Heisenberg-)Kitaev physics (in magnetic fields)

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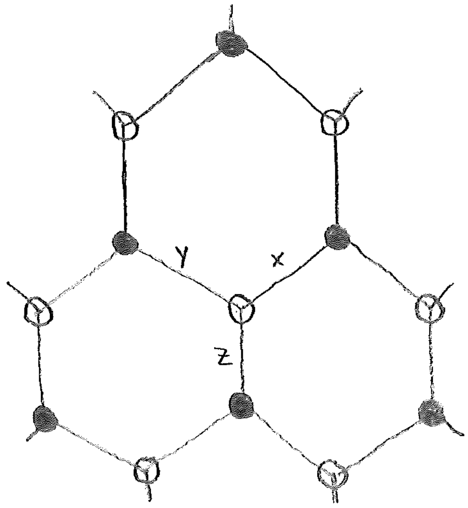
Kitaev's honeycomb model:

[Kitaev '06]



$$H = - J_x \sum_{x\text{-bonds}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-bonds}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-bonds}} \sigma_j^z \sigma_j^z$$

Majorana representation:

Majorana fermion:

$$c_1 = a + a^\dagger \quad , \qquad c_2 = \frac{a - a^\dagger}{i}$$

$$c_1^2 = c_2^2 = \{a, a^\dagger\} = \mathbb{1}$$

$$c_1 c_2 = - c_2 c_1 \quad , \qquad c_1^\dagger = c_1 \quad , \quad c_2^\dagger = c_2$$

Representation of a single spin:

$$\sigma^x \longmapsto \tilde{\sigma}^x = i b^x c \quad \in \mathcal{L}(\tilde{\mathcal{H}})$$

$$\in \mathcal{L}(\mathcal{H}) \quad \sigma^y \longmapsto \tilde{\sigma}^y = i b^y c$$

$$\sigma^z \longmapsto \tilde{\sigma}^z = i b^z c$$

b^x, b^y, b^z, c Majorana , $\dim \mathcal{H} = 2$, $\dim \tilde{\mathcal{H}} = 4$

Projection:

$$|\xi\rangle \in \mathcal{L} \subset \tilde{\mathcal{L}} \quad \Leftrightarrow \quad D|\xi\rangle = |\xi\rangle \quad , \quad D = b^x b^y b^z c$$

"$\mathbb{Z}_2$ gauge transformation"

Spin algebra:

$$(\tilde{\sigma}^\alpha)^\dagger = (i\, b^\alpha c)^\dagger = \tilde{\sigma}^\alpha \quad , \quad \alpha = x, y, z$$

$$(\tilde{\sigma}^\alpha)^2 = i^2 b^\alpha c\, b^\alpha c = \mathbb{1}$$

$$\tilde{\sigma}^x \tilde{\sigma}^y \tilde{\sigma}^z = i^3\, b^x c\, b^y c\, b^z c = i\, b^x b^y b^z c = i D$$

$$[\tilde{\sigma}^\alpha, D] = 0 \quad (\mathcal{L} \text{ preserved})$$

Representation of honeycomb model:

$$\sigma_j^\alpha \longmapsto \tilde{\sigma}_j^\alpha = i\, b_j^\alpha c_j \qquad D_j = b_j^x b_j^y b_j^z c_j$$

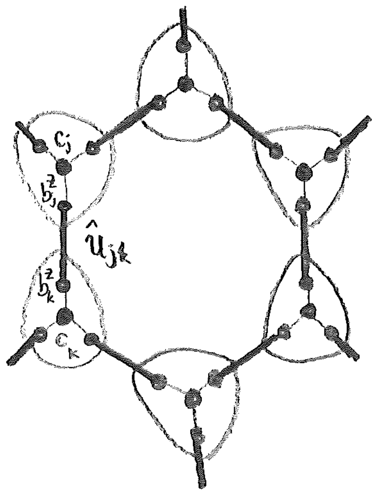
$$|\xi\rangle \in \mathcal{L} \quad \Leftrightarrow \quad D_j|\xi\rangle = |\xi\rangle \quad \forall j = 1, \dots, n$$

