Charge and spin supercurrents in triplet superconductor-ferromagnet-singlet superconductor Josephson junctions

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(Received 7 May 2013; published 20 August 2013)

We study the Josephson effect in a triplet superconductor-ferromagnet-singlet superconductor junction. We show that the interaction of tunneling Cooper pairs with the interface magnetization can permit a Josephson current at the lowest order of a tunneling Hamiltonian perturbation theory. Two conditions must be satisfied for this to occur: The magnetization of the ferromagnet has a component parallel to the **d** vector of the triplet superconductor, and the gaps of the superconductors have the same parity with respect to the interface momentum. The resulting charge current displays an unconventional dependence on the orientation of the magnetic moment and the phase difference. This is accompanied by a phase-dependent spin current in the triplet superconductor, while a phase-independent spin current is always present. The tunneling perturbation theory predictions are confirmed using a numerical Green's function method. An analytical treatment of a one-dimensional junction demonstrates that our conclusions are robust far away from the tunneling regime and reveals signatures of the unconventional Josephson effect in the critical currents.

DOI: 10.1103/PhysRevB.88.054509 PACS number(s): 74.50.+r, 74.20.Rp

I. INTRODUCTION

The physics of spin-triplet superconductors (TSs) is much richer than their spin-singlet superconductor (SS) counterparts due to the spin degree of freedom of a triplet Cooper pair. Although a bewildering variety of triplet pairing states are, in principle, possible for any crystal symmetry,[1] there are only a handful of systems where a TS state has been well established, e.g., UPt_3 and Sr_2RuO_4 (Refs. 2–5). Even in these cases, many questions remain about the precise form of the TS order parameter.[3,5,6] Much effort has therefore been directed at developing experimental tests capable of unambiguously identifying a TS. A promising route is to incorporate the candidate TS into a heterostructure device and search for signatures of the odd-parity orbital state in tunneling measurements.[7,8] Alternatively, the spin part of the Cooper pair wave function can be probed by bringing the TS into contact with a ferromagnet (FM). This is manifested by the crucial role of the relative orientation between the vector order parameters of the TS and the FM, the **d** vector and magnetization, respectively, in controlling the physics of the device.[9–13]

Theoretical investigations of the Josephson effect between TSs have revealed many remarkable consequences of their intrinsic spin structure. For example, the spin of the Cooper pair permits the existence of a Josephson spin current between TSs with misaligned **d** vectors,[14,15] similar to the spin supercurrent observed in superfluid ^{3}He (Refs. 16 and 17). Another notable proposal is the TS-FM-TS (TFT) junction,[18–21] where the coupling of the Cooper pairs' spin with the exchange field in the barrier causes a sign reversal of the current as the orientation of the exchange field with respect to the **d** vectors is varied. This is in stark contrast to the well-known 0-π transition in magnetic junctions between SSs,[22–24] which is independent of the orientation of the barrier magnetization. Such an anomalous 0-π transition would therefore be extremely strong evidence of the triplet state of the superconductors. On the other hand, creating a TFT junction from any of the TS candidate materials is experimentally challenging due to the high purity requirements needed for the TS. Even with recent success in growing superconducting thin films of Sr_2RuO_4 (Ref. 25), such devices will likely remain hypothetical for some time, due to the greater challenges posed by growing a layered heterostructure. More immediately plausible is to create a TS-FM-SS (TFS) junction by coating the Sr_2RuO_4 thin film with a ferromagnetic layer and then contacting to a conventional superconductor.

For junctions between superconductors of like parity, the lowest harmonic in the Josephson current vs phase difference ϕ relation is usually $\sin(\phi)$, which originates from tunneling processes involving only a single Cooper pair. This term is necessarily absent in a nonmagnetic Josephson junction between an SS and a TS, however, due to the orthogonal spin pairing states.[26] Instead, the singlet-triplet conversion can occur only in processes involving the coherent tunneling of even numbers of Cooper pairs, and so $\sin(2\phi)$ is the leading harmonic in the current vs phase relation.[27–29] The coupling between a single tunneling Cooper pair and the magnetic degrees of freedom in a magnetically active barrier, on the other hand, can accomplish the conversion between singlet and triplet spin states,[23] hence generating a lowest-order Josephson coupling in the sense discussed above. An example of such a magnetic interaction is the intrinsic spin-orbit coupling expected to occur at the junction interface.[30–34] This has been proposed as the origin of the unexpectedly large Josephson currents in junctions between single crystals of Sr_2RuO_4 and conventional s-wave SSs, and the pronounced dependence on the crystal face upon which the Josephson contact is made.[35–38] In contrast, relatively little work has been done for a ferromagnetic tunneling barrier. Previous studies have found a highly unconventional Josephson charge current in a TFS junction which is *even* in the phase difference and *odd* in the component of the magnetization parallel to the **d** vector of

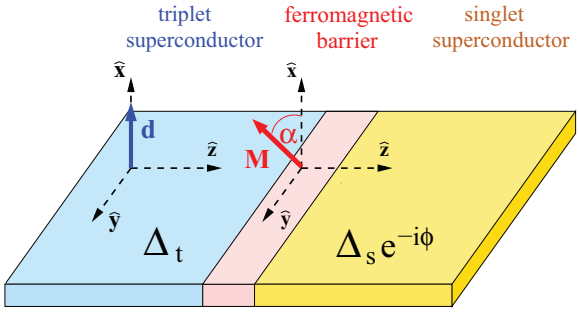

FIG. 1. (Color online) Schematic diagram of the Josephson junction considered in this paper. The **d** vector of the TS defines the x axis, and we restrict the magnetization to the x-y plane, making the angle α to the x axis. The phase difference between the TS and SS is given by ϕ.

the TS.[39–41] The origin of this current and the conditions under which it occurs nevertheless remain obscure.

It is the purpose of this paper to perform a detailed theoretical study of the Josephson effect in a TFS junction, shown schematically in Fig. 1. Similar to the TFT junction, we find highly unconventional Josephson physics which originates from the coupling of the barrier moment to the spin of the tunneling quasiparticles comprising the Cooper pairs. Using a tunneling Hamiltonian perturbation theory, we show that there is a lowest-order Josephson charge current when the orbital pairing states of the superconductors have the same parity with respect to the interface momentum, and the magnetization has a component parallel to the **d** vector of the TS. There is also a spin current in the TS, which has both a phase-dependent and a phase-independent contribution. The latter is a universal spin supercurrent which appears at TS-FM interfaces and is due to spin-dependent reflection processes.[11] We test the predictions of the perturbation theory using both lattice and continuum models of the junction. In the lattice theory we survey a wide selection of different orbital pairing states in the SS and TS. The focus of the continuum theory, on the other hand, is to understand the role of resonant tunneling through the Andreev bound states at the junction interface. Both approaches yield excellent agreement with the perturbative analysis at sufficiently high temperatures. Although deviations from perturbation theory become more severe as the temperature is lowered, it nevertheless remains qualitatively correct down to zero temperature.

This paper is organized as follows. In Sec. II we present a perturbative theory for the Josephson effect in the TFS. The predictions of this section are confirmed first in Sec. III by numerically determining the currents in a microscopic lattice model of the junction, and then by analytical calculation for a continuum model in Sec. IV. Our concluding discussion is given in Sec. V.

II. PERTURBATION THEORY

A. Hamiltonian of the junction

The Hamiltonian of the TFS junction is written

$$\mathcal{H} = \mathcal{H}_{\rm TS} + \mathcal{H}_{\rm SS} + \mathcal{H}_{\rm tun} + \mathcal{H}_{\rm ref}. \tag{1}$$

Here $\mathcal{H}_{\rm TS}$ and $\mathcal{H}_{\rm SS}$ describe the bulk TS and SS on either side of the barrier. We have

$$\mathcal{H}_{\rm TS} = \frac{1}{2}\sum_{\mathbf{k}} \Psi^\dagger_{t,\mathbf{k}} \begin{pmatrix} \epsilon_{t,\mathbf{k}}\hat{\sigma}_0 & i\mathbf{d}_{\mathbf{k}}\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_y \\ (i\mathbf{d}_{\mathbf{k}}\cdot\hat{\boldsymbol{\sigma}}\hat{\sigma}_y)^\dagger & -\epsilon_{t,\mathbf{k}}\hat{\sigma}_0 \end{pmatrix} \Psi_{t,\mathbf{k}}, \tag{2}$$

$$\mathcal{H}_{\rm SS} = \frac{1}{2}\sum_{\mathbf{k}} \Psi^\dagger_{s,\mathbf{k}} \begin{pmatrix} \epsilon_{s,\mathbf{k}}\hat{\sigma}_0 & i\Delta_{s,\mathbf{k}}e^{-i\phi}\hat{\sigma}_y \\ -i\Delta^*_{s,\mathbf{k}}e^{i\phi}\hat{\sigma}_y & -\epsilon_{s,\mathbf{k}}\hat{\sigma}_0 \end{pmatrix} \Psi_{s,\mathbf{k}}, \tag{3}$$

where $\Psi_{\nu,\mathbf{k}} = (c_{\nu,\mathbf{k},\uparrow}, c_{\nu,\mathbf{k},\downarrow}, c^\dagger_{\nu,-\mathbf{k},\uparrow}, c^\dagger_{\nu,-\mathbf{k},\downarrow})^T$ and $c_{\nu,\mathbf{k},\sigma}$ ($c^\dagger_{\nu,\mathbf{k},\sigma}$) are the fermion annihilation (creation) operators for a spin-σ quasiparticle with momentum $\mathbf{k}$ in the $\nu = s$ (t) SS (TS). The bare dispersion in each superconductor is $\epsilon_{\nu,\mathbf{k}}$. We only consider unitary equal-spin-pairing states for the TS. Without loss of generality, it is convenient to let the vector order parameter of the TS define the x axis, i.e., $\mathbf{d}_{\mathbf{k}} = \Delta_{t,\mathbf{k}}\mathbf{e}_x$. Other orientations of $\mathbf{d}_{\mathbf{k}}$ can be achieved by spin rotation of the system and do not result in new physics. The global phase difference between the TS and the SS is ϕ.

