Summary of topics - lecture financial mathematics II (Module Math Ma-MMMA)

Chapter 0 (Introduction & Motivation): Brownian Motion (BM), Black-Scholes model, BM as Gaussian process, quadratic + total variation, quadratic variation of BM. Basic stochastic calculus: stochastic integration wrt BM, Ito's formula, Ito process, Ito's formula for Ito processes, Ito-Isometry + heuristic derivations. Multivariate generalizations of Ito processes and Ito's formula, product rule for Ito processes. Local martingales.

Chapter 1 (Financial Market Models and Pricing of Derviatives): financial market model, numeraire, discounting, portfolio process + value process, self-financing condition, value process and discounted value process as stochastic integrals, arbitrage and related theorem ('All locally riskless arbitrage-free portfolios grow at the same rate').

Chapter 2 (Local volatility models): Definition local volatility model, pricing PDE and sketch of derivation, Examples: Black-Scholes, CEV. Calibration problem, Tanaka's formula, local time, derivation of Dupire's formula.

Chapter 3 (Stochastic volatility models): Stochastic Volatility model, pricing PDE, market price of risk. Examples: Heston model, Hull-White model, Stein-Stein model, SABR-model. Properties of Ornstein-Uhlenbeck and Cox-Ingersoll-Ross (CIR) process. Deriving the distributional law of the CIR process, Riccati equations. Fourier-pricing in the Heston model, exponential-affine ansatz.

Chapter 4 (Risk-neutral pricing and FTAPs): Change of measure, Radon-Nikodym theorem, Change of measure on a filtered probability space, density process. Girsanov's theorem for Brownian motion, Girsanov's theorem for changing the drift of an Ito process, Novikov's condition. Martingale representation theorem in L2 and L1.
Definitions: equivalent (local) martingale measure, market price of risk. Existence of a unique ELMM in a one-dim market model. First fundamental theorem of asset pricing in the multivariate setting. Market completeness and second fundamental theorem of asset pricing. Risk neutral pricing of derivatives.

Chapter 5: (Optimal Stopping and pricing of american options): Definition: optimal stopping problem and its value process. Value process as Snell Envelope. Characterization of optimal stopping times. Characterization of minimal optimal stopping time. Application to

American options. Free-boundary PDE for American option pricing, continuation region and exercise region.

Chapter 6 (Numerical methods in mathematical finance): Uniform random number generation, (pseudo-)random number generator, properties of good random number generator, linear congruential generator. Non-uniform random number generation, transformation lemma, acceptance-rejection method and related theorem.
Monte-Carlo-Method: Basic concept and probabilistic error bounds, role of variance, order of convergence. Curse of dimensionality and comparison to deterministic methods. Variance reduction by antithetic variables, control variates and importance sampling.
Numerical Methods for Stoch. Differential Equations (SDEs). Definition numerical scheme, weak and strong order of convergence. Euler-Maruyama scheme, its weak and strong order of convergence. Milstein scheme (incl. weak and strong order). Combined error estimates for Euler-Monte-Carlo, multilevel Monte-Carlo