Emergent symmetry in quantum critical Dirac systems

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&

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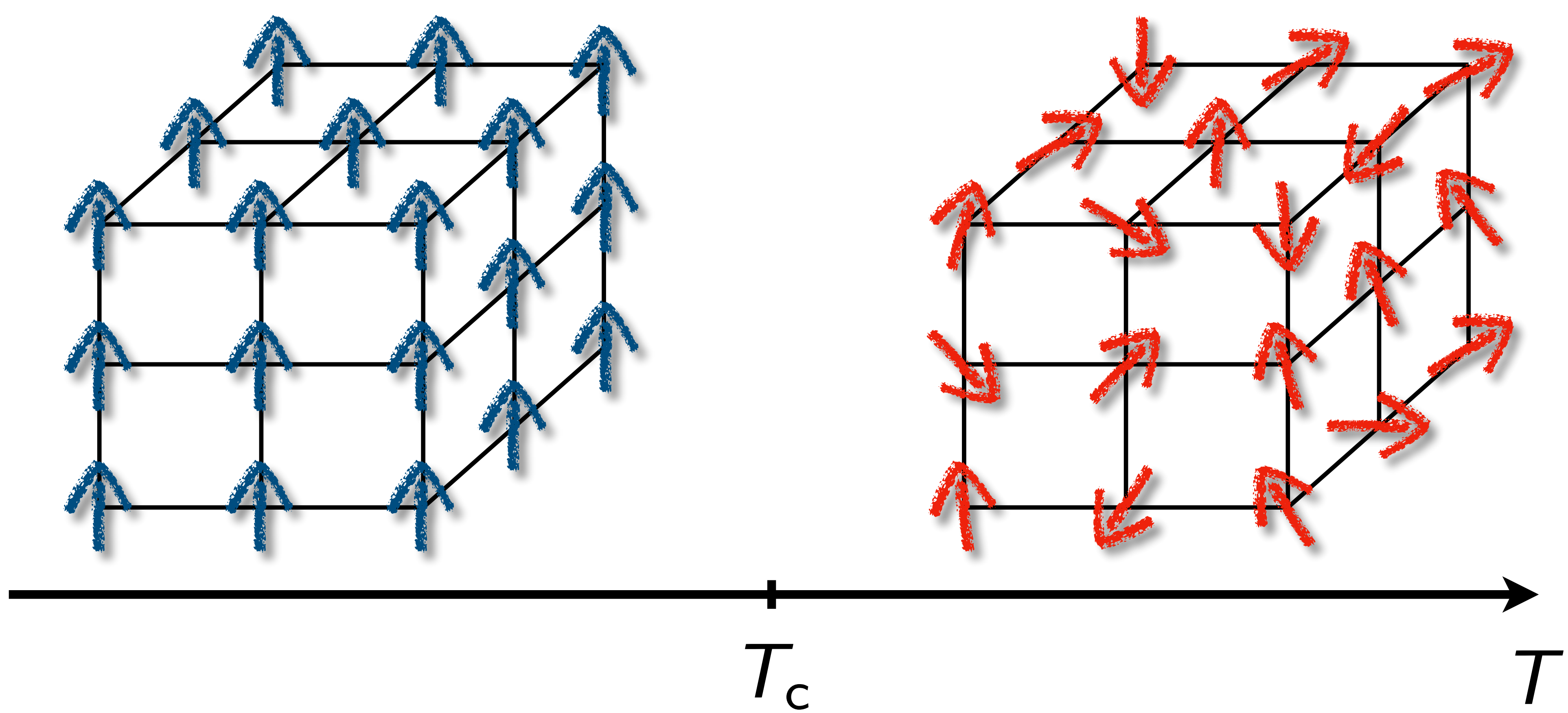
Classical criticality & universality

Universality class determined by ...

