Erratum to: Superconvergence Using Pointwise Interpolation in Convection-Diffusion Problems

Sebastian Franz*

March 3, 2023

The postprocessing operator $\hat{P}_{vec}$ defined on page 141 is not unisolvent for $p = 2$ and for $p \geq 3$ the condition should read

$$\int_{-1}^{1} (\widehat{P}_{vec}\hat{v} - \hat{v})p = 0, \quad p \in \mathcal{P}_{p-3}[-1,1] \setminus \mathbb{R},$$

instead of

$$\int_{-1}^{1} (\widehat{P}_{vec}\hat{v} - \hat{v})p = 0, \quad p \in \mathcal{P}_{p-2}[-1,1] \setminus \mathbb{R},$$

which is the operator given in [1].

A more systematic choice is the following. Let us order the $2p + 1$ functionals of the one-dimensional version of π_p^N on $[x_{i-1}, x_i]$ and $[x_i, x_{i+1}]$ as follows

$$\begin{aligned}
N_0(v) &:= v(x_{i-1}), & N_{j+1}(v) &:= \int_{x_{i-1}}^{x_i} v(x) \left(\frac{x - x_{i-1}}{h_{i-1}}\right)^j \mathrm{d}x,\ j \in \{0, \dots, p-2\}, \\
N_p(v) &:= v(x_i), & N_{2p-j-1}(v) &:= \int_{x_i}^{x_{i+1}} v(x) \left(\frac{x - x_i}{h_i}\right)^j \mathrm{d}x,\ j \in \{0, \dots, p-2\}, \\
N_{2p}(v) &:= v(x_{i+1}).
\end{aligned}$$

Then we define the postprocessing operator $P_{vec} : C[x_{i-1}, x_{i+1}] \to \mathcal{P}_{p+1}[x_{i-1}, x_{i+1}]$ locally by the $p + 2$ functionals

$$N_0(P_{vec}v - v) = 0, \quad N_{1+2j}(P_{vec}v - v) = 0,\ j \in \{0, \dots, p-1\}, \quad N_{2p}(P_{vec}v - v) = 0$$

The unisolvence of P_{vec} follows immediately from the unisolvence of π_p^N, i.e. the linear independence of N_j, $j \in \{0, \dots, 2p\}$. By using the tensor product structure we obtain the full postprocessing operator $P_{vec,M} : C(M) \to \mathcal{Q}_{p+1}(M)$.

*Institute of Scientific Computing, Technische Universität Dresden, Germany. e-mail: sebastian.franz@tu-dresden.de

References

[1] L. Tobiska. Analysis of a new stabilized higher order finite element method for advection-diffusion equations. *Comput. Methods Appl. Mech. Engrg.*, 196:538–550, 2006.