Quantum criticality in 2D Fermi systems with quadratic band touching

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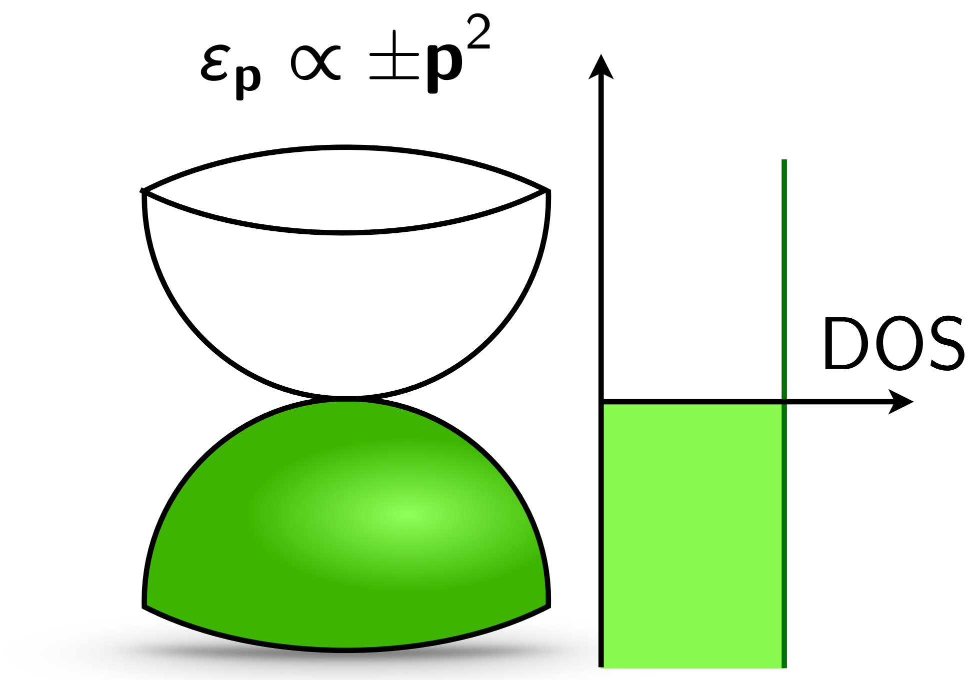
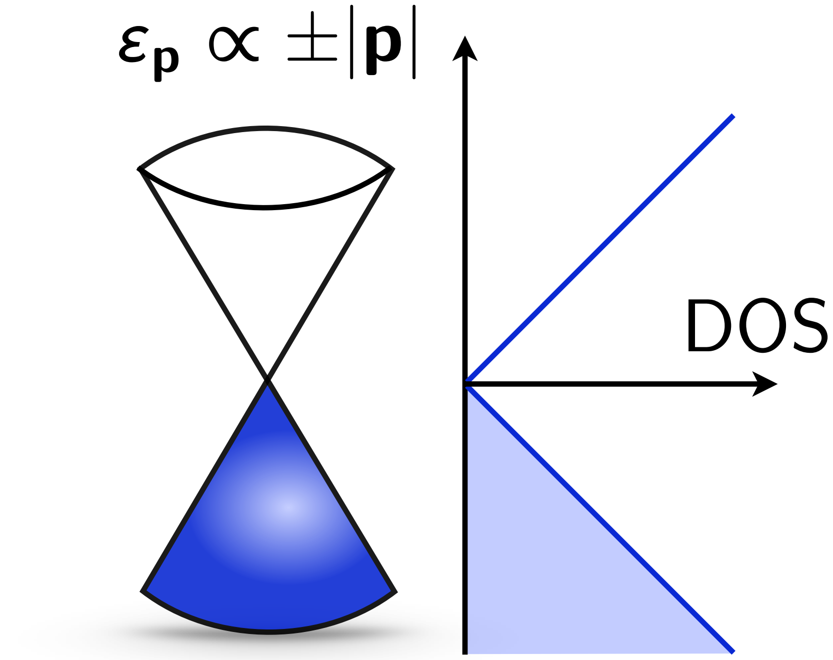
ct.qmat
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

SFB 1143

Quadratic vs. linear Fermi nodes in 2D



~ "metal"

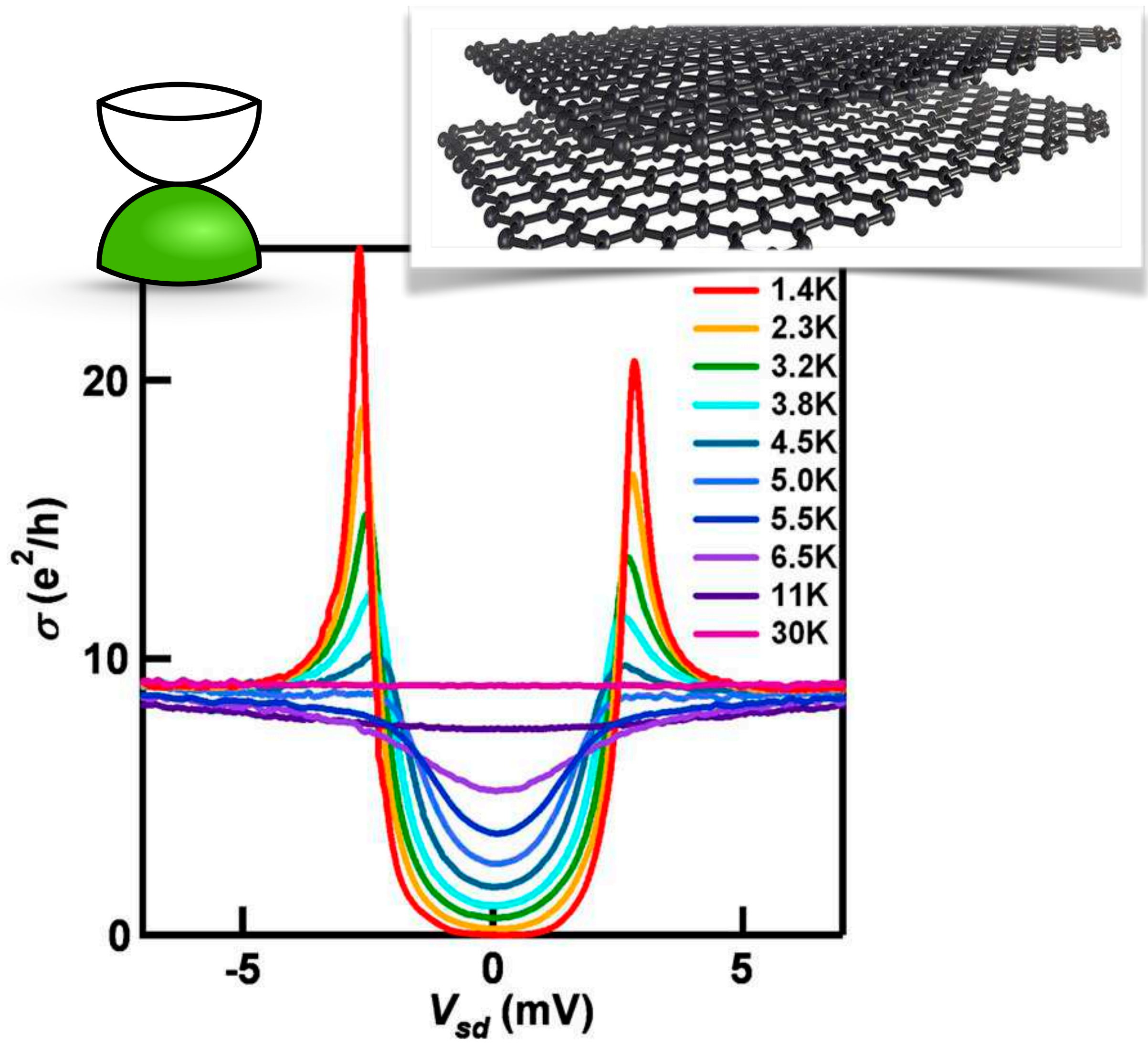


~ "semiconductor"

