preferential Attachment Trees

Martin Byrenheid

Technische Universität Dresden

Due to the work of Barabási and Albert, the concept of preferential attachment (PA) has become a popular field of research in the area of mathematics, physics and computer science alike. Although there are results on the asymptotic behavior of the maximum node degree of graphs grown according to preferential attachment, it remains an open question whether the growth process eventually stabilizes in the sense that there is one node that has the maximum degree ad infinitum or if the maximum degree node changes infinitely often. In this talk, I will present statistical results obtained from a simulative generation of preferential attachment trees that suggest that in the case of trees, the process eventually stabilizes. Furthermore, I will present theoretical results that serve as a starting point for further discussion.

Limits of inhomogeneous random graphs

Nicolas Broutin
Université Pierre et Marie Curie

For many models of random graphs, as one increases the density of edges, one usually observes a sudden change in structure at a critical density that depends on the model: precisely at that point, a macroscopic "giant component" containing a linear proportion of the nodes starts to emerge. The structure of the graph at the "critical" point of the phase transition does not yet contain any macroscopic component, but many large ones at an intermediate scale, that will quickly merge into a single "giant". Understanding the phase transition and the structure of the "critical" random graphs that one observes just before the birth of the giant has since the first papers of Erdos and Renyi been one of the most fascinating topics in random graphs, and more generally in models related to statistical physics.

I will try pain the big picture and describe some recent results about the scaling limits of all critical rank-one inhomogeneous random graphs. This generalizes theorems concerning the Erdos-Renyi case as well as the inhomogeneous case with "regular power law weights". Our approach relies on a novel representation which allows us to provide a criterion for compactness as well as the computation of the fractal dimensions (box-counting, Hausdorff and packing). This is joint work with T. Duquesne and M. Wang.

Quasi-processes for branching Markov chains

Steffen Dereich
WWU Münster

The concept of a quasi-process stems from probabilistic potential theory. Although the notion may not be that familiar nowadays, it is connected to various current developments in probability. For instance, interlacements as introduced by Sznitman are Poisson point processes with the intensity measure being a quasi-process. Furthermore, extensions of Markov families as recently derived for certain self-similar Markov processes are closely related to quasi-processes and entrance boundaries.

In this talk we start with a basic introduction of parts of classical potential theory and then focus on branching Markov chains. The main result will be a spine construction of a branching quasi-process.

The talk is based on joint work with Martin Maiwald (WWU Münster).

Branching with Reinforcement

Peter Mörters
Universität zu Köln

We investigate a continuous time branching process where particles have a fitness and give birth with a rate given by this fitness. At birth a particle either inherits the type of its parent or gets a new fitness sampled independently from a fixed continuous distribution μ. Particles with the same fitness form families and I present results on the size of typical and extremal families. The talk is based on joint work with Steffen Dereich, Cécile Mailler and Anna Senkevich.