The TS and SS are connected by the tunneling Hamiltonian, which we define

$$\mathcal{H}_{\rm tun} = \sum_{\nu=s,t}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\sigma,\sigma'} T^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{\overline{\nu},\mathbf{k},\sigma} c_{\nu,\mathbf{k}',\sigma'}, \tag{4}$$

where $\overline{\nu} = s(t)$ for $\nu = t(s)$. In order to properly model the interaction of the tunneling quasiparticles with the magnetic moment of the barrier we require explicitly spin-dependent tunneling matrix elements. Following Ref. 20, we also include a reflection Hamiltonian

$$\mathcal{H}_{\rm ref} = \sum_{\nu=s,t}\sum_{\mathbf{k},\mathbf{k}'}\sum_{\sigma} R^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{\nu,\mathbf{k},\sigma} c_{\nu,\mathbf{k}',-\sigma}, \tag{5}$$

to properly account for the interaction of the quasiparticles with the FM layer. We only include spin-flip reflection processes in $\mathcal{H}_{\rm ref}$, since spin-preserving reflection processes clearly do not contribute to either the charge or the spin currents.

1. *Ansatz for tunneling and reflection matrix elements*

Our perturbation analysis crucially relies upon the form of the tunneling and reflection matrix elements. In particular, it is necessary to include the phase shift acquired by the quasiparticles during the various tunneling or reflection processes.[20] Although it is, in principle, possible to determine the tunneling and reflection matrix elements from a more fundamental Hamiltonian, here we motivate a phenomenological form by comparison with an exactly solvable scattering problem.

A common model for the tunneling barrier is a δ-function potential,[18–20,28,29,40,42] i.e., $\hat{V}(\mathbf{r}) = U_0\hat{\sigma}_0\delta(z) + \mathbf{U}_M\cdot\hat{\boldsymbol{\sigma}}\delta(z)$, where U_0 is the charge scattering potential and $\mathbf{U}_M = U_M(\cos\alpha\hat{\mathbf{e}}_x + \sin\alpha\hat{\mathbf{e}}_y)$ is the magnetic scattering potential. It is straightforward to evaluate the scattering matrix for this potential, and hence determine the transmission and reflection coefficients, $t_{\nu,\sigma,\sigma'}(\mathbf{k},\mathbf{k}')$ and $r_{\nu,\sigma,\sigma'}(\mathbf{k},\mathbf{k}')$, respectively. The transmission and spin-flip reflection coefficients are both small when the magnetic potential is large; i.e., the dimensionless parameter $g = k_F|\mathbf{U}_M|/2E_F$ is much larger than one, where k_F is the Fermi wave vector and E_F is the Fermi energy. It is reasonable to expect that in this limit we should have $T^{\sigma,\sigma'}_{\nu,\mathbf{k},\mathbf{k}'} \sim t_{\nu,\sigma,\sigma'}(\mathbf{k},\mathbf{k}')$ and $R^{\sigma,-\sigma}_{\nu,\mathbf{k},\mathbf{k}'} \sim r_{\nu,\sigma,-\sigma}(\mathbf{k},\mathbf{k}')$. By requiring

that a tunneling or reflected quasiparticle acquires the same phase as in the exact solution, we hence have an ansatz for the matrix elements,

$$T^{\sigma,\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = \frac{1}{g^2} T^{sp}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}, \tag{6a}$$

$$T^{-\sigma,\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = \frac{i\nu e^{i\sigma\alpha}}{g} T^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}, \tag{6b}$$

$$R^{-\sigma,\sigma}_{\nu,\mathbf{k},\mathbf{k}'} = \frac{i\nu e^{i\sigma\alpha}}{g} R^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}, \tag{6c}$$

where $\nu = -1$ $(+1)$ as a factor for $\nu = t$ (s) as a subscript. Crucially for our analysis, the phase shift acquired during spin-flip tunneling or reflection depends on the initial spin σ and the angle α of the magnetic moment in the x-y plane (see Fig. 1). We assume that the $T^{sp}(k_z,k'_z)$, $T^{sf}(k_z,k'_z)$, and $R^{sf}(k_z,k'_z)$ are real functions, independent of g, and satisfy

$$T^{sp}(k_z,k'_z) = T^{sp}(-k_z,-k'_z) = T^{sp}(k'_z,k_z), \tag{7a}$$

$$T^{sf}(k_z,k'_z) = -T^{sf}(-k_z,-k'_z) = T^{sf}(k'_z,k_z), \tag{7b}$$

$$R^{sf}(k_z,k'_z) = -R^{sf}(-k_z,-k'_z) = -R^{sf}(k'_z,k_z). \tag{7c}$$

These conditions originate from both the comparison to the scattering coefficients and the requirement that $\mathcal{H}_{\rm tun}$ and $\mathcal{H}_{\rm ref}$ be Hermitian. Note that the $\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}$ in Eq. (6) ensures the conservation of momentum parallel to the barrier, a consequence of the translational invariance along the interface.[43]

Although we have motivated our ansatz Eq. (6) by comparison with the δ-function barrier, we expect our approach to be of more general validity.[21] In particular, the spin-dependent phase shifts acquired by the tunneling quasiparticles should be robust to other choices of barrier model, since they are a consequence of the orientation of the magnetization.

B. Perturbation theory

The number operator for particles in each spin sector of the two superconductors is given by $N_{\nu,\sigma} = \sum_{\mathbf{k}} c^\dagger_{\nu,\mathbf{k},\sigma} c_{\nu,\mathbf{k},\sigma}$. From this we define the associated particle currents

$$I^\nu_\sigma = -\nu\langle \partial_t N_{\nu,\sigma}\rangle. \tag{8}$$

We proceed by expanding the S matrix to lowest order in $\mathcal{H}' = \mathcal{H}_{\rm tun} + \mathcal{H}_{\rm ref}$, hence treating the tunneling and reflection processes as a perturbation of the Hamiltonian $\mathcal{H}_0 = \mathcal{H}_{\rm TS} + \mathcal{H}_{\rm SS}$ (Refs. 20 and 44). This is justified so long as the tunneling and reflection matrix elements are small, which by our ansatz [Eq. (6)] holds in the $g \gg 1$ limit. By the Kubo formula, we then have

$$I^\nu_\sigma = -i\nu \int_{-\infty}^{t} dt' \langle [\partial_t N_{\nu,\sigma}(t), \mathcal{H}'(t')]\rangle, \tag{9}$$

where the time dependence is given within the interaction picture, i.e., $\mathcal{O}(t) = e^{i\mathcal{H}_0 t}\mathcal{O}e^{-i\mathcal{H}_0 t}$, with $\mathcal{H}_0 = K_0 + \sum_\nu \mu_\nu \sum_\sigma N_{\nu,\sigma}$ and K_0 is the associated grand canonical Hamiltonian. Since we are only interested in the dc Josephson effect, we take the same chemical potential in both the TS and the SS, i.e., $\mu_s = \mu_t = \mu$. We hence find for the terms in the commutator in Eq. (9),

$$\partial_t N_{\nu,\sigma}(t) = i\left\{B^{-\sigma,\sigma}_\nu(t) - B^{\sigma,-\sigma}_\nu(t)\right\} + i\sum_\varsigma \left\{A^{\varsigma,\sigma}_\nu(t) - A^{\sigma,\varsigma}_{\bar\nu}(t)\right\}, \tag{10}$$

$$\mathcal{H}'(t) = \sum_\nu \sum_\varsigma B^{\varsigma,-\varsigma}_\nu(t) + \sum_\nu \sum_{\varsigma,\varsigma'} A^{\varsigma,\varsigma'}_\nu(t), \tag{11}$$

where we introduce the operators

$$A^{\varsigma,\varsigma'}_\nu(t) = \sum_{\mathbf{k},\mathbf{k}'} T^{\varsigma,\varsigma'}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{\bar\nu,\mathbf{k},\varsigma}(t) c_{\nu,\mathbf{k}',\varsigma'}(t), \tag{12}$$

$$B^{\varsigma,\varsigma'}_\nu(t) = \sum_{\mathbf{k},\mathbf{k}'} R^{\varsigma,\varsigma'}_{\nu,\mathbf{k},\mathbf{k}'} c^\dagger_{\nu,\mathbf{k},\varsigma}(t) c_{\nu,\mathbf{k}',\varsigma'}(t). \tag{13}$$

The time dependence of the fermion operators in these expressions is given by $c_{\nu,\mathbf{k},\sigma}(t) = e^{iK_0 t} c_{\nu,\mathbf{k},\sigma} e^{-iK_0 t}$.

Following standard arguments,[15,20,44] we write the current Eq. (9) as

$$I^\nu_\sigma = 2\nu \mathrm{Im}\left\{\Phi^{\rm ret}_{\nu,\sigma}(\omega=0) + \Psi^{\rm ret}_{\nu,\sigma}(\omega=0)\right\}, \tag{14}$$

where the retarded correlation functions $\Phi^{\rm ret}_{\nu,\sigma}(\omega)$ and $\Psi^{\rm ret}_{\nu,\sigma}(\omega)$ give the contributions from tunneling and reflection processes, respectively. They are obtained by analytic continuation $i\omega_n \to \omega + i0^+$ of the corresponding Matsubara functions,

$$\Phi_{\nu,\sigma}(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau} \sum_{\varsigma,\varsigma',\varsigma''} \left\langle T_\tau A^{\varsigma,\sigma}_\nu(\tau) A^{\varsigma',\varsigma''}_\nu(0)\right\rangle, \tag{15}$$

$$\Psi_{\nu,\sigma}(i\omega_n) = \int_0^\beta d\tau e^{i\omega_n\tau} \left\langle T_\tau B^{-\sigma,\sigma}_\nu(\tau) B^{-\sigma,\sigma}_\nu(0)\right\rangle. \tag{16}$$