... dimension of system

... symmetry of order parameter

... range of interactions



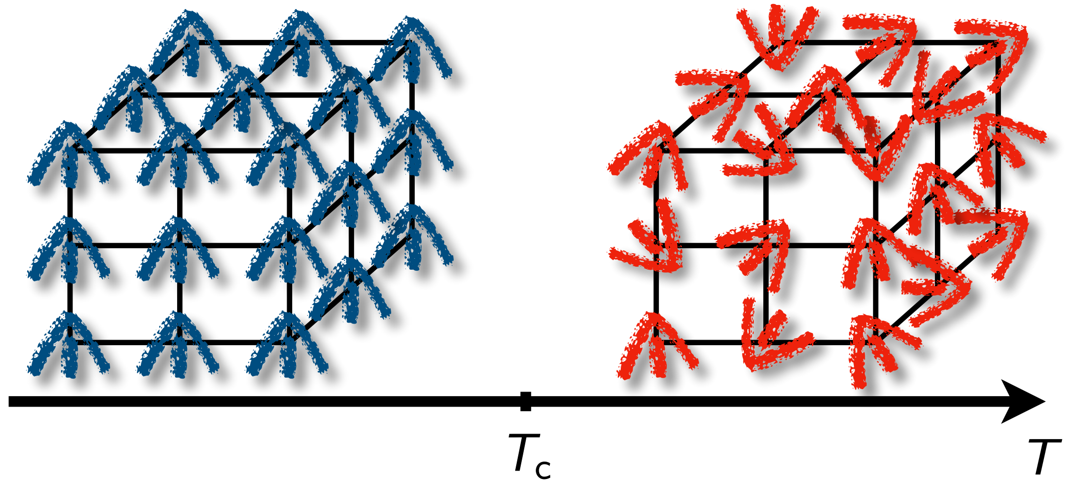
Heisenberg system: O(3) universality

$\eta = 0.038$

$\nu = 0.711$

[Campostrini *et al.*, PRB '02]

Effect of perturbations?



Cubic anisotropy: cubic universality class $\eta = 0.033$ [Carmona *et al.*, PRB '00]

$\nu = 0.706$

$\Rightarrow$ no emergent O(3) symmetry

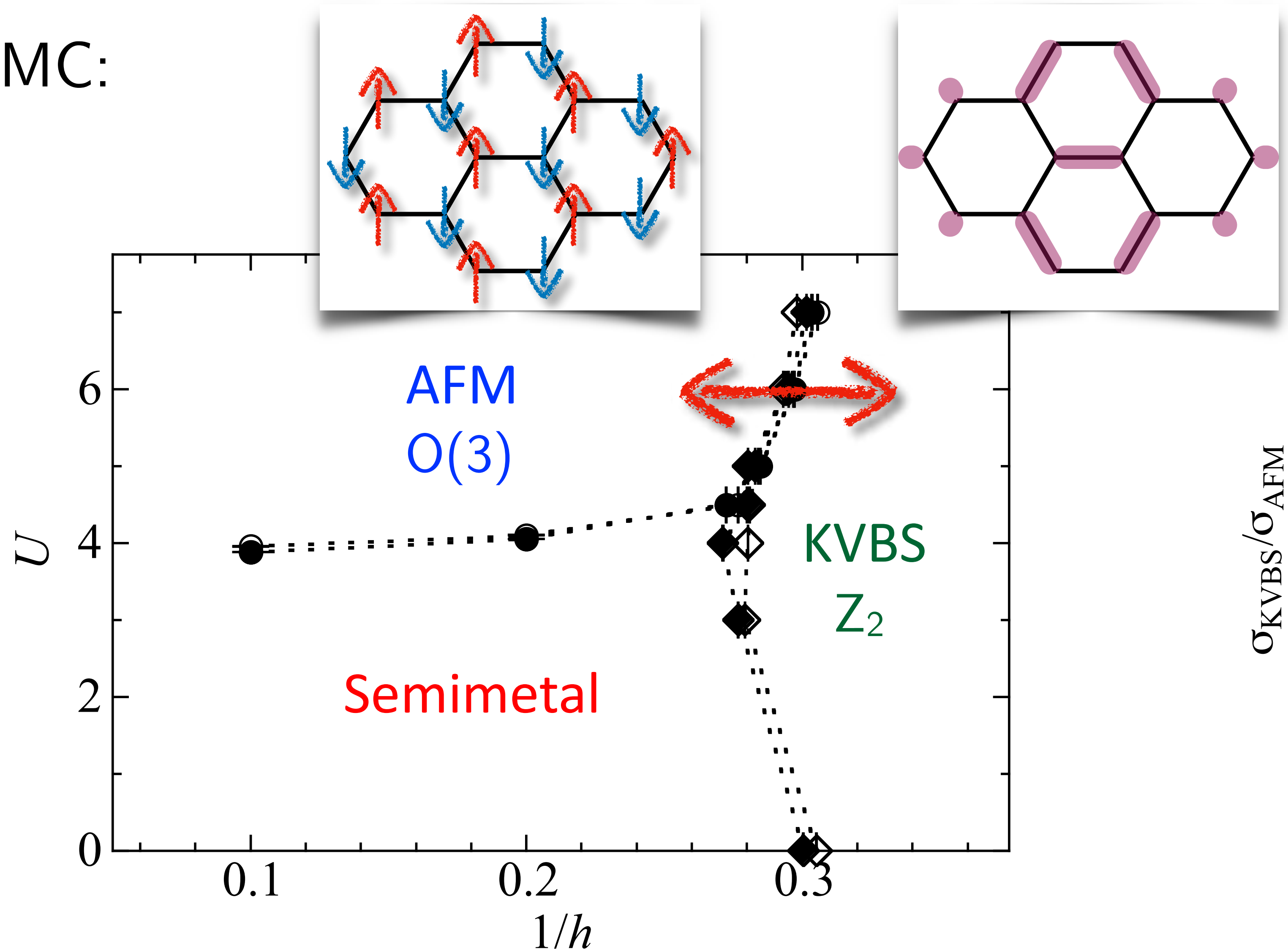
[Calabrese *et al.*, PRB '03]