Expectation: unstable stable ... for weak interactions

Examples: Bilayer vs. single-layer graphene

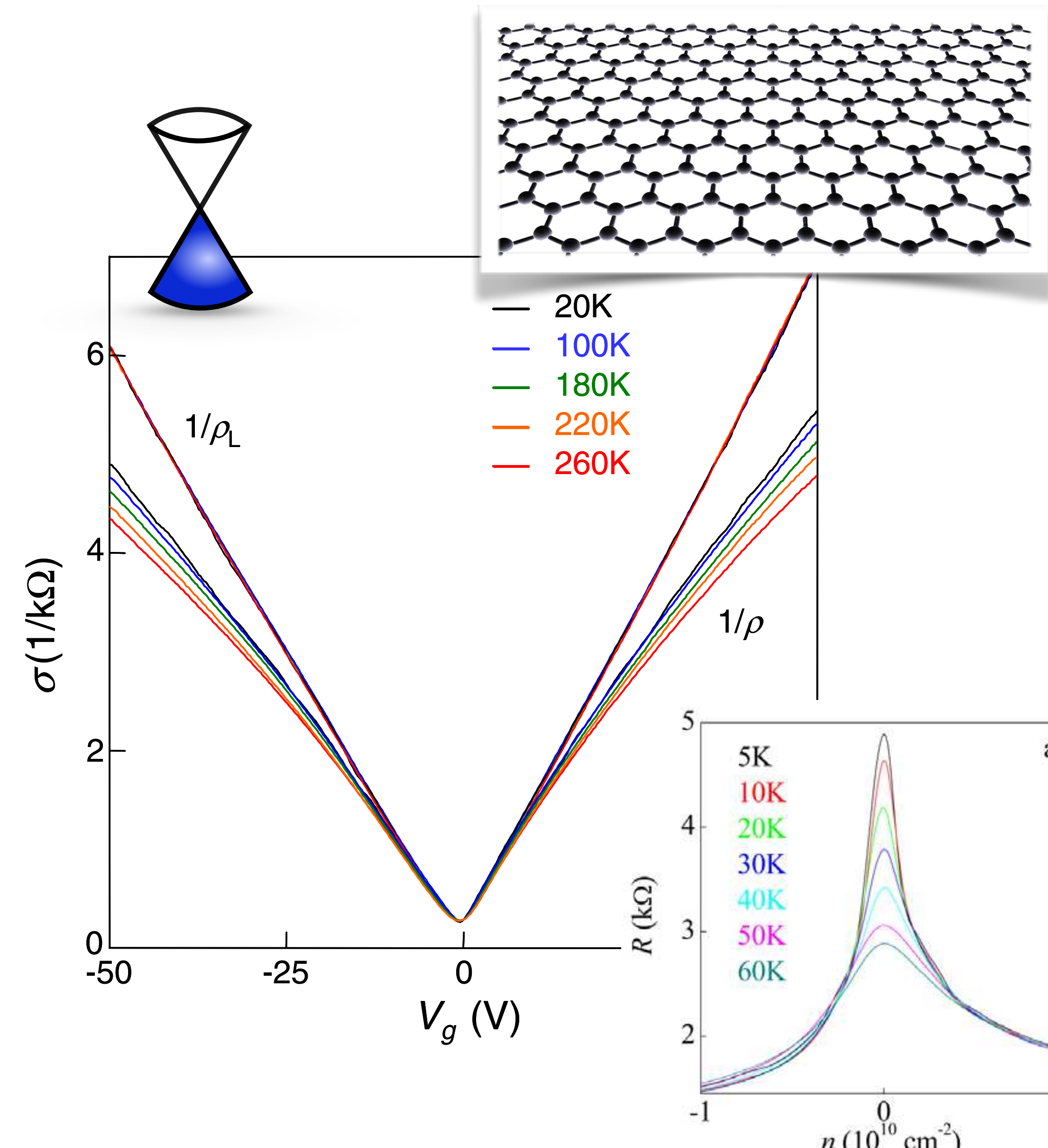
Electrical conductivity vs. doping:



Spontaneous gap
below 10...30 K

[Bao *et al.*, PNAS '12]

...

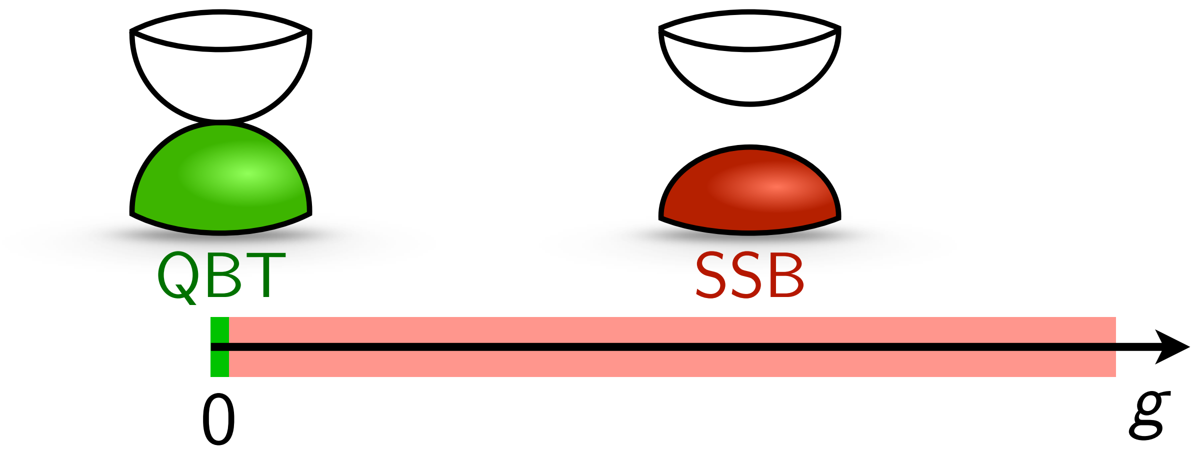


Stable Dirac semimetal
down to ~5 K

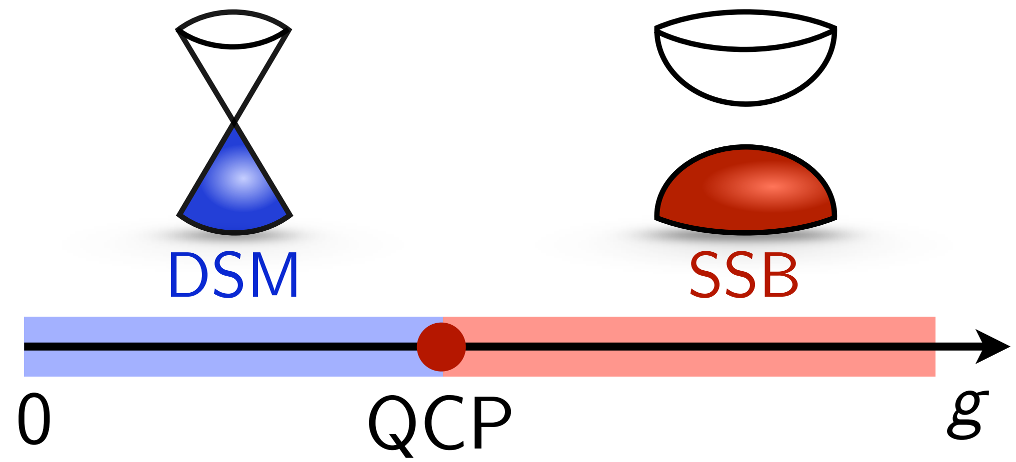
[Morozov *et al.*, PRL '08]
[Elias *et al.*, Nat. Phys. '11]

Interaction effects: General picture

Quadratic band touching (isotropic):



Dirac semimetal:



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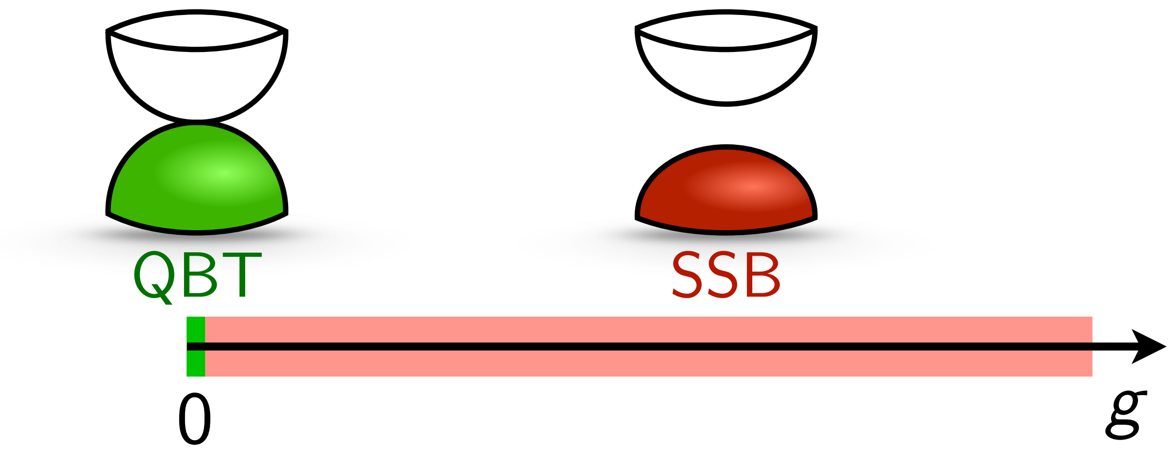
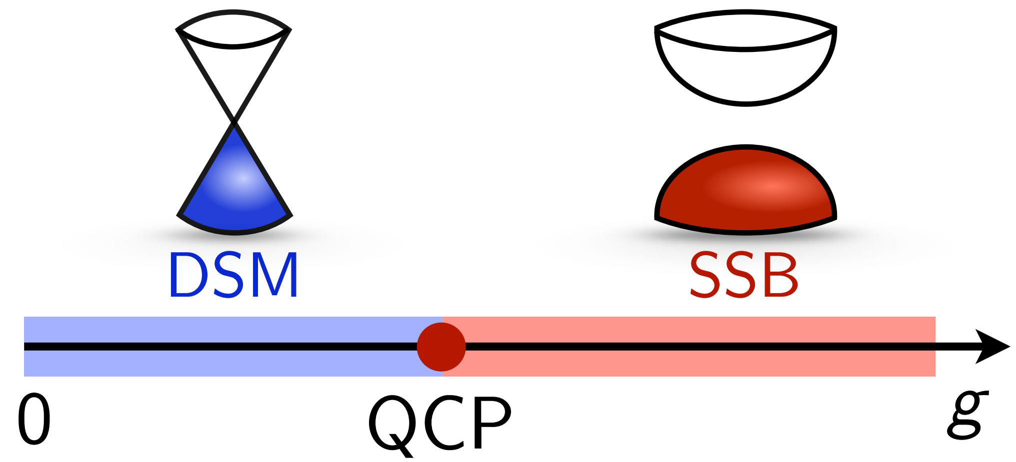


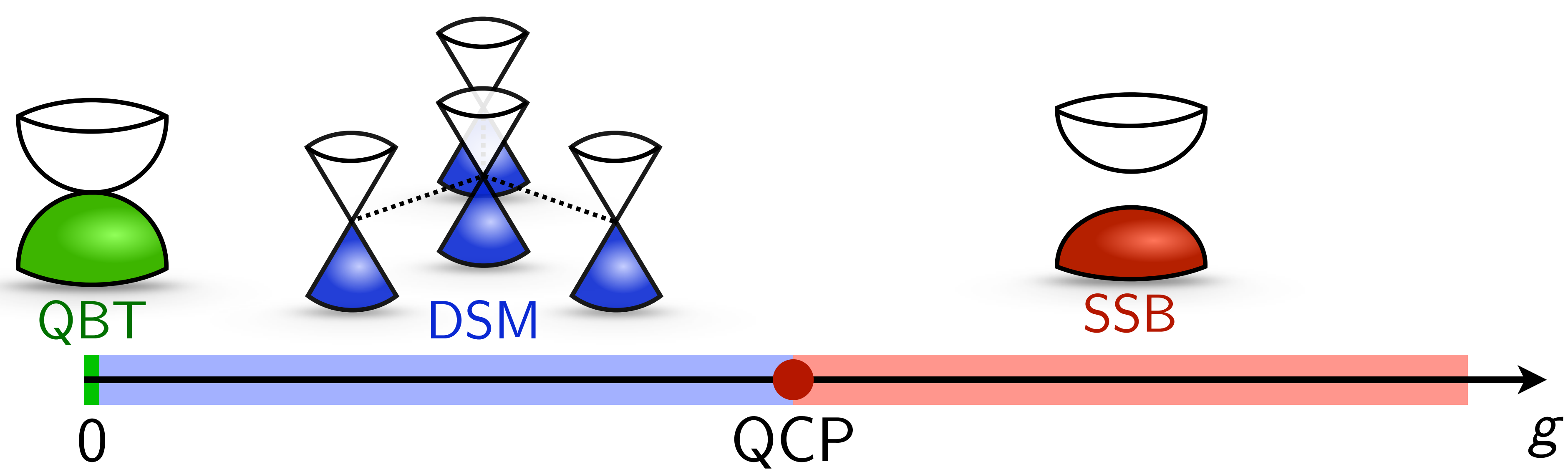
Dirac semimetal:



Today:

Quadratic band touching with C_3 symmetry:

... as in bilayer graphene

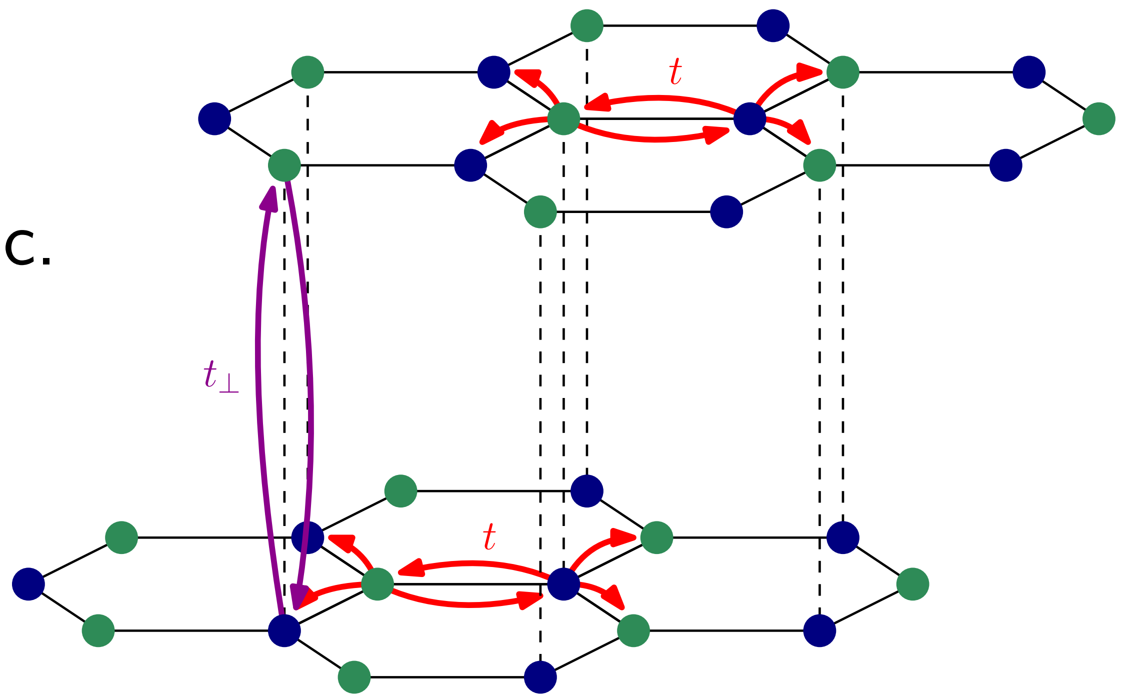


... with emergent Lorentz symmetry

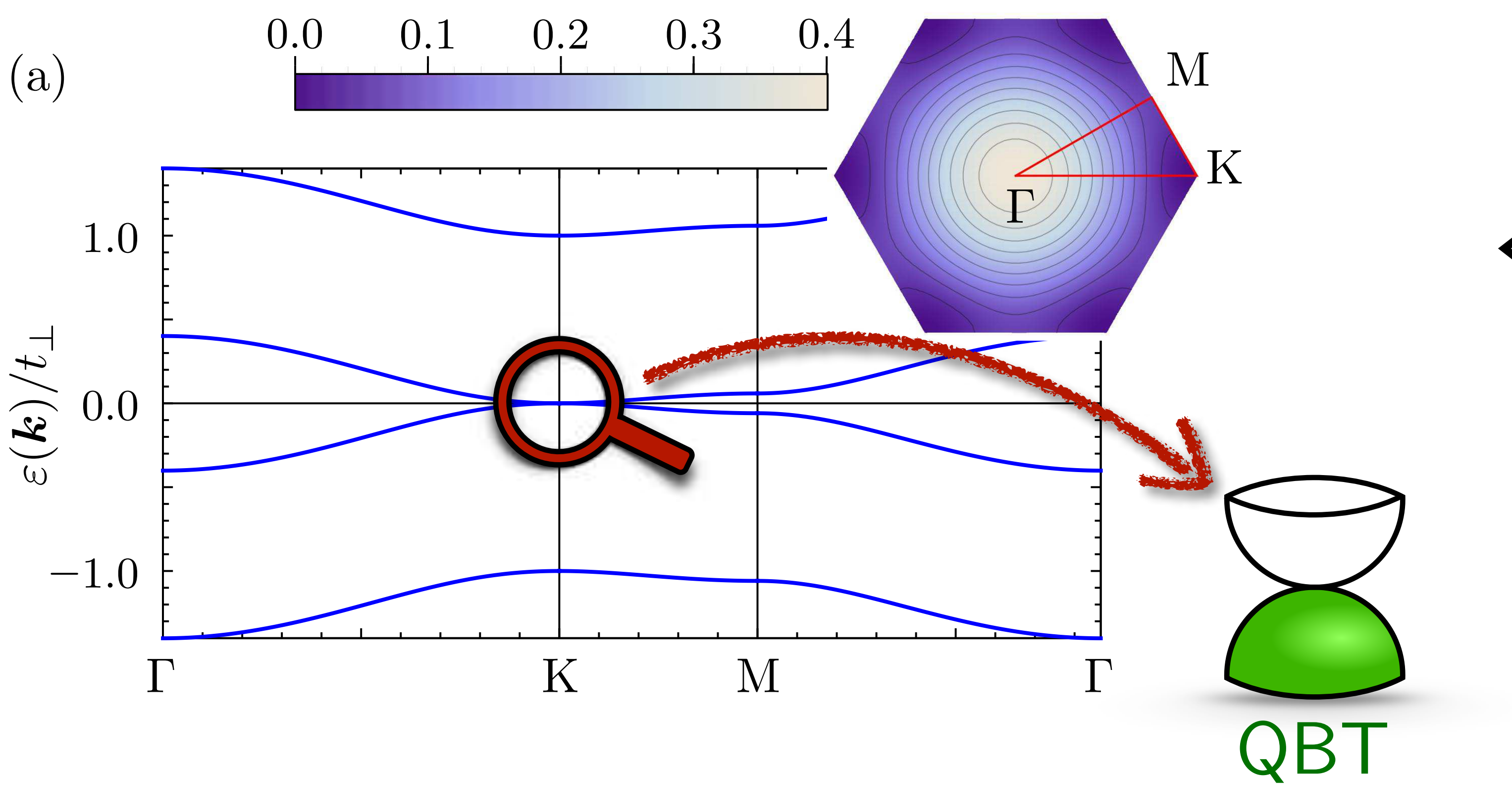
Lattice model

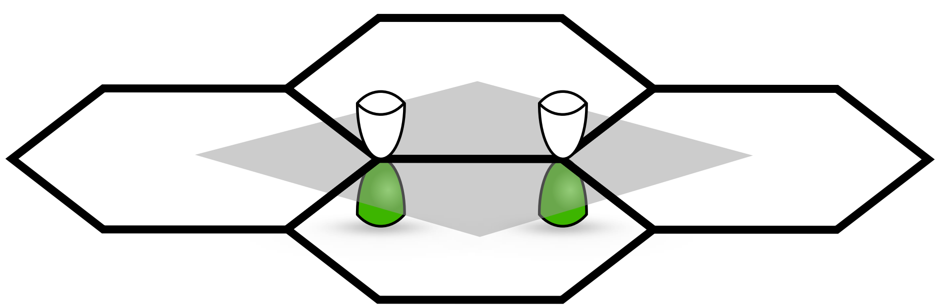
Hamiltonian:

$$H_0 = -t \sum_{\langle ij \rangle} \sum_{m=1}^{2} a_{im}^\dagger b_{jm} - t_\perp \sum_i a_{i1}^\dagger b_{i2} + \text{H.c.}$$



Spectrum:





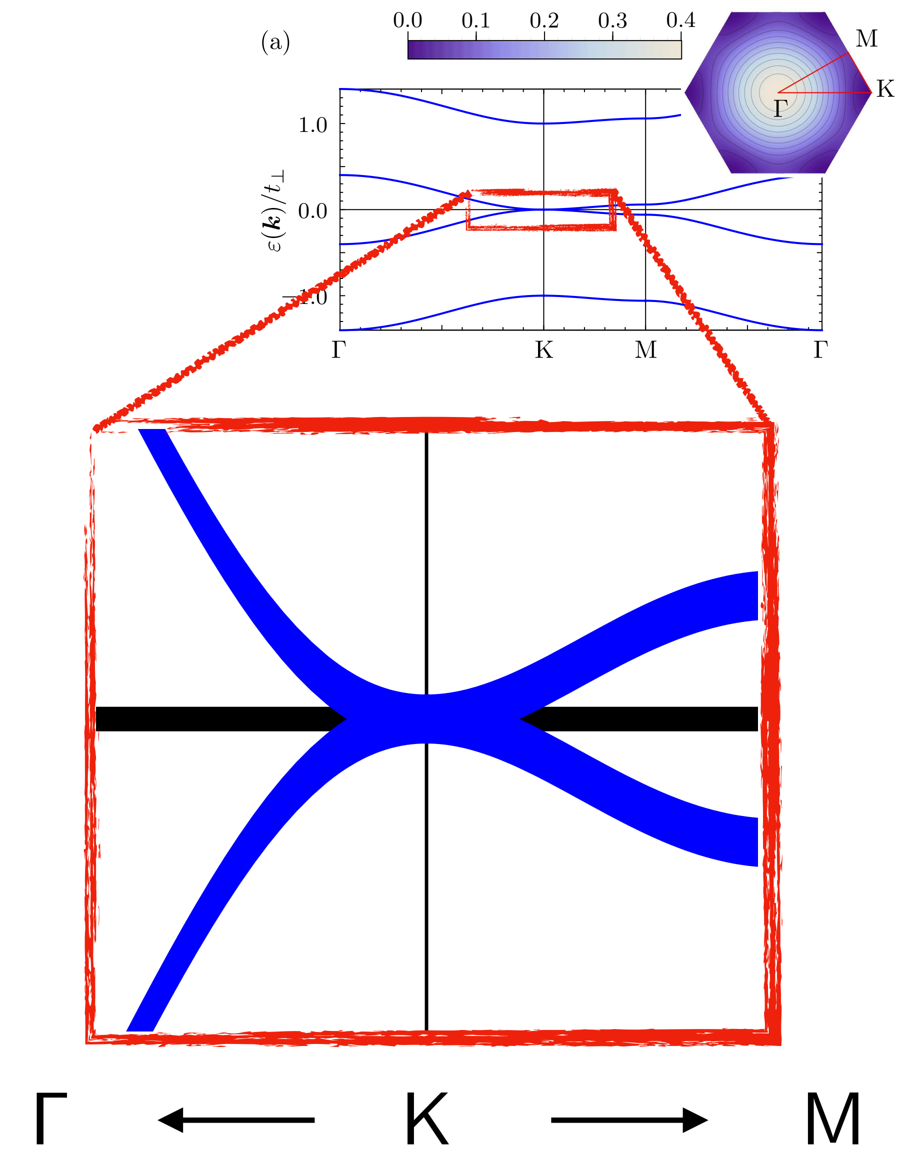
Low-energy theory

anisotropy!