The Matsubara functions are evaluated by using Wick's theorem to expand the two-particle correlators. We hence find the particle currents in the TS,

$$I^t_\sigma = -\frac{1}{g^2}\sum_{\mathbf{k},\mathbf{k}'} [R^{sf}(k_z,k'_z)]^2 \delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Im}\left\{e^{2i\sigma\alpha}\frac{\Delta^*_{t,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{t,\mathbf{k}}E_{t,\mathbf{k}'}}\right\} F_{t,t}(\mathbf{k},\mathbf{k}') - \frac{2}{g^3}\sum_{\mathbf{k},\mathbf{k}'} T^{sp}(k_z,k'_z)T^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Re}\left\{e^{i(\phi+\sigma\alpha)}\frac{\Delta^*_{s,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{s,\mathbf{k}}E_{t,\mathbf{k}'}}\right\} F_{s,t}(\mathbf{k},\mathbf{k}'), \tag{17}$$

and the SS,

$$I^s_\sigma = \frac{2\cos(\alpha)}{g^3}\sum_{\mathbf{k},\mathbf{k}'} T^{sp}(k_z,k'_z)T^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Re}\left\{e^{i\phi}\frac{\Delta^*_{s,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{s,\mathbf{k}}E_{t,\mathbf{k}'}}\right\} F_{s,t}(\mathbf{k},\mathbf{k}'). \tag{18}$$

Here we utilize the convenient shorthand notation[15]

$$F_{\nu,\nu'}(\mathbf{k},\mathbf{k}') = \frac{f(E_{\nu,\mathbf{k}}) - f(E_{\nu',\mathbf{k}'})}{E_{\nu,\mathbf{k}} - E_{\nu',\mathbf{k}'}} + \frac{1 - f(E_{s,\mathbf{k}}) - f(E_{t,\mathbf{k}'})}{E_{s,\mathbf{k}} + E_{t,\mathbf{k}'}}, \tag{19}$$

where

$$E_{\nu,\mathbf{k}} = \sqrt{(\epsilon_{\nu,\mathbf{k}} - \mu)^2 + |\Delta_{\nu,\mathbf{k}}|^2} \tag{20}$$

is the dispersion in the ν superconductor and $f(E)$ is the Fermi distribution function. Equations (17) and (18) are the central results of our perturbation theory analysis, as they give all lowest-order contributions to the particle current in each superconductor. Note that in the TS we have a contribution from reflection processes, whereas in the SS only tunneling processes contribute to the current. The lowest-order (i.e., single-Cooper-pair) tunneling processes involve both a spin-preserving and a spin-flip tunneling event, which is necessary to transform the spin-singlet Cooper pairs of the SS into the $S_z = \pm\hbar$ triplet Cooper pairs of the TS and vice versa. The particle current in the TS [Eq. (17)] depends on the spin σ through the spin-dependent phase shifts acquired during the scattering; in contrast, the particle current in the SS [Eq. (18)] is the same for each spin orientation, as required by the singlet pairing state.

The important role of spin-flip tunneling implies a transfer of spin to the FM tunneling barrier. In our calculation, however, we regard the magnetic moment of the FM barrier to have fixed magnitude and direction. Including the response of the FM to the injected spin current is a challenging problem, requiring a nonequilibrium treatment that is beyond the scope of the current paper.

C. Charge and spin currents

We can use either Eq. (17) or Eq. (18) to calculate the Josephson charge current $I_c = -e(I^\nu_\uparrow + I^\nu_\downarrow)$, as by charge conservation this is the same in each superconductor. We hence find

$$I_c = 4e\frac{\cos(\alpha)}{g^3}\sum_{\mathbf{k},\mathbf{k}'} T^{sp}(k_z,k'_z)T^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Re}\left\{e^{i\phi}\frac{\Delta^*_{s,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{s,\mathbf{k}}E_{t,\mathbf{k}'}}\right\}F_{s,t}(\mathbf{k},\mathbf{k}'). \quad (21)$$

The charge current depends strongly upon the orientation of the magnetic moment through the $\cos(\alpha)$ factor. This implies that reversing the direction of the barrier moment also reverses the sign of the current, as was previously observed in Ref. 40. The origin of this factor is the interference of the particle currents in each spin sector of the TS, which are phase-shifted with respect to one another by $\pm 2\alpha$ as a consequence of the spin-flip tunneling; see the second term in Eq. (17).

We extract the current vs phase relationship by examining the summand in Eq. (21). In order to have a lowest-order Josephson effect, we require that the product $T^{sp}(k_z,k'_z)T^{sf}(k_z,k'_z)\Delta^*_{s,\mathbf{k}}\Delta_{t,\mathbf{k}'}\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel}$ not be odd in any component of $\mathbf{k}$ or $\mathbf{k}'$. From Eq. (7) we hence deduce that the gaps $\Delta^*_{s,\mathbf{k}}$ and $\Delta_{t,\mathbf{k}'}$ have opposite parity with respect to the z component of the wave vector; equivalently, the gaps must have the same parity with respect to the interface momentum. For an s-wave SS, therefore, there is a Josephson current,

$$I_c \propto \frac{1}{g^3}\cos(\alpha)\cos(\phi), \quad (22)$$

when the TS has p_z-wave symmetry. The proportionality constant is determined by the details of the junction, such as the normal-state dispersion or the structure of the interface. On the other hand, Eq. (21) is vanishing for a p_y-wave TS, and so a Josephson current appears only in the next order of perturbation theory. The current vs phase relation is then $I_c \propto \sin(2\phi)$, as for the nonmagnetic barrier.[26] A list of the Josephson charge current vs phase relationships predicted by Eq. (21) for different combinations of orbital pairing states is given in Table I.

The spin current is present only in the TS, as the singlet Cooper pairs do not carry spin. The spin current is polarized along the z axis and is given by

$$\begin{aligned} I_{s,z} &= \frac{\hbar}{2}(I^t_\uparrow - I^t_\downarrow) \\ &= -\hbar\frac{\sin(2\alpha)}{g^2}\sum_{\mathbf{k},\mathbf{k}'}[R^{sf}(k_z,k'_z)]^2\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Re}\left\{\frac{\Delta^*_{t,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{t,\mathbf{k}}E_{t,\mathbf{k}'}}\right\}F_{t,t}(\mathbf{k},\mathbf{k}') \\ &\quad -\hbar\frac{2\sin(\alpha)}{g^3}\sum_{\mathbf{k},\mathbf{k}'}T^{sp}(k_z,k'_z)T^{sf}(k_z,k'_z)\delta_{\mathbf{k}_\parallel,\mathbf{k}'_\parallel} \times \mathrm{Im}\left\{e^{i\phi}\frac{\Delta^*_{s,\mathbf{k}}\Delta_{t,\mathbf{k}'}}{E_{s,\mathbf{k}}E_{t,\mathbf{k}'}}\right\}F_{s,t}(\mathbf{k},\mathbf{k}'). \end{aligned} \quad (23)$$

The first term in Eq. (23) is the spin current due to spin-flip reflection from the interface, where a spin $S_z = \pm\hbar$ Cooper pair is reflected as a $S_z = \mp\hbar$ Cooper pair.[11,20,21] As this involves two spin-flip reflection events, the Cooper pair acquires a phase shift of $\pm 2\alpha$ and the magnitude of this term is $\sim g^{-2}$. Furthermore, due to its origin in the reflection processes, it is sensitive to the orbital structure of the triplet gap; e.g., the sign reversal of the p_z-wave gap upon specular reflection gives the Cooper pairs an additional π phase shift relative to the p_y-wave case, and there is hence a sign difference in the respective reflection spin currents. In contrast, the second term

TABLE I. Table showing the Josephson charge and spin currents appearing in the lowest order of our perturbation theory for different combinations of TS and SS gap symmetries. In the interest of brevity we restrict ourselves to gaps lying in the y-z plane. The Josephson current amplitudes are expected to satisfy $I_c \sim g^{-3}$, $I_{s,r} \sim g^{-2}$, $I_{s,t} \sim g^{-3}$ in the tunneling limit $g \gg 1$. The numerical values of these terms depend upon the details of the junction, e.g., the normal-state dispersion, the structure factor of the gaps, the properties of the tunneling region, etc.

TS gap symmetry	SS gap symmetry	Charge current [Eq. (21)]	Spin current [Eq. (23)]
p_z, $p_z + ip_y$	s, $d_{y^2-z^2}$	$I_c\cos(\alpha)\cos(\phi)$	$I_{s,r}\sin(2\alpha) + I_{s,t}\sin(\alpha)\sin(\phi)$
p_y	s, $d_{y^2-z^2}$	0	$I_{s,r}\sin(2\alpha)$
p_z	d_{yz}	0	$I_{s,r}\sin(2\alpha)$
p_y	d_{yz}	$I_c\cos(\alpha)\cos(\phi)$	$I_{s,r}\sin(2\alpha) + I_{s,t}\sin(\alpha)\sin(\phi)$
$p_z + ip_y$	d_{yz}	$I_c\cos(\alpha)\sin(\phi)$	$I_{s,r}\sin(2\alpha) + I_{s,t}\sin(\alpha)\cos(\phi)$

in Eq. (23) originates from the interference of the spin-↑ and -↓ tunneling particle currents, similar to the charge current, and is $\sim g^{-3}$. We therefore expect that the reflection spin current dominates the tunneling spin current in the tunneling limit $g \gg 1$. Whereas the reflection spin current is always present, the condition for the tunneling term in Eq. (23) to be nonzero is the same as for a finite lowest-order charge current. Again considering the case of an s-wave SS, we thus find that for a p_z-wave TS there is the spin current

$$I_{s,z} \propto \frac{\gamma}{g^2}\sin(2\alpha) + \frac{1}{g^3}\sin(\alpha)\sin(\phi), \tag{24}$$

where γ is a numerical constant; for a p_y-wave TS, in contrast, only the first term is present, and has opposite sign. The Josephson spin currents are given in Table I for various combinations of orbital pairing states.

III. LATTICE MODEL OF THE JUNCTION

In this section we test the predictions of the perturbation theory by calculating the charge and spin Josephson currents in the tunneling regime of a two-dimensional microscopic lattice model of the TFS junction. Using the recursive Green's function method,[45,46] we examine representative examples of the different orbital combinations listed in Table I.