General analysis (classical criticality): emergent O(N) only for $N < 3$

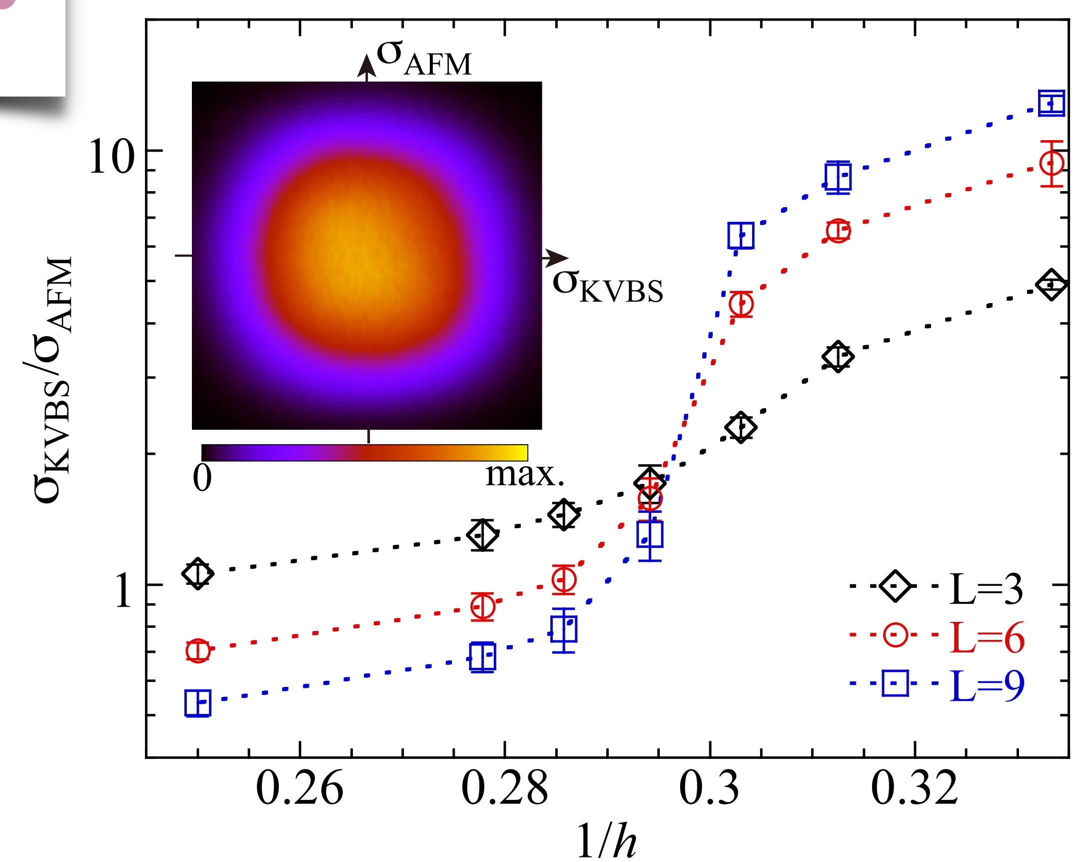
This talk: Emergent O(N) at quantum critical points

Emergent O(N) in 2D Dirac systems?

QMC:



standard deviation



Evidence for …

[Sato, Hohenadler, Assaad, PRL ’17]

… continuous transition

… emergent SO(4)

… c.f. emergent SO(5) at DQCP between AFM and Néel on square lattice

→ duality to (gauged) Dirac fermions

[Nahum *et al.*, PRL ’15]

Emergent O(N) in 2D Dirac systems?