Spectrum near **K:**

$$\varepsilon_{\mathbf{K}+\mathbf{p}} \approx \pm \frac{3t^2}{4t_\perp}\mathbf{p}^2 \left(1 - \frac{|\mathbf{p}|}{2\sqrt{3}}\cos(3\varphi)\right) + \mathcal{O}(p^4)$$

... for $\tan\varphi = p_y/p_x$



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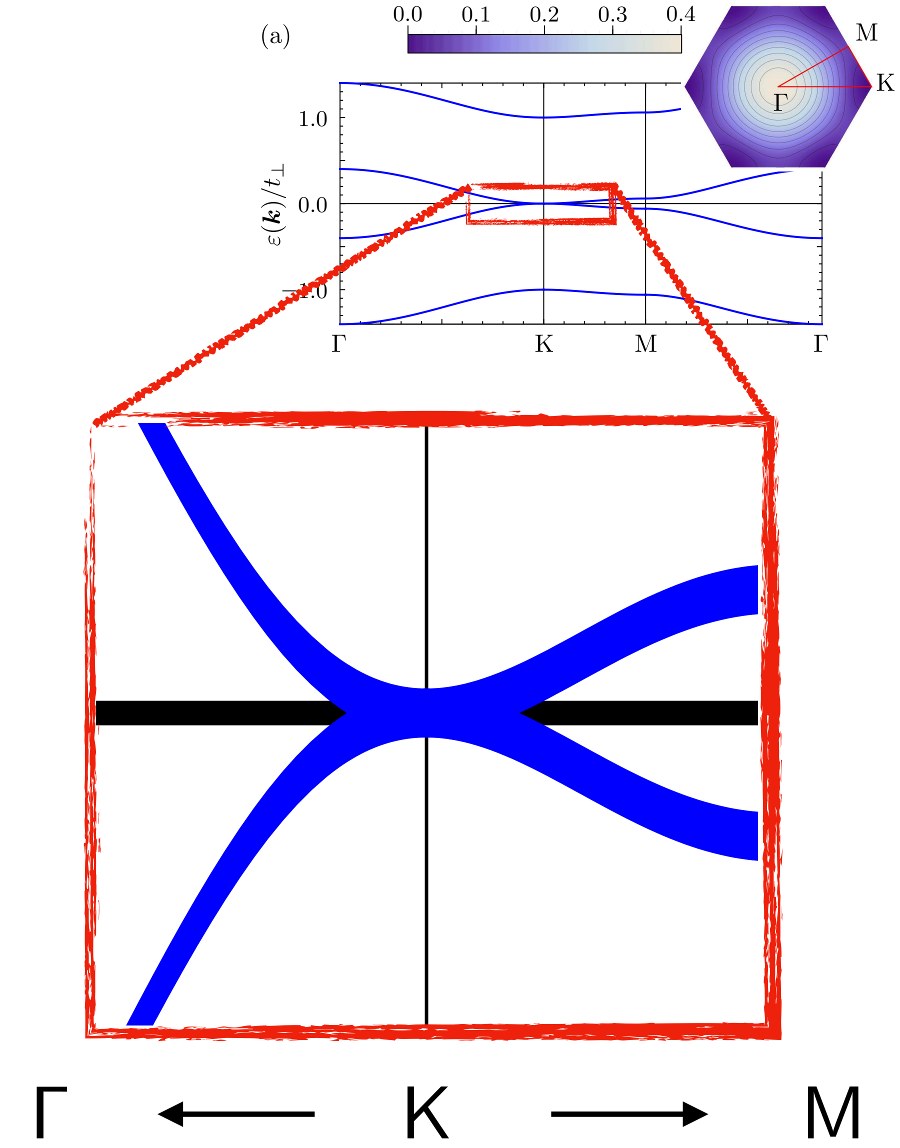

Lagrangian:

layer

$\propto p^2$

$$\mathcal{L}_0 = \Psi^\dagger\Big[\partial_\tau + (\partial_x^2-\partial_y^2)(\sigma^1\otimes\mathbb{1}_2) + 2\partial_x\partial_y(\sigma^2\otimes\mathbb{1}_2)$$

valley

$$-\frac{1}{2\sqrt{3}}\left((\partial_x^3+\partial_x\partial_y^2)(\sigma^1\otimes\sigma^3)+(\partial_x^2\partial_y+\partial_y^3)(\sigma^2\otimes\sigma^3)\right)\Big]\Psi$$

$\propto p^3$

... irrelevant, but not unimportant!

Interactions

Density-density-type interaction:

... most dominant interaction in t-V model

[Vafek, PRB '10]

$$\mathcal{L}_{\text{int}} = -\frac{g}{2}\left[\Psi^\dagger(\sigma^3 \otimes \sigma^3)\Psi\right]^2$$

... closed under RG

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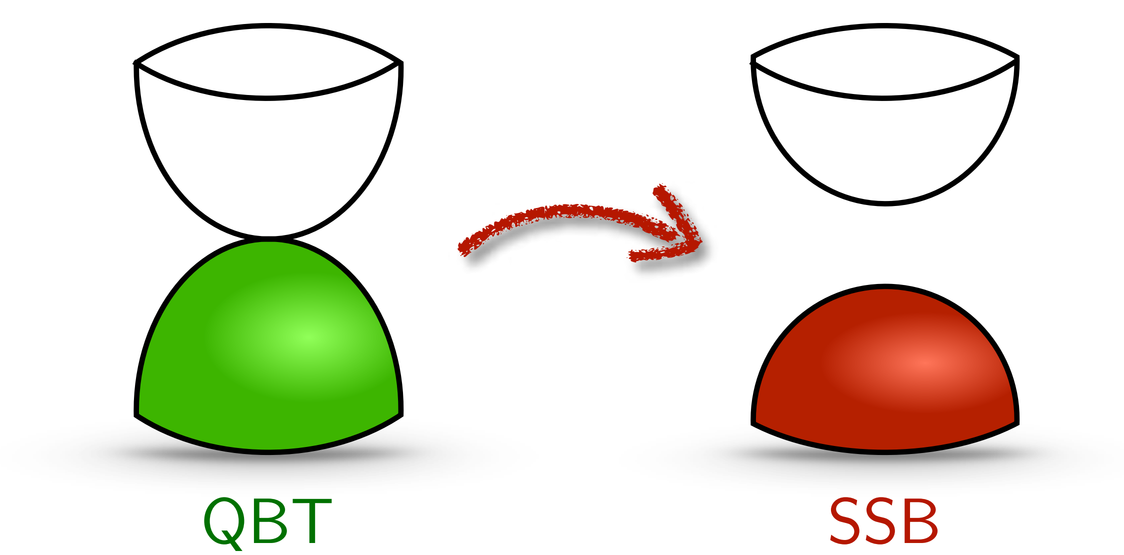
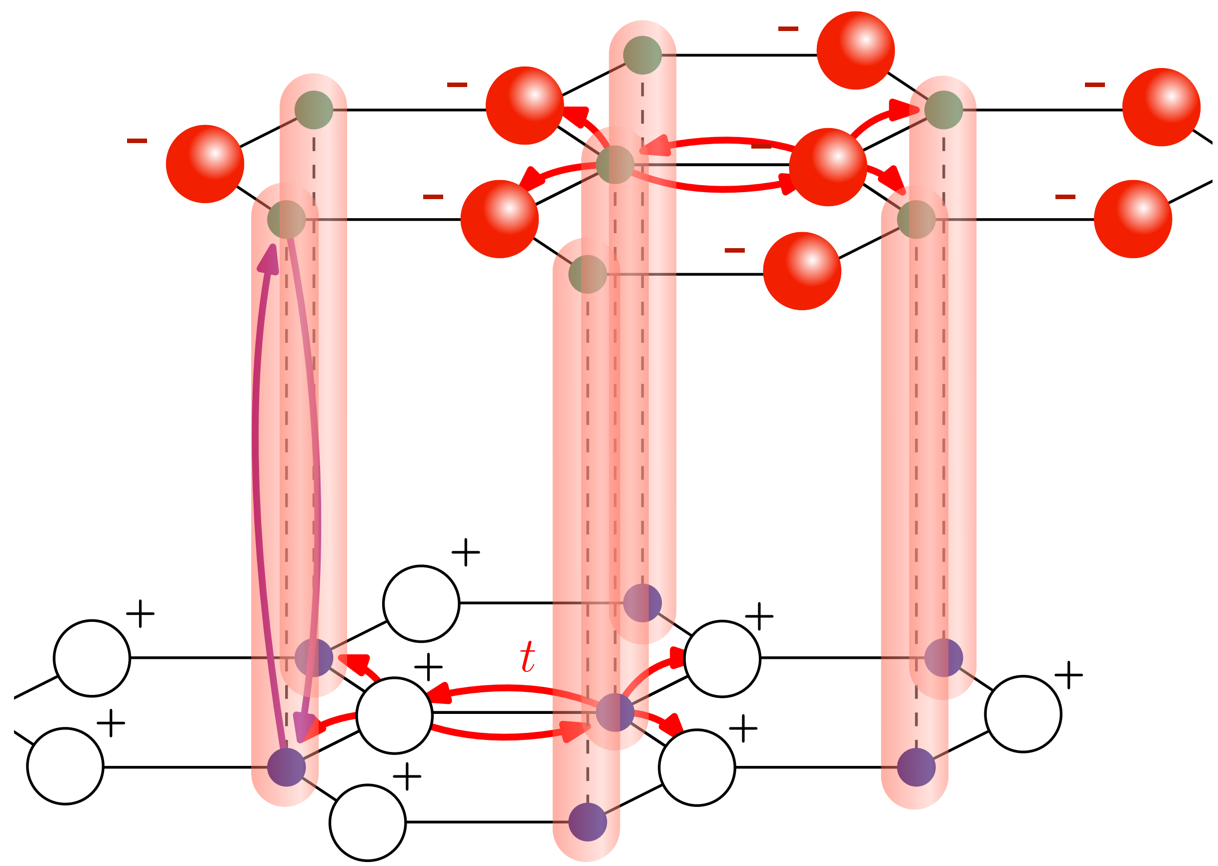
Ordered state $\langle \Psi^\dagger(\sigma^3 \otimes \sigma^3)\Psi \rangle \neq 0$:



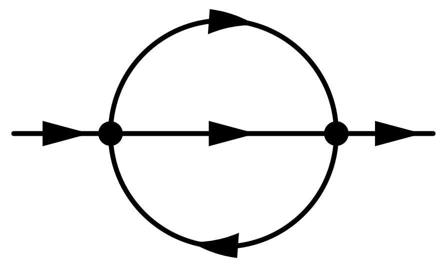
Gap opening



Charge layer polarized

Renormalization group

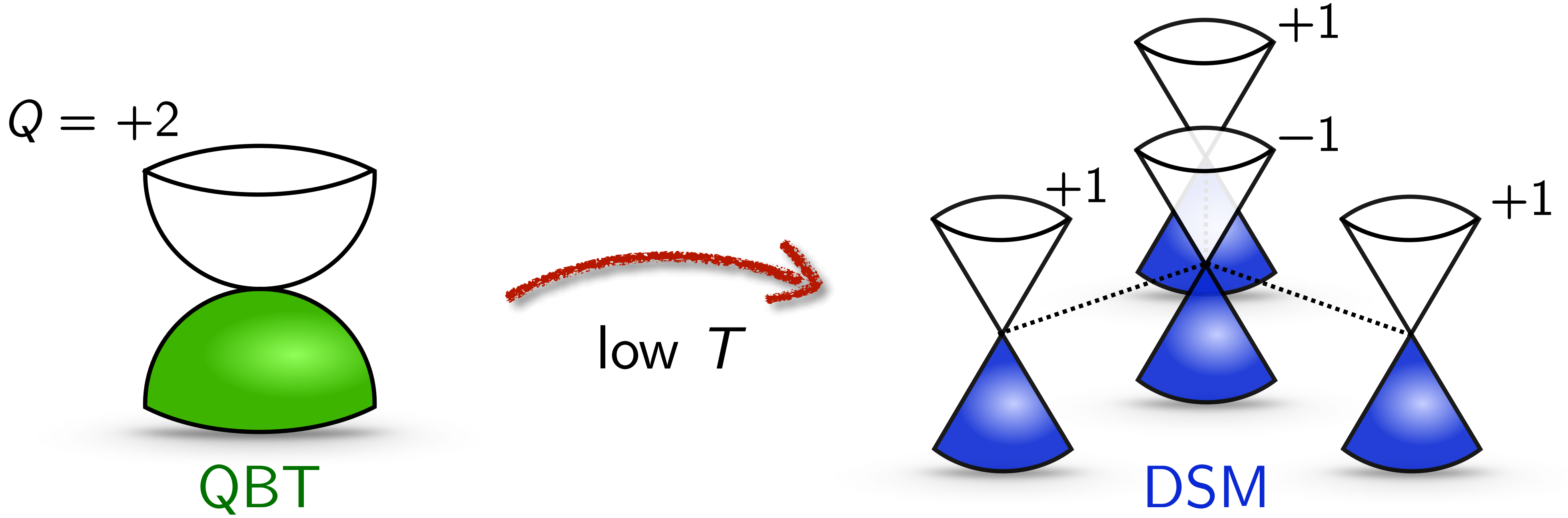
Self-energy:



$$\propto \Psi^\dagger \left[-\partial_x(\sigma^1 \otimes \sigma^3) + \partial_y(\sigma^2 \otimes \sigma^3)\right] \Psi$$

$\propto p$
relevant!

Spectrum:



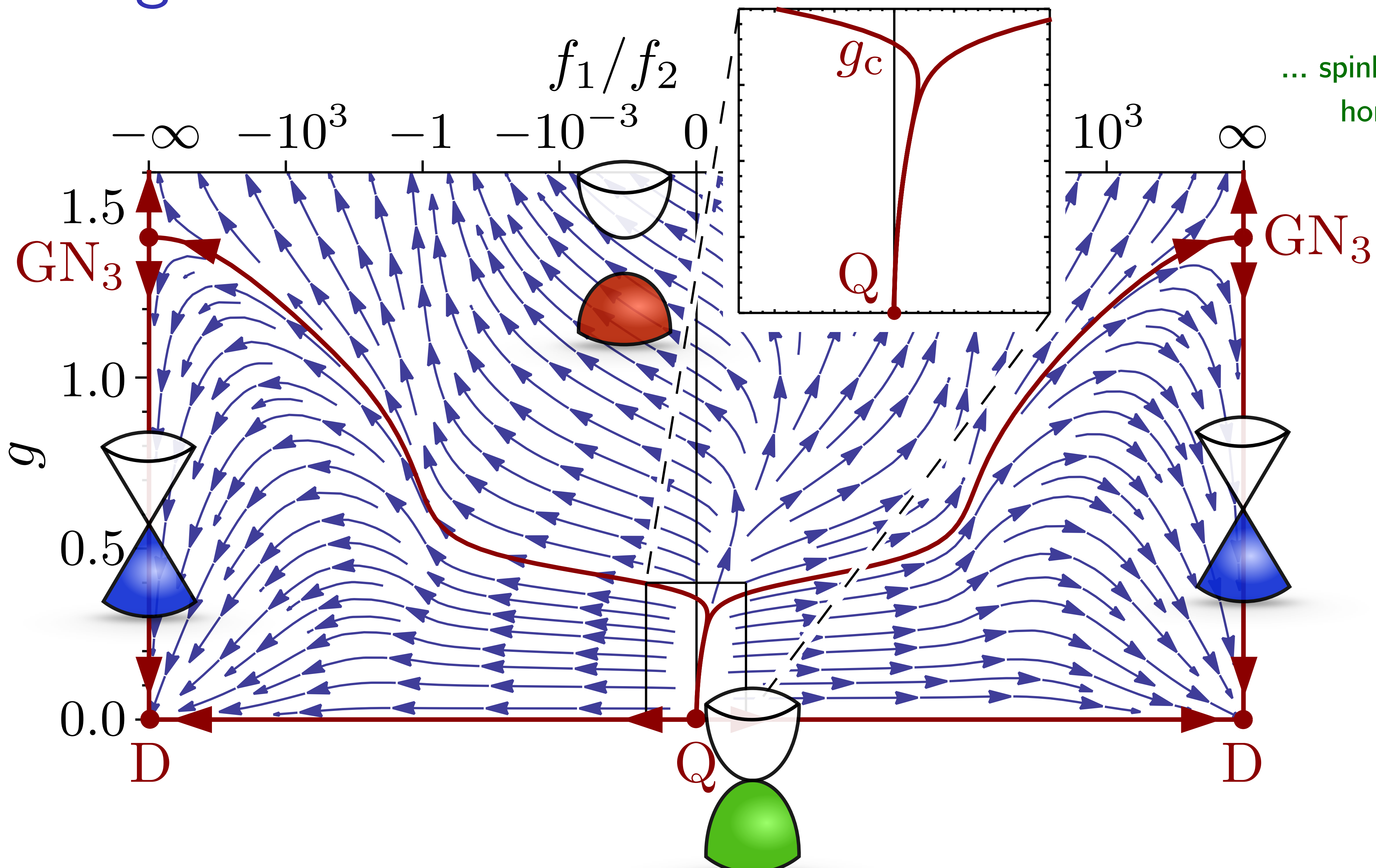
[Pujari *et al.*, PRL '16]