A. Formulation

We write the Hamiltonian describing the TFS junction in real space as

$$H = \frac{1}{2}\sum_{\mathbf{r},\mathbf{r}'} \Psi^\dagger(\mathbf{r}) \begin{pmatrix} \hat{H}_0(\mathbf{r},\mathbf{r}') & \hat{\Delta}(\mathbf{r},\mathbf{r}') \\ -\hat{\Delta}^*(\mathbf{r},\mathbf{r}') & -\hat{H}_0^*(\mathbf{r},\mathbf{r}') \end{pmatrix} \Psi(\mathbf{r}'), \tag{25}$$

where $\Psi(\mathbf{r}) = (\psi_\uparrow(\mathbf{r}),\psi_\downarrow(\mathbf{r}),\psi_\uparrow^\dagger(\mathbf{r}),\psi_\downarrow^\dagger(\mathbf{r}))^T$ is the vector of field operators and the sum in Eq. (25) is over the lattice sites. For the normal-state Hamiltonian $\hat{H}_0(\mathbf{r},\mathbf{r}')$ we assume a two-dimensional square lattice tight-binding model in the y-z plane,

$$\hat{H}_0(\mathbf{r},\mathbf{r}') = [-t\{\delta_{\mathbf{r},\mathbf{r}'+\mathbf{z}} + \delta_{\mathbf{r},\mathbf{r}'-\mathbf{z}} + \delta_{\mathbf{r},\mathbf{r}'+\mathbf{y}} + \delta_{\mathbf{r},\mathbf{r}'-\mathbf{y}}\} + \mu\delta_{\mathbf{r},\mathbf{r}'}]\hat{\sigma}_0 + \mathbf{U}_M(\mathbf{r})\cdot\hat{\boldsymbol{\sigma}}\,\delta_{\mathbf{r},\mathbf{r}'}. \tag{26}$$

The vectors are represented as $\mathbf{r} = n\mathbf{z} + m\mathbf{y}$, where $\mathbf{z}$ and $\mathbf{y}$ are the unit vectors of the tight-binding lattice in the z and y directions, respectively. In the y direction, we apply the periodic boundary condition. The exchange potential $\mathbf{U}_M(\mathbf{r})$ is only nonzero in the ferromagnetic barrier region of the junction. A sketch of the lattice model is shown in Fig. 2. The pair potentials for the different pairing symmetries considered here have the following real-space forms:

$$\hat{\Delta}(\mathbf{r},\mathbf{r}') = \begin{cases} \Delta e^{-i\phi}\delta_{\mathbf{r},\mathbf{r}'} i\hat{\sigma}_y, & s \text{ wave}, \\ (\Delta e^{-i\phi}/2) i\hat{\sigma}_y[-\delta_{\mathbf{r},\mathbf{r}'+\mathbf{z}+\mathbf{y}} - \delta_{\mathbf{r},\mathbf{r}'-\mathbf{z}-\mathbf{y}} + \delta_{\mathbf{r},\mathbf{r}'+\mathbf{z}-\mathbf{y}} + \delta_{\mathbf{r},\mathbf{r}'-\mathbf{z}+\mathbf{y}}], & d_{zy} \text{ wave}, \\ i(\Delta/2)\hat{\sigma}_z[\delta_{\mathbf{r},\mathbf{r}'+\mathbf{z}} - \delta_{\mathbf{r},\mathbf{r}'-\mathbf{z}}], & p_z \text{ wave}, \\ i(\Delta/2)\hat{\sigma}_z[\delta_{\mathbf{r},\mathbf{r}'+\mathbf{y}} - \delta_{\mathbf{r},\mathbf{r}'-\mathbf{y}}], & p_y \text{ wave}, \\ (\Delta/2)\hat{\sigma}_z[i\delta_{\mathbf{r},\mathbf{r}'+\mathbf{z}} - i\delta_{\mathbf{r},\mathbf{r}'-\mathbf{z}} - \delta_{\mathbf{r},\mathbf{r}'+\mathbf{y}} + \delta_{\mathbf{r},\mathbf{r}'-\mathbf{y}}], & p_z + ip_y \text{ wave}. \end{cases} \tag{27}$$

The pairing potentials for the TS assume that $\mathbf{d}$ is directed along the x axis in spin space.

The charge and the spin densities in the nth column along the z direction are defined, respectively, by

$$\rho(n) = e\sum_{m=1}^{M}\sum_{\alpha}\psi_\alpha^\dagger(\boldsymbol{r})\psi_\alpha(\mathbf{r}), \tag{28}$$

$$\mathbf{s}(n) = \frac{\hbar}{2}\sum_{m=1}^{M}\sum_{\alpha,\beta}\psi_\alpha^\dagger(\mathbf{r})\hat{\boldsymbol{\sigma}}_{\alpha,\beta}\psi_\beta(\mathbf{r}). \tag{29}$$

From the equations of motion,

$$\partial_t\rho(n) = \frac{i}{\hbar}[H,\rho(n)], \tag{30}$$

$$\partial_t\mathbf{s}(n) = \frac{i}{\hbar}[H,\mathbf{s}(n)], \tag{31}$$

we derive the current conservation laws,

$$\partial_t\rho(n) = -I_{ce}(n) + I_{ce}(n-1) - S_{cd}(n), \tag{32}$$

$$\partial_t\mathbf{s}(n) = -\mathbf{I}_{se}(n) + \mathbf{I}_{se}(n-1) - \mathbf{S}_{sd}(n) - \mathbf{S}_v(n). \tag{33}$$

On the right-hand side of these equations we have terms that can be interpreted as the divergence of a current. The

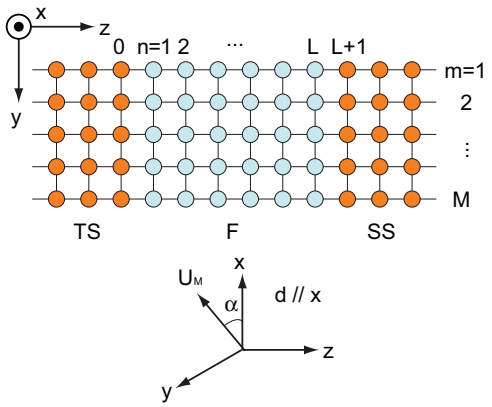

FIG. 2. (Color online) Schematic diagram of the two-dimensional lattice model of the TFS junction. The length of the ferromagnetic layer and the junction width are L and M unit cells, respectively. The on-site magnetic potential $\mathbf{U}_M(\mathbf{r})$ in the ferromagnetic layer is oriented in the x-y plane.

first of these terms are the familiar kinetic currents which originate from the commutator of the densities with the hopping Hamiltonian:

$$I_{ce}(n) = \frac{iet}{\hbar}\sum_{m=1}^{M}\sum_{\alpha}[\psi_{\alpha}^{\dagger}(\mathbf{r}+\mathbf{z})\psi_{\alpha}(\mathbf{r}) - \psi_{\alpha}^{\dagger}(\mathbf{r})\psi_{\alpha}(\mathbf{r}+\mathbf{z})], \tag{34}$$

$$\mathbf{I}_{se}(n) = \frac{it}{2}\sum_{m=1}^{M}\sum_{\alpha,\beta}[\psi_{\alpha}^{\dagger}(\mathbf{r}+\mathbf{z})\hat{\sigma}_{\alpha,\beta}\psi_{\beta}(\mathbf{r}) - \psi_{\alpha}^{\dagger}(\mathbf{r})\hat{\sigma}_{\alpha,\beta}\psi_{\beta}(\mathbf{r}+\mathbf{z})]. \tag{35}$$

The remaining terms in Eqs. (32) and (33) are the source terms. Specifically, $S_{cd}(n)$ is the source term for the electric current due to the pair potential, $\mathbf{S}_{sd}(n)$ is the source term for the spin current due to the pair potential, and $\mathbf{S}_{v}(n)$ is the source term for the spin current due to the exchange potential. They are represented by

$$S_{cd}(n) = \frac{-ie}{\hbar}\sum_{m=1}^{M}\sum_{\alpha,\beta}\sum_{\mathbf{r}'}[\psi_{\alpha}^{\dagger}(\mathbf{r})\psi_{\beta}^{\dagger}(\mathbf{r}')\hat{\Delta}_{\beta,\alpha}(\mathbf{r}',\mathbf{r}) + \psi_{\alpha}(\mathbf{r})\psi_{\beta}(\mathbf{r}')\hat{\Delta}_{\beta,\alpha}^{*}(\mathbf{r}',\mathbf{r})], \tag{36}$$

$$\mathbf{S}_{sd}(n) = \frac{-i}{2}\sum_{m=1}^{M}\sum_{\alpha,\beta,\lambda}\sum_{\mathbf{r}'}[\psi_{\alpha}^{\dagger}(\mathbf{r})\psi_{\lambda}^{\dagger}(\mathbf{r}')\hat{\Delta}_{\lambda,\beta}(\mathbf{r}',\mathbf{r})\boldsymbol{\sigma}_{\beta,\alpha}^{*} + \psi_{\alpha}(\mathbf{r})\psi_{\lambda}(\mathbf{r}')\hat{\Delta}_{\lambda,\beta}^{*}(\mathbf{r}',\mathbf{r})\boldsymbol{\sigma}_{\beta,\alpha}], \tag{37}$$

$$\mathbf{S}_{v}(n) = -\sum_{m=1}^{M}\sum_{\alpha,\beta}[\psi_{\alpha}^{\dagger}(\mathbf{r})\{\mathbf{U}_{M}(\mathbf{r})\times\boldsymbol{\sigma}\}_{\alpha,\beta}\psi_{\beta}(\mathbf{r})]. \tag{38}$$