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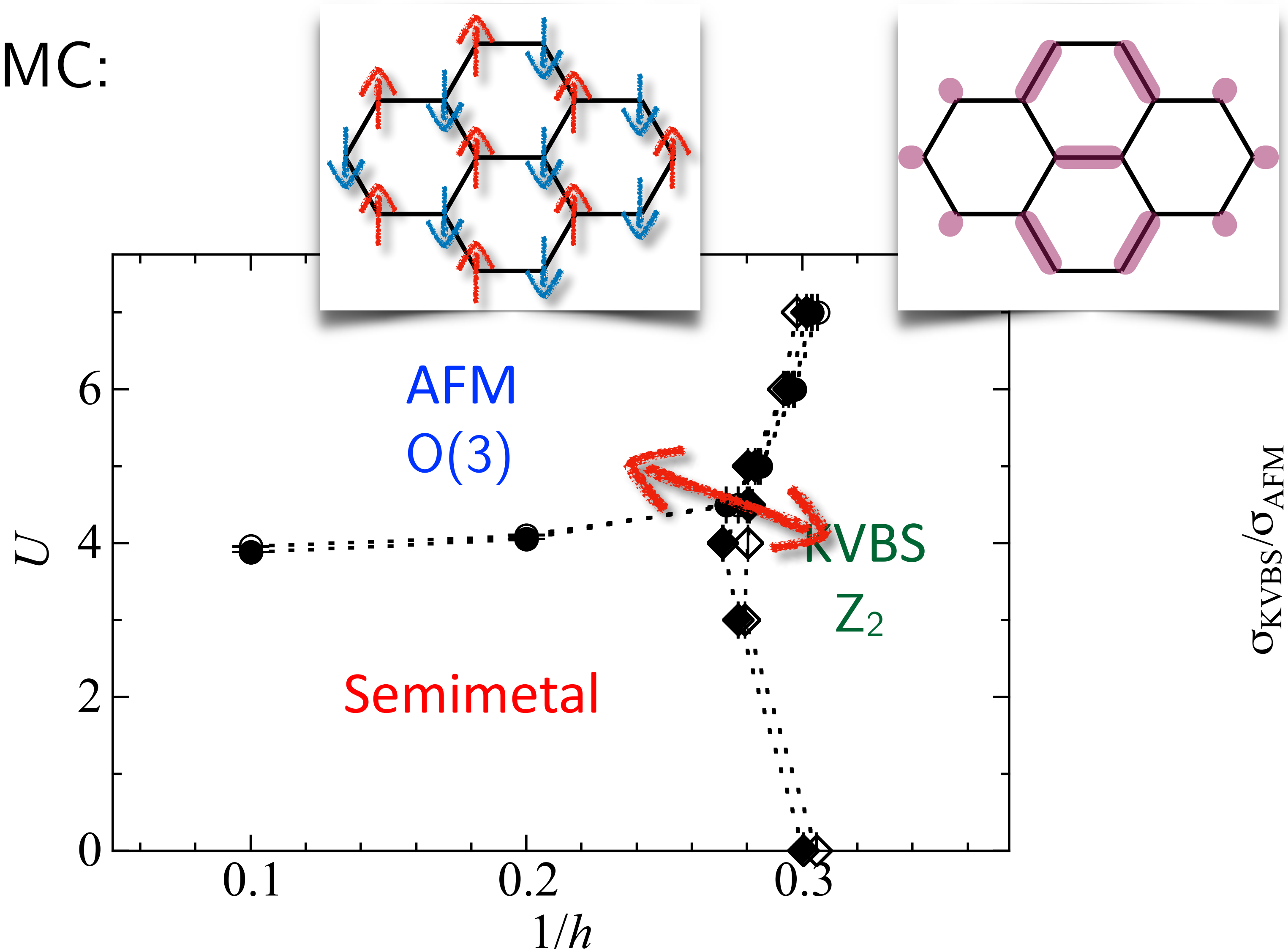