Hamiltonian:

$$\tilde{\sigma}_j^\alpha \tilde{\sigma}_k^\alpha = (i\, b_j^\alpha c_j)(i\, b_k^\alpha c_k) = -i \underbrace{(i\, b_j^\alpha b_k^\alpha)}_{\equiv \hat{u}_{jk} = \hat{u}_{jk}^\dagger} c_j c_k$$

$$\tilde{H} = i \sum_{\alpha = x,y,z} \sum_{\langle jk \rangle_\alpha} J_\alpha\, \hat{u}_{jk}\, c_j c_k$$

"$\mathbb{Z}_2$ gauge theory with Majorana fermions"



$\mathbb{Z}_2$ gauge field $\hat{u}_{jk}$: $\quad u_{jk} = \pm 1$ eigenvalues

$$[\hat{u}_{jk}, \hat{u}_{j'k'}] = [i b_j^\alpha b_k^\alpha, i b_{j'}^\beta b_{k'}^\beta] = 0$$

$$[\hat{u}_{jk}, \tilde{H}] = \Big[\hat{u}_{jk}, i \sum_\alpha \sum_{\langle j'k' \rangle_\alpha} J_\alpha \hat{u}_{j'k'} c_{j'} c_{k'}\Big] = 0$$

$\Rightarrow$ gauge field is "static", u_{jk} are good quantum numbers

Simultaneous diagonalization:

$$\tilde{H}_u = i \sum_\alpha \sum_{\langle jk \rangle_\alpha} J_\alpha u_{jk} c_j c_k \quad \text{with} \quad u_{jk} = \pm 1$$

Lieb theorem :

$u_{jk} = +1 \ \forall (j,k)$ in ground state

$\Rightarrow$ translation invariance

Reciprocal space:

$$\tilde{H}_{u=1} = \frac{1}{2} \sum_{\vec{q}} \begin{pmatrix} c_{1,-\vec{q}} & c_{2,-\vec{q}} \end{pmatrix} \begin{pmatrix} 0 & i f(\vec{q}) \\ -i f^*(\vec{q}) & 0 \end{pmatrix} \begin{pmatrix} c_{1,\vec{q}} \\ c_{2,\vec{q}} \end{pmatrix}$$

with $f(\vec{q}) = 2 \sum_{\alpha} J_\alpha \, e^{i \vec{q} \cdot \vec{\delta}_\alpha}$ nearest-neighbor vector

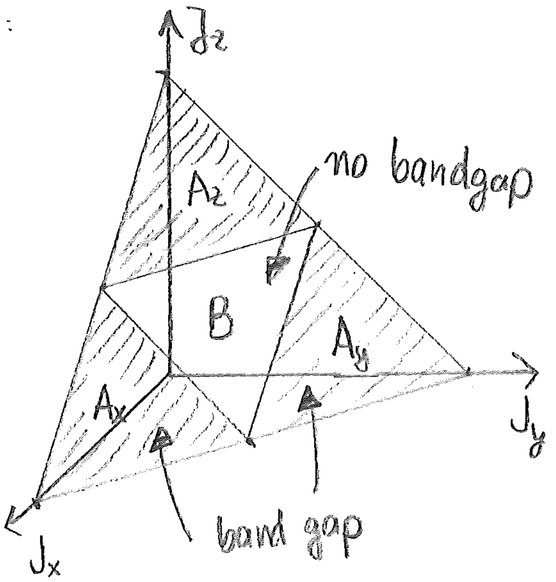
Majorana spectrum:

$$\varepsilon(\vec{q}) = \pm \, |f(\vec{q})|$$

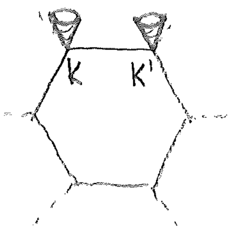
Band gap?

$$\exists\, \vec{q} \in BZ: \ \varepsilon(\vec{q}) = 0 \iff |J_x| \le |J_y| + |J_z| \,,\ |J_y| \le |J_x| + |J_z| \,,\ |J_z| \le |J_x| + |J_y|$$

Phase diagram:



Example: $J_x = J_y = J_z = J$



$$\varepsilon(\vec{q}) = \sqrt{3 J^2 |\delta\vec{q}|^2} + \mathcal{O}(\delta q^3)$$

$$\delta\vec{q} = \vec{q} - (\pm \vec{K})$$

"Dirac cones"