The magnetic source term $\mathbf{S}_{v}(n)$ has a straightforward physical interpretation as the torque exerted by the fixed magnetic potential in the FM layers. The pairing source terms, $S_{cd}(n)$ and $\mathbf{S}_{sd}(n)$, have a more subtle origin: They account for the discrepancy between the fixed pairing potentials [Eq. (27)] in the superconducting regions and the value of these pairing potentials under a self-consistent mean-field treatment.[47] This discrepancy acts like a source or sink of Cooper pairs, which must be accounted for when calculating the current. The expectation values of these terms hence vanish under a self-consistent treatment. Currents due to the source terms in the TS are defined

$$I_{cd}(n) = -\sum_{n+1\leqslant i\leqslant n_0} S_{cd}(i), \tag{39}$$

$$\mathbf{I}_{sd}(n) = -\sum_{n+1\leqslant i\leqslant n_0} \mathbf{S}_{ds}(i), \tag{40}$$

$$\mathbf{I}_{v}(n) = -\sum_{n+1\leqslant i\leqslant L} \mathbf{S}_{v}(i), \tag{41}$$

where n_0 should be in the ferromagnetic layer. We are therefore able to rewrite the continuity equations Eqs. (32) and (33) as

$$\partial_t\rho(n) = -I_c(n) + I_c(n-1), \tag{42}$$

$$\partial_t\mathbf{s}(n) = -\mathbf{I}_{s}^{\text{total}}(n) + \mathbf{I}_{s}^{\text{total}}(n-1), \tag{43}$$

where

$$I_c(n) = I_{ce}(n) + I_{cd}(n), \tag{44}$$

$$\mathbf{I}_{s}^{\text{total}}(n) = \mathbf{I}_{s}(n) + \mathbf{I}_{v}(n), \tag{45}$$

$$\mathbf{I}_{s}(n) = \mathbf{I}_{se}(n) + \mathbf{I}_{sd}(n). \tag{46}$$

The averages of the currents are expressed in terms of the Matsubara Green's function $\check{G}$ defined by

$$\check{G}(\mathbf{r},\mathbf{r}',\tau-\tau') = -\langle T_{\tau}\Psi(\mathbf{r})\Psi^{\dagger}(\mathbf{r}')\rangle = T\sum_{\omega_n}\check{G}(\mathbf{r},\mathbf{r}',\omega_n)e^{-i\omega_n(\tau-\tau')}, \tag{47}$$

where $\omega_n = (2n+1)\pi T$ are the Matsubara frequencies at temperature T. Specifically, we write[14]

$$I_{ce}(n) = \frac{-iet}{2\hbar}\sum_{m=1}^{M}T\sum_{\omega_n}\mathrm{Tr}[\check{G}(\mathbf{r}+\mathbf{z},\mathbf{r},\omega_n) - \check{G}(\mathbf{r},\mathbf{r}+\mathbf{z},\omega_n)], \tag{48}$$

$$S_{ce}(n) = \frac{ie}{\hbar}\sum_{m=1}^{M}T\sum_{\omega_n}\sum_{\mathbf{r}'}\mathrm{Tr}\left[\check{G}(\mathbf{r},\mathbf{r}',\omega_n)\begin{pmatrix}0 & \Delta(\mathbf{r}',\mathbf{r})\\ \Delta^{*}(\mathbf{r}',\mathbf{r}) & 0\end{pmatrix}\right], \tag{49}$$

$$\mathbf{I}_{sq}(n) = \frac{-it}{4}\sum_{m=1}^{M}T\sum_{\omega_n}\mathrm{Tr}\left[(\check{G}(\mathbf{r}+\mathbf{z},\mathbf{r},\omega_n) - \check{G}(\mathbf{r},\mathbf{r}+\mathbf{z},\omega_n))\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}^{*}\end{pmatrix}\right], \tag{50}$$

$$\mathbf{S}_{sd}(n) = \frac{i}{2}\sum_{m=1}^{M}T\sum_{\omega_n}\sum_{\mathbf{r}'}\mathrm{Tr}\left[\check{G}(\mathbf{r},\mathbf{r}',\omega_n)\begin{pmatrix}0 & \Delta(\mathbf{r}',\mathbf{r})\\ \Delta^{*}(\mathbf{r}',\mathbf{r}) & 0\end{pmatrix}\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}^{*}\end{pmatrix}\right], \tag{51}$$

$$\mathbf{S}_{v}(n) = \frac{-1}{2}\sum_{m=1}^{M}T\sum_{\omega_n}\mathrm{Tr}\left[\check{G}(\mathbf{r},\mathbf{r},\omega_n)\begin{pmatrix}\mathbf{U_M}(\mathbf{r})\times\boldsymbol{\sigma} & 0\\ 0 & -\mathbf{U}_{M}(\mathbf{r})\times\boldsymbol{\sigma}^{*}\end{pmatrix}\right]. \tag{52}$$

The recursive Green's function method enables us to numerically calculate the Green's function and hence evaluate the above equations.[45,46]

The charge current $I_c(n)$ in Eq. (44) is independent of n, as required by the charge conservation law. Spin must also be conserved, and so we find that $\mathbf{I}_s^{\text{total}}(n)$ is vanishing for all n because the spin current cannot flow in the SS. This result contradicts the prediction of Sec. II that there is a Josephson spin current in the TS. The paradox can be resolved by noting that neither the perturbation theory nor the Green's function method can properly account for the transfer of spin to the ferromagnetic barrier, as in both theories the magnetic moment is assumed fixed by the constant exchange potential $\mathbf{U}_M(\mathbf{r})$. Rigorously accounting for the conservation of spin in such a situation naturally leads to the conclusion of vanishing spin current. It is nevertheless reasonable to identify the Josephson spin current with the current $\mathbf{I}_s(n)$, which produces a torque on the ferromagnetic barrier, and to hence regard $\mathbf{I}_v(n)$ as a compensating current necessary to maintain the constant exchange potential. Although $\mathbf{I}_s(n)$ depends on n in the FM, it is independent of n in the TS. In the following we only consider this spin current in the TS in order to make contact with the perturbation theory.

In the following we present results for a junction of width $M = 10$ and ferromagnetic barrier length $L = 10$. In units of the transfer integral t we take $|\mathbf{U}_M| = 0.1$ for the exchange potential, and $\mu = 2$ for the chemical potential. The pairing potential has weak-coupling temperature dependence, with zero-temperature magnitude $\Delta_0 = 0.01$. We have confirmed that the transport properties are qualitatively insensitive to choices of these parameters. Since $\mathbf{U}_M \times \mathbf{d} \parallel \mathbf{z}$, as shown in Fig. 2, the x and y components of the spin current are zero. To compare with the analytical predictions of Sec. II, we fix the temperature at $T = 0.5T_c$ in the tunneling regime.

B. *s*-wave singlet superconductor

In this section we present results for the Josephson currents in a TFS junction between an s-wave SS and each of the three different TS states listed in Eq. (27). Commencing with the p_z-wave TS (the p_z-F-s junction), in Fig. 3 we plot the charge and spin currents as functions of the phase ϕ and angle α. As can be seen in panels (a) and (c), the dominant term in the charge current is

$$I_c = \widetilde{I}_c \cos(\phi)\cos(\alpha), \tag{53}$$

with a much weaker contribution $\propto \sin(2\phi)$ visible at $\alpha = 0.5\pi$ in panel (a). This is clearly consistent with the perturbation theory predictions. The spin current also agrees with the perturbative analysis, with the numerical results well described by

$$I_{s,z} = \widetilde{I}_s \sin(2\alpha) + \widetilde{I}'_s \sin(\phi)\sin(\alpha), \tag{54}$$

where $\widetilde{I}_s \gg \widetilde{I}'_s$. Indeed, in panel (d) the spin current vs α curves at different ϕ almost overlap due to the very weak ϕ dependence. The much smaller coefficient of the ϕ-dependent term was anticipated in our tunneling Hamiltonian analysis.

We now consider the results for the p_y-wave TS, which are shown in Fig. 4. In contrast to the p_z-wave TS, the dominant contribution to the charge current is $\propto \sin(2\phi)$, there is only very weak dependence upon α, and the maximum critical

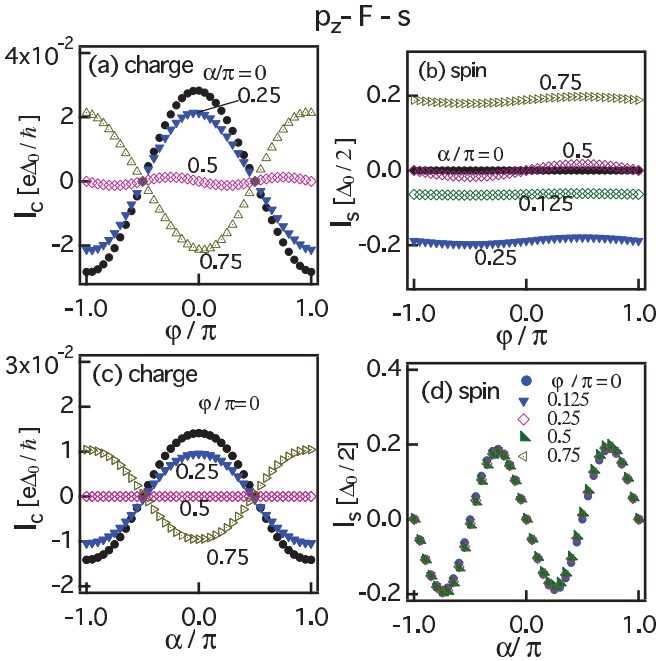

FIG. 3. (Color online) Josephson currents in the p_z-F-s junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. We choose the parameters as $M = L = 10$, $\mu = 2t$, $|\mathbf{U}_M| = 0.1t$, $\Delta_0 = 0.01t$, and $T = 0.5T_c$.

current is much smaller. Since the $\propto \sin(2\phi)$ term originates from coherent tunneling of two Cooper pairs,[26] these results are consistent with our prediction of vanishing charge current due to single-Cooper-pair tunneling processes. For the spin

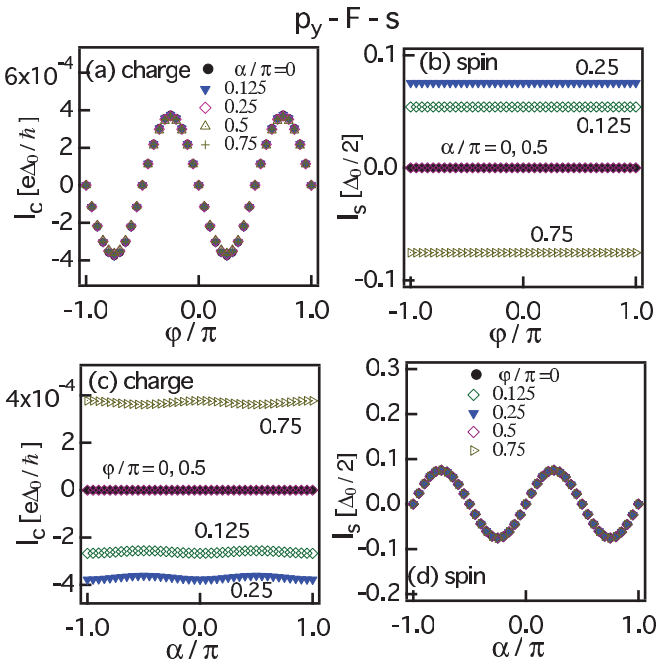

FIG. 4. (Color online) Josephson currents in the p_y-F-s junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. The parameter values are fixed as in Fig. 3.