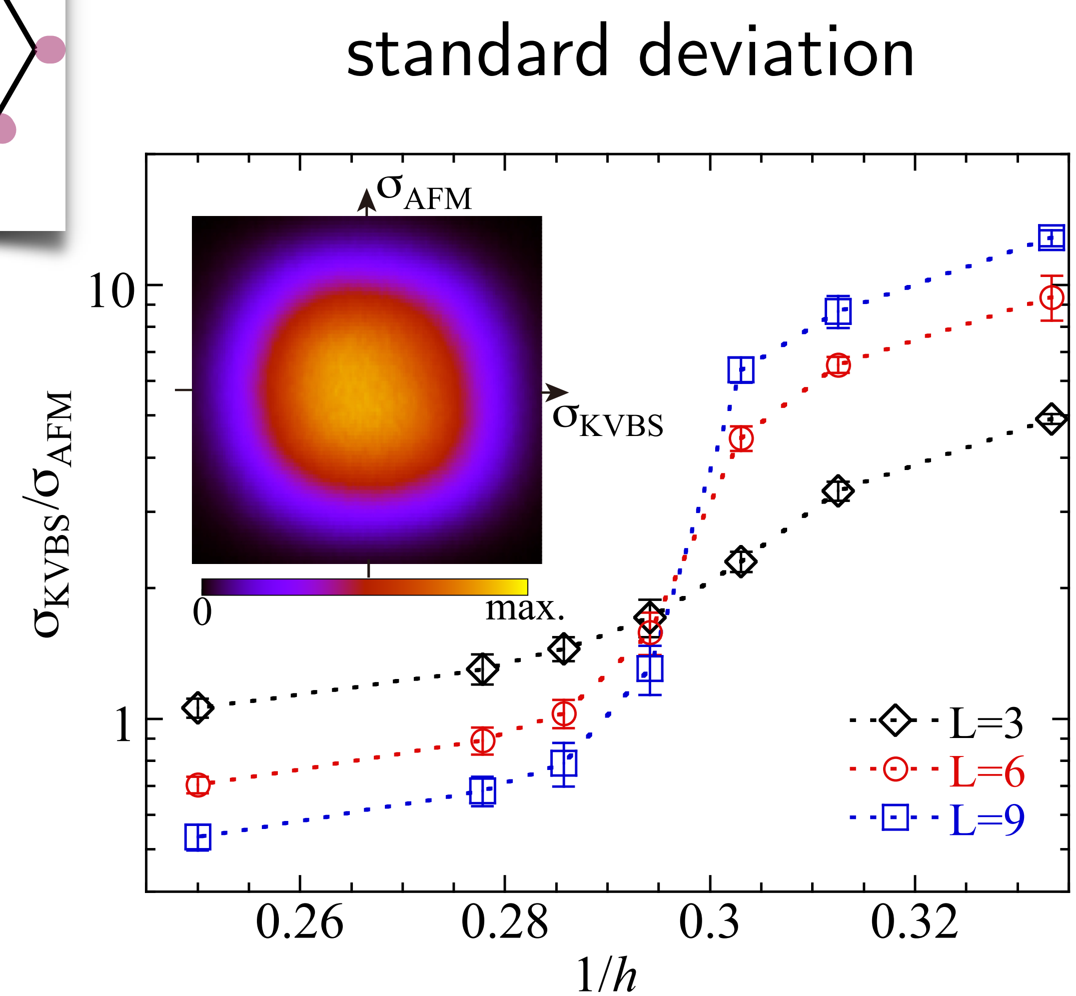

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Anticommuting Dirac masses

Massive Dirac Hamiltonian:

$$\mathcal{H} = \alpha_i p_i + m_a \beta_a^\phi + m_b \beta_b^\chi$$

$i = 1, \ldots, d$
$a = 1, \ldots, N_1$
$b = 1, \ldots, N_2$
α_i, β_a^ϕ, β_b^χ Dirac matrices

Compatible masses:

$$\{\beta_a^\phi, \beta_b^\chi\} = 0$$

$\{\beta_a^\phi, \beta_{a'}^\phi\} = 2\delta_{aa'}\mathbb{1}_{d_\gamma}$
$\{\beta_b^\chi, \beta_{b'}^\chi\} = 2\delta_{bb'}\mathbb{1}_{d_\gamma}$
$\{\beta_a^\phi, \alpha_i\} = \{\beta_b^\chi, \alpha_i\} = 0$

Generators of $\mathrm{O}(N_1) \oplus \mathrm{O}(N_2) \subseteq \mathrm{O}(N_1 + N_2)$:

$$M_{aa'}^\phi = \frac{i}{2}[\beta_a^\phi, \beta_{a'}^\phi] \qquad M_{bb'}^\chi = \frac{i}{2}[\beta_b^\chi, \beta_{b'}^\chi] \qquad M_{ab}^{\phi\chi} = \frac{i}{2}[\beta_a^\phi, \beta_b^\chi]$$

How many masses possible?

$$d_\gamma \geq 2^{\lfloor (N_1 + N_2 + d)/2 \rfloor}$$

... by naive counting
$d_\gamma \geq \frac{1}{2}\dim_{\mathbb{R}}(\mathcal{C}\ell(d, N_1 + N_2))$
[Herbut, PRB '12]

Example: Spin-1/2 fermions on honeycomb lattice

$d = 2,\ d_\gamma = 8 \ \Rightarrow\ N_1 + N_2 \le 5$: 56 quintuplets of anticommuting masses

[Ryu *et al.*, PRB '09]