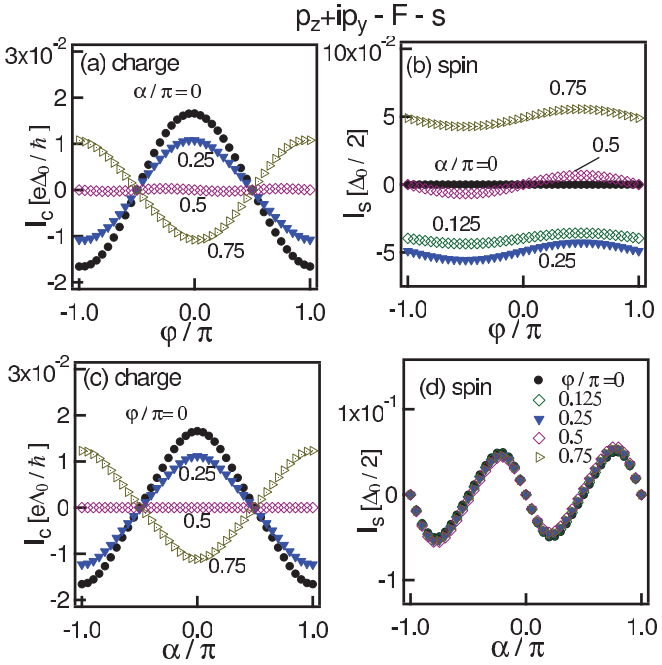

FIG. 5. (Color online) Josephson currents in the $(p_z + ip_y)$-F-s junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. The parameter values are fixed as in Fig. 3.

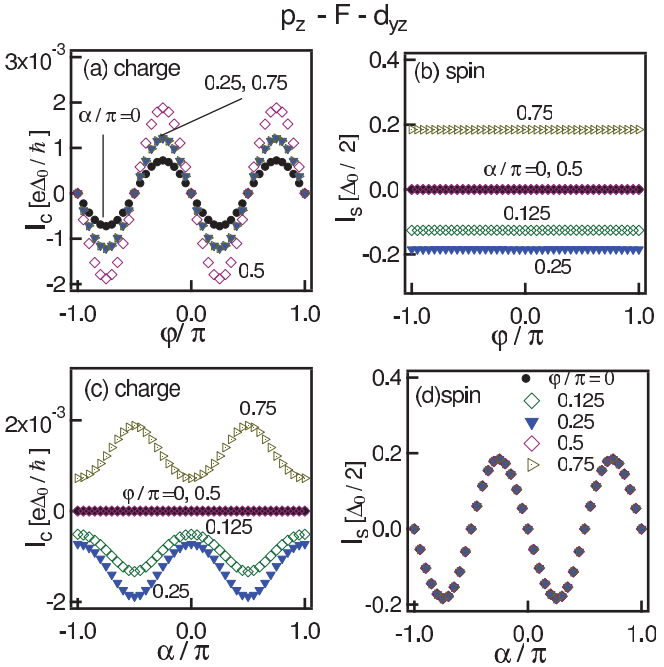

FIG. 6. (Color online) Josephson currents in the p_z-F-d_{yz} junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. The parameter values are fixed as in Fig. 3.

current we find $I_{s,z} = \widetilde{I}_s \sin(2\alpha)$ to excellent approximation, in perfect agreement with the perturbation theory predictions. Note that the spin current has opposite sign compared to the p_z-wave junction as expected.

In Fig. 5 we present the currents for the $(p_z + ip_y)$-wave TS state. As predicted in Sec. II, the results for this junction are very similar to those for the p_z-wave TS, as the lowest-order Josephson coupling proceeds through the p_z component of the chiral p-wave gap. The results are therefore summarized by Eqs. (53) and (54).

C. d_{yz}-wave singlet superconductor

The parity requirement for the superconducting gaps leads us to expect qualitatively different behavior upon replacing the s-wave superconductor with a d_{yz}-wave superconductor due to the even and odd dependence on k_y, respectively. To test this, we repeat the above analysis for a d_{yz}-wave pairing symmetry in the SS. Starting with the p_z-wave TS, in Fig. 6 we find that the charge current is approximately given by

$$I_c = \widetilde{I}_c \sin(2\phi) + \widetilde{I}'_c \sin(2\phi)\cos(2\alpha), \qquad (55)$$

with $\widetilde{I}_c$ and $\widetilde{I}'_c$ of comparable magnitude. This is clearly consistent with the predicted absence of single-Cooper-pair tunneling processes when only one of the order parameters is odd in k_y. The spin current is independent of ϕ and has the approximate form $I_{S,z} = \widetilde{I}_s \sin(2\alpha)$, characteristic of the contribution due to spin-flip reflection.

In Fig. 7 we show the charge and spin currents for the p_y-wave TS pairing symmetry. Since both the singlet and the triplet gaps are odd in k_y, we expect that a charge current is realized at the lowest order of perturbation theory. Indeed, as can be seen in panels (a) and (c), the numerical results for the charge current are well approximated by Eq. (53), and

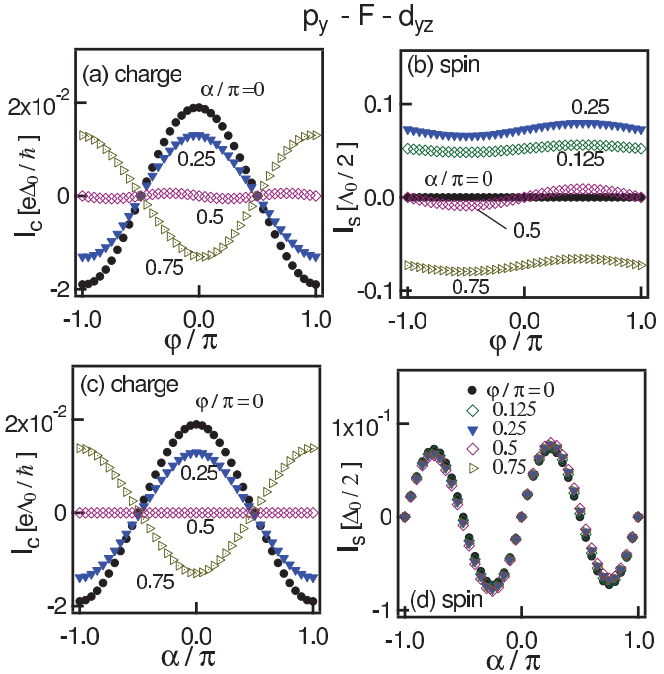

FIG. 7. (Color online) Josephson currents in the p_y-F-d_{yz} junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. The parameter values are fixed as in Fig. 3.

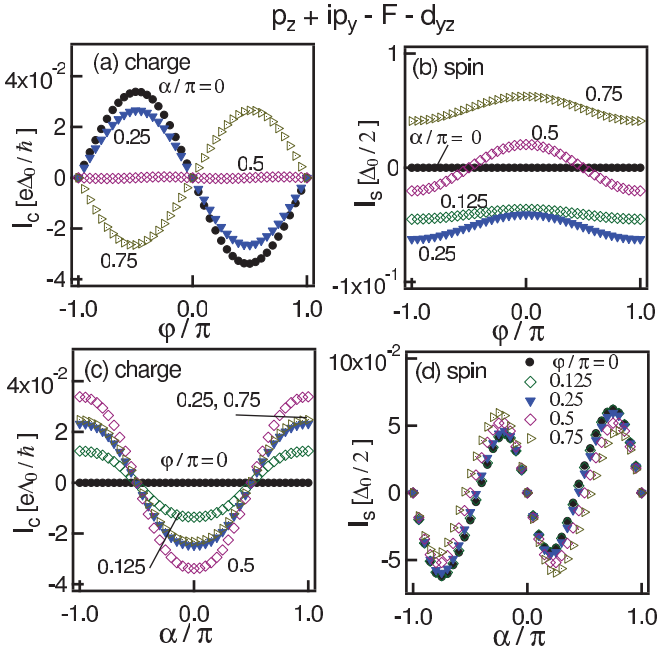

FIG. 8. (Color online) Josephson currents in the $(p_z + ip_y)$-F-d_{yz} junction. (a) Charge and (b) z-component spin currents as a function of ϕ for fixed α. (c) Charge and (d) z-component spin currents as a function of α for fixed ϕ. The parameter values are fixed as in Fig. 3.

the charge current is an order of magnitude larger than for the p_z-wave junction. As expected, there is also a ϕ-dependent contribution to the spin current, which is again of the form Eq. (54).

To conclude our survey, in Fig. 8 we plot the charge and spin currents for the chiral p-wave TS state. In this junction the d_{yz} symmetry couples to the ip_y component of the $(p_z + ip_y)$-wave symmetry, and so the factor of i gives an additional $\pi/2$ phase shift. Therefore, the lowest-order currents in this junction should be approximately obtained from the pure p_y-wave case with the replacement $\phi \to \phi + \pi/2$. Indeed the numerical results in Fig. 8 are consistent with the relations

$$I_c = \widetilde{I}_c \sin(\phi)\cos(\alpha), \tag{56}$$

$$I_{s,z} = \widetilde{I}_s \sin(2\alpha) + \widetilde{I}'_s \cos(\phi)\sin(\alpha). \tag{57}$$

This agrees with the predicted currents in Table I.

In contrast to the charge current, the spin current hardly changes upon replacing the s-wave SS with the d_{xy}-wave SS. This reflects the dominance of reflection processes, which are insensitive to the superconductor on the other side of the junction.

IV. ROLE OF ANDREEV BOUND STATES

The perturbation theory of Sec. II gives a good description of the transport when multiple-Cooper-pair tunneling processes make an insignificant contribution to the current, i.e., when higher-order terms in the perturbation expansion can be neglected. Although this is always the case sufficiently close to the transition temperature of the superconductors, resonant tunneling through Andreev bound states can cause large deviations from perturbation theory predictions at zero temperature. This is often particularly pronounced in junctions where the gap of the superconductors is odd in the momentum component perpendicular to the interface.[28,48] It is therefore likely that in some of the junctions studied above, e.g., the junction between a p_z-wave TS and an s-wave SS, there will be strong contributions to the current from higher harmonics in ϕ and α at low temperatures. In order to estimate the importance of this effect, here we consider an analytically tractable model of the TFS junction where the current is due entirely to tunneling through Andreev bound states.

A. One-dimensional model junction

We study a one-dimensional continuum model of the TFS junction with p_z- and s-wave orbital symmetries for the TS and SS, respectively. Our analysis here closely follows that of Refs. 18 and 19 for a TFT junction. The energies of the Andreev bound states are obtained by means of solving the Bogoliubov–de Gennes equation $H\Psi(z) = E\Psi(z)$, with Hamiltonian

$$H = \begin{pmatrix} -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U_0\delta(z) - \mu & -e^{-i\alpha}U_M\delta(z) & -i\Theta(-z)\Delta_t\frac{\partial}{\partial z} & \Theta(z)\Delta_s e^{-i\phi} \\ -e^{i\alpha}U_M\delta(z) & -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + U_0\delta(z) - \mu & -\Theta(z)\Delta_s e^{-i\phi} & i\Theta(-z)\Delta_t\frac{\partial}{\partial z} \\ -i\Theta(-z)\Delta_t\frac{\partial}{\partial z} & -\Theta(z)\Delta_s e^{i\phi} & \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - U_0\delta(z) + \mu & e^{i\alpha}U_M\delta(z) \\ \Theta(z)\Delta_s e^{i\phi} & i\Theta(-z)\Delta_t\frac{\partial}{\partial z} & e^{-i\alpha}U_M\delta(z) & \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} - U_0\delta(z) + \mu \end{pmatrix}. \tag{58}$$

For simplicity, we assume that the effective mass m and chemical potential μ are the same on either side of the junction, and hence the Fermi wave vectors $k_{F,\nu}$ in each superconductor are also the same; i.e., $k_{F,\nu} = k_F$. The TS and SS are described by the pairing potentials Δ_t and Δ_s, respectively. The barrier is modeled as a δ function with charge scattering potential U_0 and magnetic scattering potential $\mathbf{U_M} = U_M(\cos\alpha\hat{\mathbf{e}}_x + \sin\alpha\hat{\mathbf{e}}_y)$.

The Andreev bound states have energy lying within the bulk gap of the two superconductors, i.e., $|E| < \min\{k_F|\Delta_t|, |\Delta_s|\}$, and are hence exponentially localized at the interface. Within the Andreev approximation, where the superconducting gap is assumed to be negligible compared to the Fermi energy, an appropriate ansatz for the wave function of these states is

$$\Psi_\nu(z) = e^{-\nu\kappa_\nu z}(\Psi_{\nu,+}e^{+ik_F z} + \Psi_{\nu,-}e^{-ik_F z})\Theta(\nu z). \tag{59}$$

Following the notation of Sec. II, $\nu = s$ (t) denotes the singlet (triplet) side, and as a factor $\nu = 1$ (-1). κ_ν is the inverse decay

length in the v superconductor. The spinors $\Psi_{v,\pm}$ are defined

$$\Psi_{t,\pm} = (a_{t,\pm},\ b_{t,\pm},\ \pm e^{\pm i\gamma_t} a_{t,\pm},\ \mp e^{\pm i\gamma_t} b_{t,\pm})^T, \tag{60a}$$

$$\Psi_{s,\pm} = (a_{s,\pm},\ b_{s,\pm},\ -e^{i(\phi\mp\gamma_s)} b_{s,\pm},\ e^{i(\phi\mp\gamma_s)} a_{s,\pm})^T, \tag{60b}$$

where the subscript $\{+,-\}$ indicates the direction of propagation and the phases γ_v are given by

$$\cos\gamma_t = \frac{E}{k_F|\Delta_t|}, \quad \sin\gamma_t = \frac{\hbar^2\kappa_t}{m|\Delta_t|}, \tag{61a}$$

$$\cos\gamma_s = \frac{E}{|\Delta_s|}, \quad \sin\gamma_s = \frac{\hbar^2\kappa_s k_F}{m|\Delta_s|}. \tag{61b}$$

The $a_{v,\pm}$ and $b_{v,\pm}$ appearing in Eq. (60) are constants to be determined by the boundary conditions obeyed by the wave function at the interface. In addition to continuity of the wave function across the junction,

$$\Psi_t(z=0^-) = \Psi_s(z=0^+), \tag{62}$$

we require that the derivative obeys

$$\partial_z\Psi_s(z)|_{z=0^+} - \partial_z\Psi_t(z)|_{z=0^-} = 2k_F \begin{pmatrix} Z & -ge^{-i\alpha} & 0 & 0 \\ -ge^{i\alpha} & Z & 0 & 0 \\ 0 & 0 & Z & -ge^{i\alpha} \\ 0 & 0 & -ge^{-i\alpha} & Z \end{pmatrix} \Psi_s(z=0^+), \tag{63}$$

in order to conserve probability. Here we use the dimensionless parameters

$$Z = \frac{k_F U_0}{2\mu}, \quad g = \frac{k_F U_M}{2\mu}, \tag{64}$$

to characterize the strength of barrier potentials. The boundary conditions give eight equations for the coefficients $a_{v,\pm}$ and $b_{v,\pm}$ and have nontrivial solution when the energy of the bound state satisfies the equation

$$\begin{aligned} 0 = {} & 2 + 8g^2 + 8g^2\cos(2\alpha) - 2\cos(2\phi) - W\cos^2(\gamma_t) \\ & - [4 + 8g^2 + 8g^2\cos(2\alpha)]\cos^2(\gamma_s) \\ & + [4 + W + 16g^2]\cos^2(\gamma_t)\cos^2(\gamma_s) \\ & - 2[2g^2 + 2Z^2 + 1]\sin(2\gamma_t)\sin(2\gamma_s) \\ & + 16g\cos(\alpha)\sin(\phi)\cos(\gamma_t)\cos(\gamma_s) \\ & - 16g\cos(\alpha)\sin(\phi)\sin(\gamma_t)\sin(\gamma_s), \end{aligned} \tag{65}$$

where

$$W = 4(2Z^2 + 1) + 16(g^2 - Z^2)^2. \tag{66}$$

B. Analytical solution at $k_F\Delta_t = \Delta_s$

The parameters that characterize triplet and singlet superconductors are, of course, independent from each other, as the two superconductors cannot be made from the same material. Therefore, there are numerous choices to realize the parameters in Eq. (64), according to the materials from which the junction is made. Nevertheless, in the limit when the pairing potentials of the singlet and triplet superconductor are the same, i.e., $k_F\Delta_t = \Delta_s$, it is possible to express the Josephson charge and spin currents entirely in terms of the Andreev bound-state energies. Although this is a highly idealized situation, it clearly reveals the influence of resonant tunneling on the currents, which we expect to remain qualitatively valid for a more realistic model of the junction.

When $k_F\Delta_t = \Delta_s$, the bound state energy parameters are equal $\gamma_L = \gamma_R$, and Eq. (65) is drastically simplified,

$$\frac{1}{D^2}\left(\frac{E}{|\Delta_s|}\right)^4 - 4A\left(\frac{E}{|\Delta_s|}\right)^2 + 4B^2 = 0, \tag{67}$$

where

$$D = [g^4 + 2g^2(1 - Z^2) + (1 + Z^2)^2]^{-1/2}, \tag{68a}$$

$$A = \frac{1}{4}\left[\frac{1}{D^2} - g^2\sin^2(\alpha) - 2g\cos(\alpha)\sin(\phi)\right], \tag{68b}$$

$$B = \frac{g}{2}\cos(\alpha) - \frac{1}{4}\sin(\phi). \tag{68c}$$

The positive Andreev bound-state energies are hence found to be

$$\frac{E_{a,b}}{|\Delta_s|} = \sqrt{D}|\sqrt{DA + B} \pm \sqrt{DA - B}|. \tag{69}$$

The Josephson charge and z-spin currents are defined[11,49]

$$I_c = -\frac{e}{\hbar}\sum_{l=a,b}\frac{\partial E_l}{\partial\phi}\tanh\frac{E_l}{2k_BT}, \tag{70a}$$

$$I_{s,z} = \frac{1}{4}\sum_{l=a,b}\frac{\partial E_l}{\partial\alpha}\tanh\frac{E_l}{2k_BT}. \tag{70b}$$

Inserting Eq. (69) into Eq. (70) we hence obtain

$$\begin{aligned} I_c = {} & \frac{e|\Delta_s|}{8\hbar}\sum_{\sigma=\pm}\sqrt{\frac{D}{DA + \sigma B}} \\ & \times [2Dg\cos(\alpha)\cos(\phi) + \sigma\cos(\phi)] \\ & \times \left(\tanh\frac{E_a}{2k_BT} + \sigma\,\mathrm{sign}(B)\tanh\frac{E_b}{2k_BT}\right), \end{aligned} \tag{71a}$$

$$\begin{aligned} I_{s,z} = {} & -\frac{|\Delta_s|}{16}\sum_{\sigma=\pm}\sqrt{\frac{D}{DA + \sigma B}} \\ & \times \left[\frac{D}{2}g^2\sin(2\alpha) - Dg\sin(\alpha)\sin(\phi) + \sigma g\sin(\alpha)\right] \\ & \times \left(\tanh\frac{E_a}{2k_BT} + \sigma\,\mathrm{sign}(B)\tanh\frac{E_b}{2k_BT}\right). \end{aligned} \tag{71b}$$

We plot the Josephson currents at zero and finite temperature in the upper and lower panels of Fig. 9, respectively. To obtain the finite temperature results we assume that both gaps display BCS weak-coupling temperature dependence with critical temperature T_c. We find that the barrier potential Z mainly affects the amplitude but not the form of the current-phase relation, so hereafter we present only results for $Z = 0$.