Five tuplet	Partner five-tuplet by C conjugation
{Re VBS, Im VBS, Re SSC, Im SSC, CDW}	{Re VBS, Im VBS, Néel$_x$, Néel$_y$, Néel$_z$}
{Im VBS, CDW, Re VBS$_x$, Re VBS$_y$, Re VBS$_z$}	{Im VBS, Néel$_z$, Im TSC$_{32z}$, Re TSC$_{32z}$, Re VBS$_z$}
{Re VBS, CDW, Im VBS$_x$, Im VBS$_y$, Im VBS$_z$}	{Re VBS, Néel$_z$, Re TSC$_{02z}$, Im TSC$_{02z}$, Im VBS$_z$}
{Re SSC, Im SSC, QSHE$_x$, QSHE$_y$, QSHE$_z$}	{Néel$_x$, Néel$_y$, Im TSC$_z$, Re TSC$_z$, QSHE$_z$}
{Re VBS, Re SSC, Re TSC$_{02x}$, Im TSC$_{02y}$, Re TSC$_{02z}$}	{Re VBS, Néel$_x$, Re TSC$_{02x}$, Im TSC$_{02x}$, Im VBS$_x$}
{Re VBS, Im SSC, Im TSC$_{02x}$, Re TSC$_{02y}$, Im TSC$_{02z}$}	{Re VBS, Néel$_y$, Im TSC$_{02y}$, Re TSC$_{02y}$, Im VBS$_y$}
{Im VBS, Im SSC, Re TSC$_{32x}$, Im TSC$_{32y}$, Re TSC$_{32z}$}	{Im VBS, Néel$_y$, Re TSC$_{32y}$, Im TSC$_{32y}$, Re VBS$_y$}
{Im VBS, Re SSC, Im TSC$_{32x}$, Re TSC$_{32y}$, Im TSC$_{32z}$}	{Im VBS, Néel$_x$, Im TSC$_{32x}$, Re TSC$_{32x}$, Re VBS$_x$}
{CDW, Im SSC, Im TSC$_x$, Re TSC$_y$, Im TSC$_z$}	{Néel$_z$, Néel$_y$, Im TSC$_x$, Re TSC$_x$, QSHE$_x$}
{CDW, Re SSC, Re TSC$_x$, Im TSC$_y$, Re TSC$_z$}	{Néel$_z$, Néel$_x$, Re TSC$_y$, Im TSC$_y$, QSHE$_y$}
{Im VBS$_x$, QSHE$_y$, Im VBS$_z$, Re TSC$_{32y}$, Im TSC$_{32y}$}	{Re TSC$_{02z}$, Re TSC$_z$, Im VBS$_z$, Re TSC$_{32x}$, Im TSC$_{32y}$}
{Im VBS$_x$, QSHE$_y$, Re VBS$_x$, Néel$_x$, QSHE$_z$}	{Re TSC$_{02z}$, Re TSC$_z$, Im TSC$_{32z}$, Re SSC, QSHE$_z$}
{Im VBS$_x$, Re TSC$_{32y}$, Im TSC$_{32z}$, Im TSC$_{02x}$, Im TSC$_x$}	{Re TSC$_{02z}$, Re TSC$_{32x}$, Re VBS$_x$, Im TSC$_{02y}$, Im TSC$_x$}
{Im VBS$_x$, Re TSC$_{32z}$, Re TSC$_{02x}$, Re TSC$_x$, Im TSC$_{32y}$}	{Re TSC$_{02z}$, Re VBS$_y$, Re TSC$_{02x}$, Re TSC$_y$, Im TSC$_{32y}$}