The zero-temperature results for the charge and spin currents at $\alpha = 0$ are in good agreement with the perturbation theory predictions. Rotating the magnetic moment towards the y-z plane, however, we observe that the current-phase relations display sharp jumps at the zero-energy crossings of the Andreev bound states. The Fourier decomposition with respect to the phase difference hence contains a large contribution from higher harmonics, and thus multiple Cooper

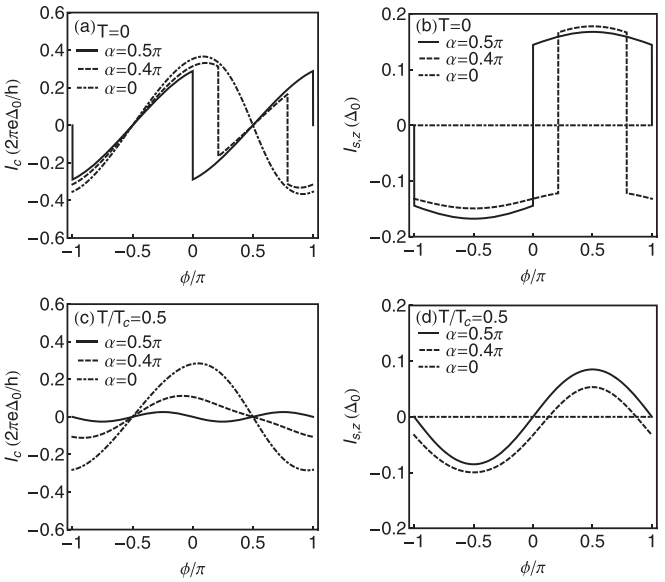

FIG. 9. (a) Josephson charge and (b) spin current for the case $g = 1$, $Z = 0$, at zero temperature $T = 0$. (c) Josephson charge and (d) spin current for the same parameter set at finite temperature $T/T_c = 0.5$.

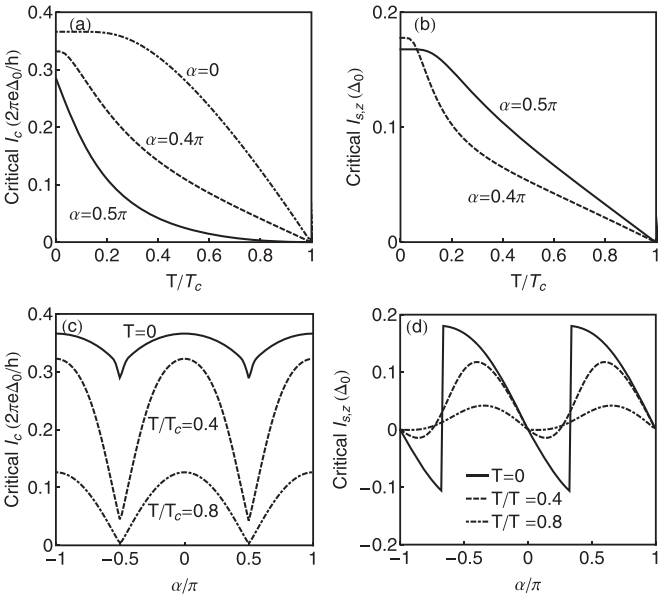

FIG. 10. (a) Critical Josephson charge and (b) spin current as a function of temperature at several different values of α. (c) Critical Josephson charge and (d) spin current as a function of α at several different temperatures. In all panels we take $g = 1$ and $Z = 0$.

pair tunneling processes are important in the zero-temperature limit. Indeed, we see in Fig. 9(a) that the maximum current at $\alpha = 0.5\pi$ is comparable to that at $\alpha = 0$, whereas in the tunneling regime we expect that the latter should be much larger than the former. In contrast, the currents at half the critical temperature [panels (c) and (d)] are in much better agreement with the perturbation theory predictions and the lattice model calculations. In particular, the amplitude of the charge current-phase relation at $\alpha = 0.5\pi$ is now much smaller than that at $\alpha = 0$.

The greatest deviations between the exact and perturbative results for the current at zero temperature occurs for angles near $\alpha = \pm 0.5\pi$, where we find jump discontinuities in the current due to zero-energy crossings of the Andreev bound states which are present for $||\alpha| - 0.5\pi| \leqslant \arcsin(1/2g)$. For $g \gg 1$, the zero-temperature current is well approximated by

$$I_c = \frac{e\Delta_0}{2\hbar}\frac{1}{g^2}\mathrm{sgn}(\cos(\alpha))\cos(\phi). \tag{72}$$

Note that the amplitude of the current is g^{-2}, instead of g^{-3}, as predicted in Eq. (22): Enhancement of the low-temperature current above the perturbation theory predictions is a well-known consequence of the presence of zero-energy Andreev states.[28] We now turn to the spin current: In the large-g limit the phase-dependent component becomes negligible, and we find

$$I_{s,z} = -\frac{\Delta_0}{4}\frac{1}{g}\mathrm{sign}(\cos(\alpha))\sin(\alpha). \tag{73}$$

Although the current is again enhanced beyond the perturbation theory predictions, the two approaches agree that reflection processes dominate in the limit of a strong magnetic barrier.

The critical charge current, defined as the maximum current with respect to ϕ, is a readily-accessible experimental quantity. We plot this alongside the critical spin current in Fig. 10 as a function of the temperature and the angle α. As can be seen in panel (a), the temperature dependence of the critical charge current qualitatively changes with the orientation of the magnetization: For $\alpha = 0$ it grows linearly with decreasing temperature immediately below T_c and saturates at $T \approx 0.2T_c$. In contrast, when $\alpha = 0.5\pi$, it grows superlinearly with decreasing T, but remains much smaller than the $\alpha = 0$ current until $T \approx 0.2T_c$, below which it displays a rapid increase. This "low-temperature anomaly" in the $\alpha = 0.5\pi$ critical charge current reflects the importance of resonant tunneling through the zero-energy Andreev bound states far below T_c.[50] Similarly, in panel (c) we observe that near T_c the critical current as a function of α follows the perturbation predictions of $\max\{|I_c|\} \propto |\cos(\alpha)|$; as the temperature is decreased towards $T = 0$, however, the critical current at $\alpha = \pm 0.5\pi$ increases due to the resonant tunneling, and there is hence overall relatively weak dependence of the critical current on the magnetization orientation. The critical spin current [Figs. 10(b) and 10(d)] also shows strong temperature and α dependence.

V. CONCLUSIONS

In this paper we have studied the unconventional Josephson charge and spin currents in a TFS junction. Using complementary theoretical methods, we have established that single-Cooper-pair tunneling currents are possible when the magnetization of the FM has a component parallel to the **d** vector of the TS, and when the orbital pairing states of the superconductors have the same parity with respect to the interface momentum. We hence see that spin and orbital degrees of freedom both play a critical role in this junction;

this is also the case when a lowest-order Josephson effect between a TS and an SS is mediated by spin-orbit coupling at the barrier.[30–33] At a microscopic level, the perturbation theory analysis reveals that the spin-dependent phase shifts of the tunneling Cooper pairs are responsible for the charge and spin Josephson currents, due to the interference of the spin-$\uparrow$ and spin-$\downarrow$ particle currents in the TS. Surprisingly, the interference of the particle currents is also responsible for a phase-dependent spin current in the TS, even though spin currents are forbidden in the SS. Similar interference effects occur in the TFT junction[20] and junctions between TSs with misaligned $\mathbf{d}$ vectors.[14,15]

Our analysis has confirmed previous observations of a highly unusual charge current in the TFS junction.[39–41] Not only is it linearly proportional to the magnetization of the FM as $\propto \mathbf{M}\cdot\mathbf{d}$, but it also has unconventional $\cos\phi$ dependence on the phase difference (if time-reversal symmetry is not broken in one of the superconductors). This implies a contribution to the junction free energy,

$$\propto \mathbf{M}\cdot\mathbf{d}\Delta_s\Delta_t\sin\phi. \tag{74}$$

In this expression we can regard $\mathbf{d}\Delta_s\Delta_t\sin\phi$ as an intrinsic interface spin which appears at junctions between a TS and an SS, even when the barrier is nonmagnetic. If the barrier is ferromagnetic, the coupling of its magnetic moment to this intrinsic spin therefore generates the lowest-order Josephson effect. Remarkably, such an intrinsic interface spin is indeed known to exist in nonmagnetic TS-SS junctions.[28,29,42,51] For an s-wave SS, this spin only appears for exactly the same p-wave TS orbital configurations which would allow a lowest-order Josephson current in the corresponding TFS junction.

The unusual form of the Josephson current in the TFS offers strong tests for a triplet state. For instance, the observation of the linear dependence of the current on $\mathbf{M}$ would be clear evidence of triplet pairing. On the other hand, a domain structure in the FM could significantly reduce the Josephson current, as the currents across domains with opposite magnetization would have opposite sign. This can be turned to our advantage, however, as a magnetic flux line trapped at the boundary between two such domains would be quantized in half-integer multiples of Φ_0. This is a key signature of the lowest-order Josephson coupling in the TFS junction, and could be directly imaged with SQUID microscopy, or deduced from the Fraunhofer pattern. More speculatively, the coupling Eq. (74) could spontaneously induce a magnetization in a barrier sufficiently close to a magnetic instability, if the free energy gain due to the Josephson coupling can offset the cost of magnetic energy.[52,53]

In our study we have neglected the likely variation of the superconducting order parameter close to the junction interface. Since our results depend only on the bulk properties of the superconductors, however, we do not expect qualitative modification of our results. A more serious limitation of our calculation is that we have not accounted for the torque exerted by the spin current on the barrier's magnetic moment. Regarding the $\mathbf{d}$ vector as fixed, we anticipate that the polarization of the spin current $\propto\mathbf{d}\times\mathbf{M}$ would cause a precession of the magnetization about the $\mathbf{d}$ vector, with eventual decay into the stable configuration.[11] If the $\mathbf{d}$ vector is only weakly pinned, on the other hand, there may be a significant reconstruction of the TS pairing state close to the interface. Although this is a very interesting problem, it is beyond the scope of the current paper.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with G. Annunziata, M. Sigrist, Y. Tanaka, C. Timm, V. M. Yakovenko, and I. Žutić. Y.A. was supported by KAKENHI on Innovative Areas "Topological Quantum Phenomena" (Grant No. 22103002) from MEXT of Japan.

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