{Im VBS$_x$, Re TSC$_{32z}$, Im TSC$_{32z}$, Im VBS$_y$, QSHE$_z$}	{Re TSC$_{02z}$, Re VBS$_y$, Re VBS$_x$, Im TSC$_{02z}$, QSHE$_z$}
{Im VBS$_x$, Re TSC$_x$, Im TSC$_x$, CDW, Re VBS$_x$}	{Re TSC$_{02z}$, Re TSC$_y$, Im TSC$_x$, Néel$_z$, Im TSC$_{32z}$}
{QSHE$_y$, Im VBS$_z$, QSHE$_x$, Re VBS$_z$, Néel$_z$}	{Re TSC$_z$, Im VBS$_z$, Im TSC$_z$, Re VBS$_z$, CDW}
{QSHE$_y$, Re TSC$_{02y}$, Re TSC$_y$, Im SSC, Im TSC$_{32y}$}	{Re TSC$_z$, Re TSC$_{02y}$, Re TSC$_x$, Néel$_y$, Im TSC$_{32y}$}
{QSHE$_y$, Re TSC$_{02y}$, Im TSC$_{02y}$, Re VBS$_x$, Re VBS$_z$}	{Re TSC$_z$, Re TSC$_{02y}$, Im TSC$_{02x}$, Im TSC$_{32z}$, Re VBS$_z$}
{QSHE$_y$, Re TSC$_{32y}$, Im TSC$_{02y}$, Im TSC$_y$, Re SSC}	{Re TSC$_z$, Re TSC$_{32x}$, Im TSC$_{02x}$, Im TSC$_y$, Néel$_x$}
{Re VBS$_y$, Néel$_y$, QSHE$_x$, Im VBS$_y$, QSHE$_z$}	{Re TSC$_{32z}$, Im SSC, Im TSC$_z$, Im TSC$_{02z}$, QSHE$_z$}
{Re VBS$_y$, Re TSC$_y$, Im TSC$_y$, CDW, Im VBS$_y$}	{Re TSC$_{32z}$, Re TSC$_x$, Im TSC$_y$, Néel$_z$, Im TSC$_{02z}$}
{Re VBS$_y$, Re TSC$_{32y}$, Im TSC$_y$, Im TSC$_{02z}$, Im TSC$_{02x}$}	{Re TSC$_{32z}$, Re TSC$_{32x}$, Im TSC$_y$, Im VBS$_y$, Im TSC$_{02y}$}
{Re VBS$_y$, Re TSC$_{02x}$, Im TSC$_{02x}$, QSHE$_x$, Re VBS$_z$}	{Re TSC$_{32z}$, Re TSC$_{02x}$, Im TSC$_{02y}$, Im TSC$_z$, Re VBS$_z$}
{Néel$_y$, Re TSC$_{32y}$, Im TSC$_{02y}$, Im TSC$_z$, Im TSC$_x$}	{Im SSC, Re TSC$_{32x}$, Im TSC$_{02x}$, QSHE$_x$, Im TSC$_x$}
{Im VBS$_z$, Re TSC$_{32y}$, Im TSC$_{02z}$, Im TSC$_z$, Im TSC$_{32x}$}	{Im VBS$_z$, Re TSC$_{32x}$, Im VBS$_y$, QSHE$_x$, Im TSC$_{32x}$}
{Re TSC$_{02y}$, Re TSC$_y$, Im TSC$_{32z}$, Im TSC$_{32x}$, Im VBS$_y$}	{Re TSC$_{02y}$, Re TSC$_x$, Re VBS$_x$, Im TSC$_{32x}$, Im TSC$_{02z}$}
{Re TSC$_y$, Re TSC$_{02x}$, Im TSC$_z$, Im TSC$_{32x}$, Néel$_x$}	{Re TSC$_x$, Re TSC$_{02x}$, QSHE$_x$, Im TSC$_{32x}$, Re SSC}

$O(N_1) \oplus O(N_2)$ Gross-Neveu-Yukawa theory

Action:

$$S = \int d^d\vec{x}d\tau \Big[\Psi^\dagger(\partial_\tau + \mathcal{H}_0)\Psi + g_1\phi_a\Psi^\dagger\beta_a^\phi\Psi + g_2\chi_b\Psi^\dagger\beta_b^\chi\Psi$$

$$+ \frac{1}{2}(\partial_\mu\phi_a)^2 + \frac{1}{2}(\partial_\mu\chi_a)^2 + \lambda_1\left(\phi_a^2\right)^2 + \lambda_2\left(\chi_b^2\right)^2 + 2\lambda_3\phi_a^2\chi_a^2 \Big]$$

... both r_1 and r_2 tuned to zero

$O(N_1 + N_2)$ symmetry $\quad \Leftrightarrow \quad g_1 = g_2, \;\; \lambda_1 = \lambda_2 = \lambda_3$

Scaling dimensions:

$$[g_1] = [g_2] = \frac{3-d}{2}$$

$$[\lambda_1] = [\lambda_2] = [\lambda_3] = 3 - d$$

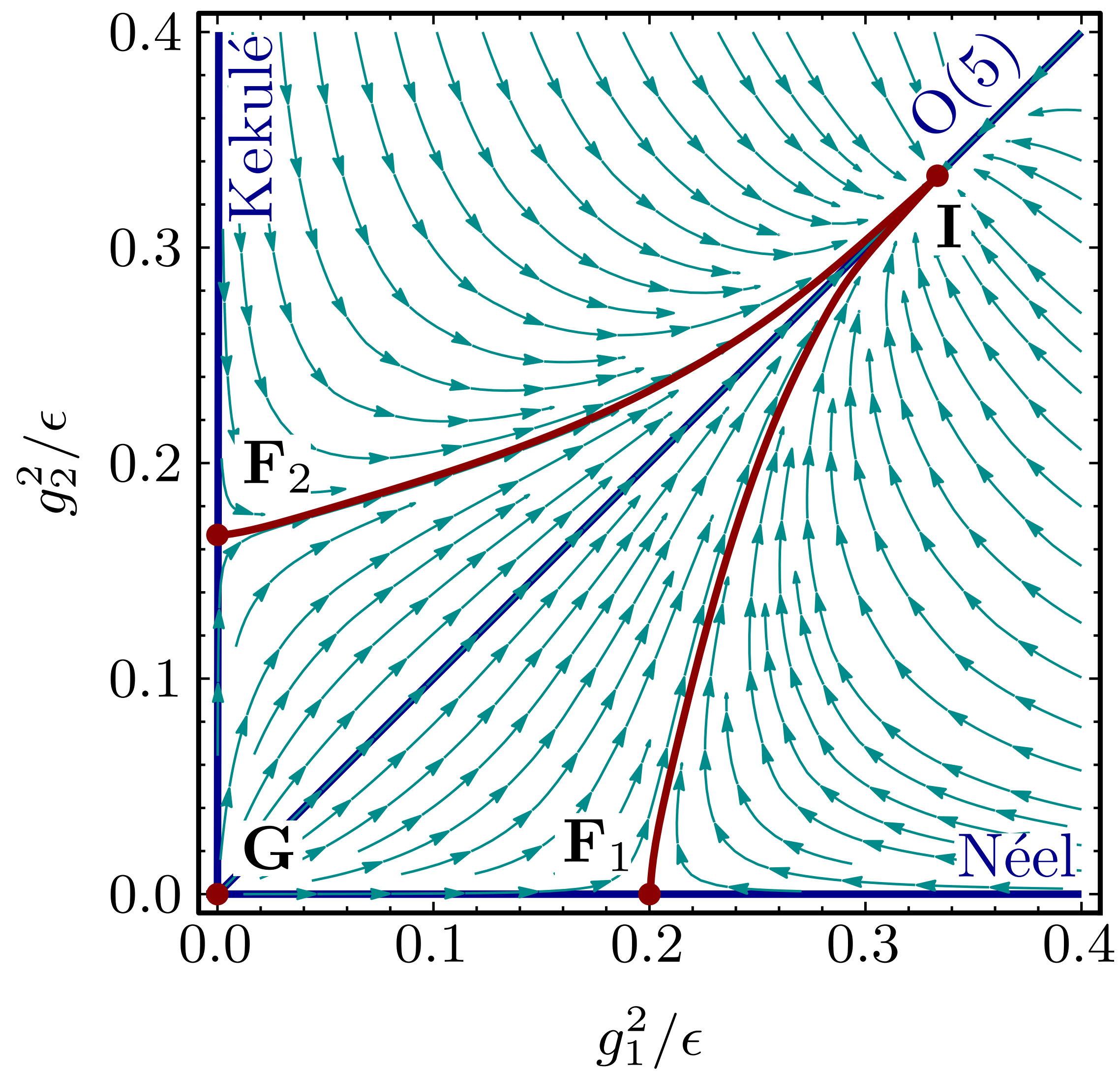
... become simultaneously marginal in $d = 3$

$\Rightarrow$ standard ε expansion in $d = 3 - \varepsilon$ possible

2D: $\varepsilon = 1$
3D: $\varepsilon = 0$

RG flow in Yukawa sector

[L.J., Scherer, Herbut, PRB(R) '18]



RG exponents: $$(\theta_1, \theta_2, \ldots) = \left(-1, -\frac{2N_\text{f}+4}{2N_\text{f}+4-N}, \ldots\right)\epsilon + \mathcal{O}(\epsilon^2)$$

... always negative for all N and (compatible) N_f

Dirac multicritical point: Emergent O(N) for all N!

Emergent O(N) in multicritical Dirac system

Critical exponents:

[L.J., Scherer, Herbut, PRB(R) '18]

$N_{\mathrm{f}} = 2$		ν	η_ϕ	η_Ψ	ω_1
Chiral Ising	FRG	1.018	0.760	0.032	0.872
	ϵ^1	31/42	4/7	1/14	1
Chiral XY	FRG	1.160	0.875	0.062	0.878
	ϵ^1	4/5	2/3	1/6	1
Chiral Heisenberg	FRG	1.296	1.015	0.084	0.924
	ϵ^1	97/110	4/5	3/10	1
Chiral O(4)	FRG	1.364	1.159	0.091	1.017
	ϵ^1	1	1	1/2	1
Chiral O(5)	FRG	1.356	1.285	0.089	1.132
	ϵ^1	31/26	4/3	5/6	1

... chiral universality classes accessible even without explicit O(N) symmetry

... immediately testable in numerics

Conclusions

Classical criticality ($d = 3$):

$$\text{Emergent O}(N) \text{ only for } N < 3$$

[Calabrese *et al.*, PRB '03]

Quantum criticality with N_f gapless Dirac fermions ($d + z = 3$):

$$\text{Emergent O}(N) \text{ for all } N \text{ and } N_\mathrm{f}$$

[L.J., Scherer, Herbut, PRB(R) '18]