... technical obstacles: two-loop, nonrelativistic, anisotropic propagator
... trick: real-space evaluation
[Groote *et al.*, NPB '99]

Flow diagram: $N = 2$

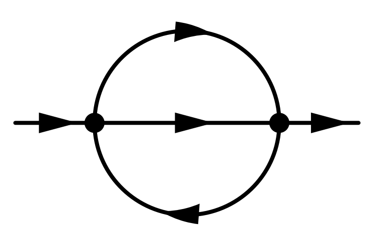
$f_3 = \frac{1}{2\sqrt{3}}$

... spinless fermions on honeycomb bilayer



Effective Hamiltonian: $\mathcal{H}_0(\mathbf{p}) \propto f_1 \mathcal{O}(p) + f_2 \mathcal{O}(p^2) + f_3 \mathcal{O}(p^3)$

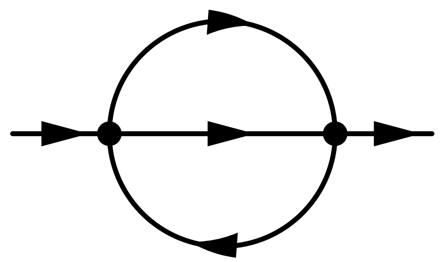
irrelevant

generated by  $\propto \frac{f_3}{f_2}$

[Ray, Vojta, LJ, PRB '18]

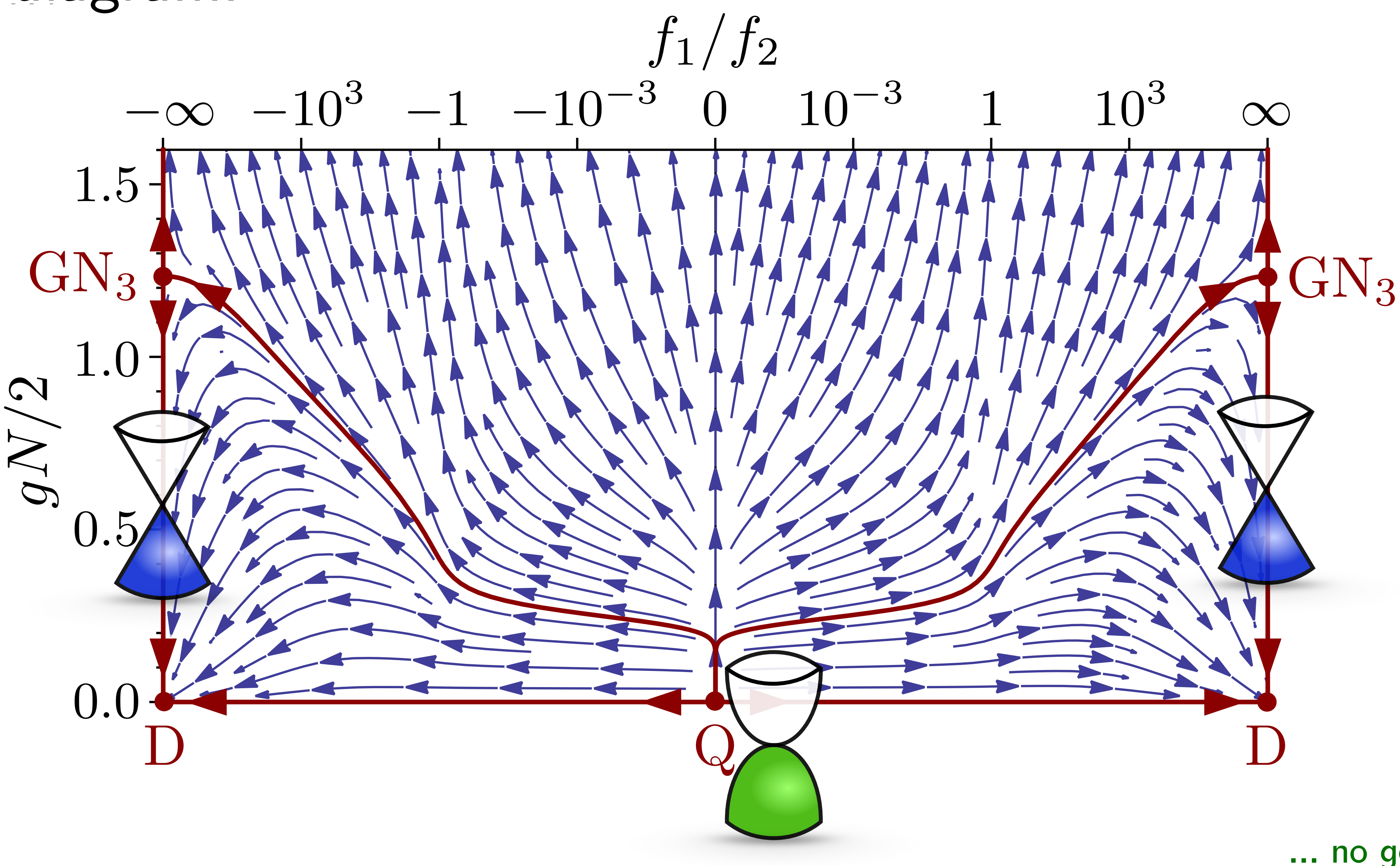
Cross-check: Mean-field limit ($N \to \infty$)

Self-energy:



$$\propto \frac{1}{N} \to 0$$

Flow diagram:

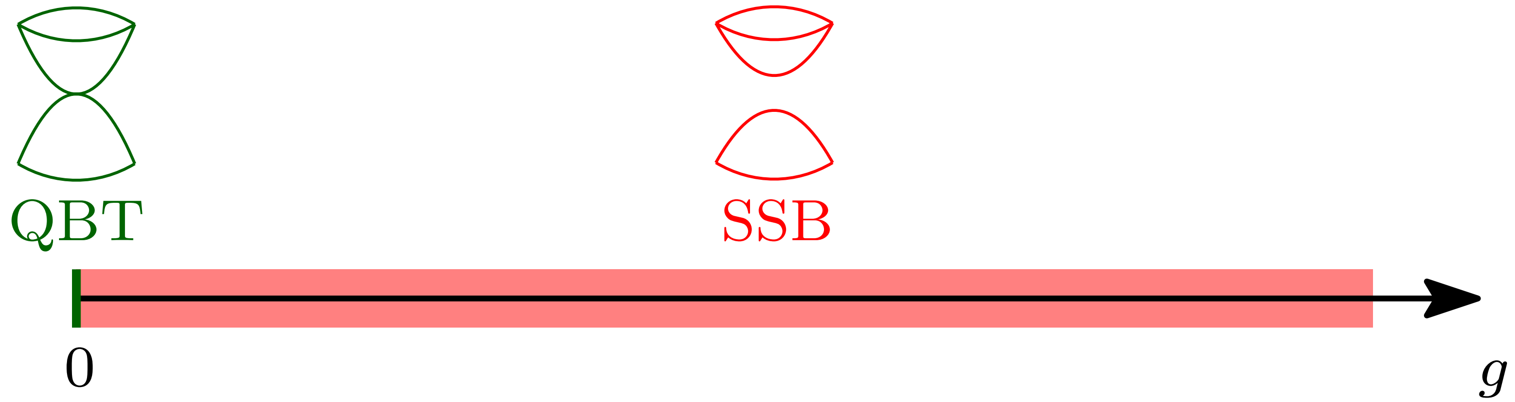


$$f_3 = \frac{1}{2\sqrt{3}}$$

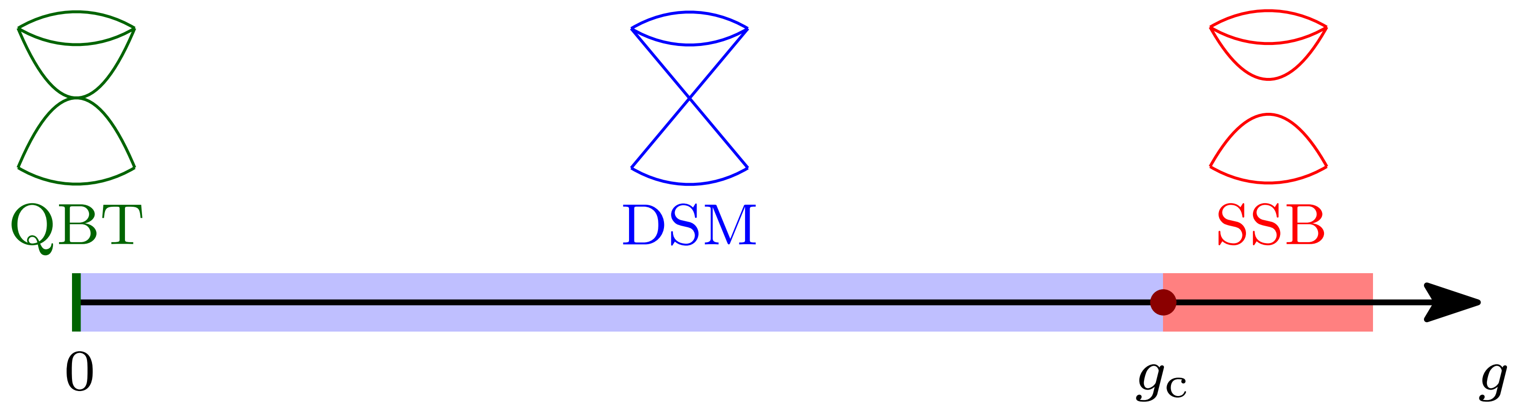
... no generation of Dirac cones

Low-temperature phase diagram

(a) O(2)



(b) $C_3, \quad t_{\mathrm{w}} = 0$



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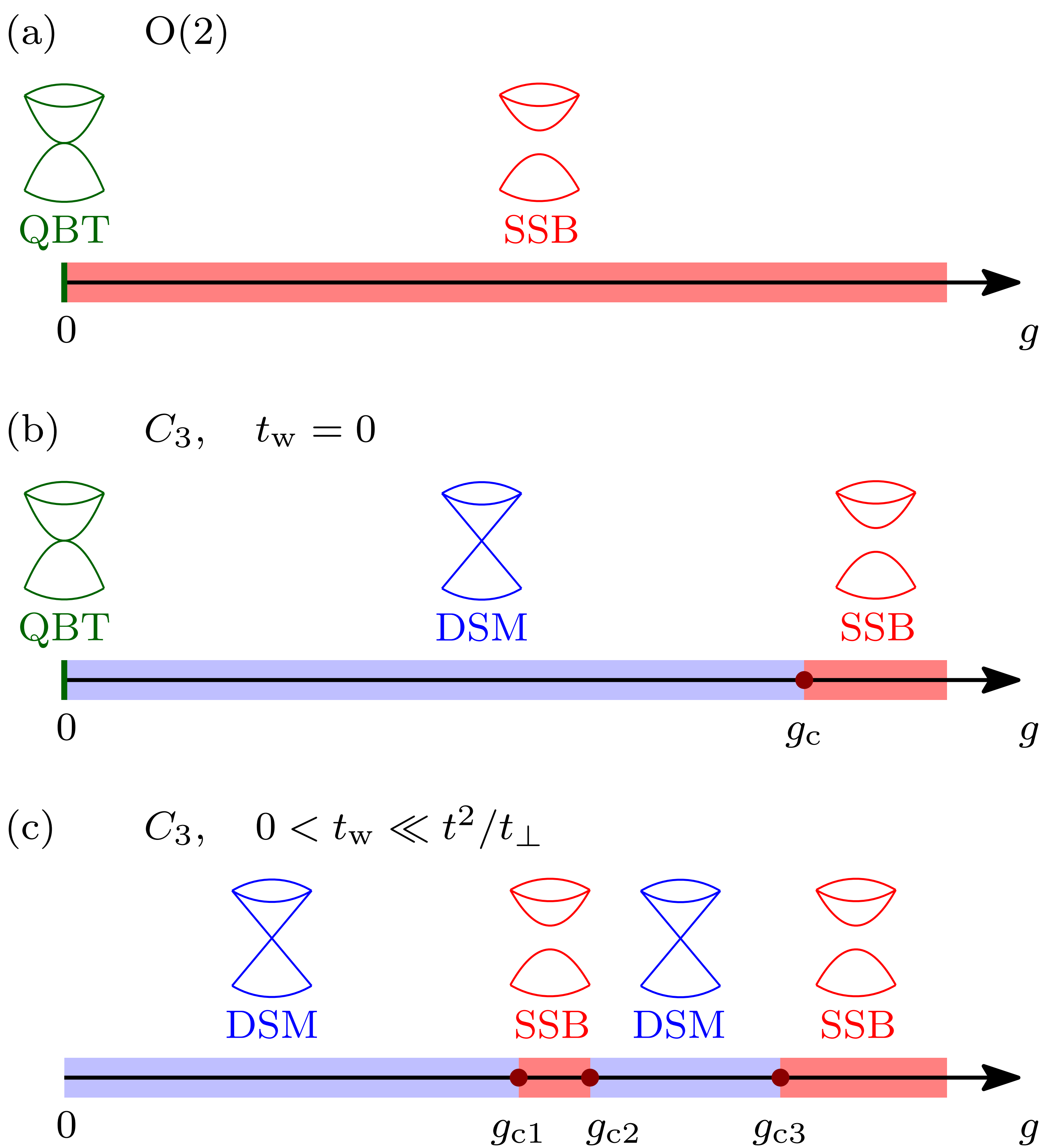
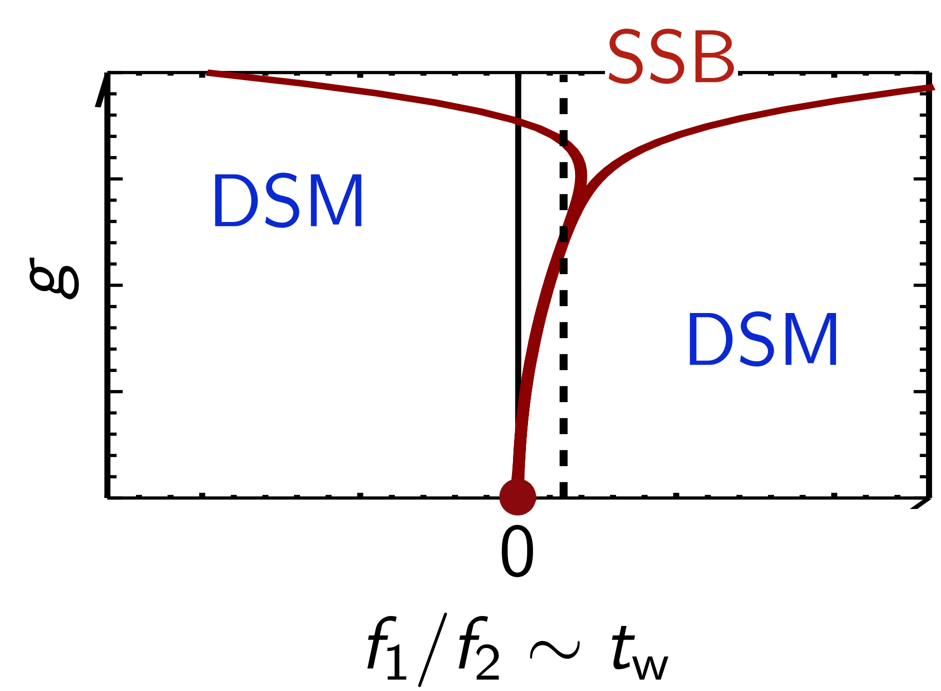




Quantum critical behavior: Spinless fermions

Order parameter: $\langle \phi \rangle \propto \langle \Psi^\dagger (\sigma^3 \otimes \sigma^3) \Psi \rangle$ Ising ... layer-inversion symmetry breaking

Universality class: Gross-Neveu-Ising ... with 8 two-component fermions

Correlation length:

$$\xi \propto |\delta g|^{-\nu} \quad \text{with} \quad \nu \approx 1$$

Correlator at criticality:

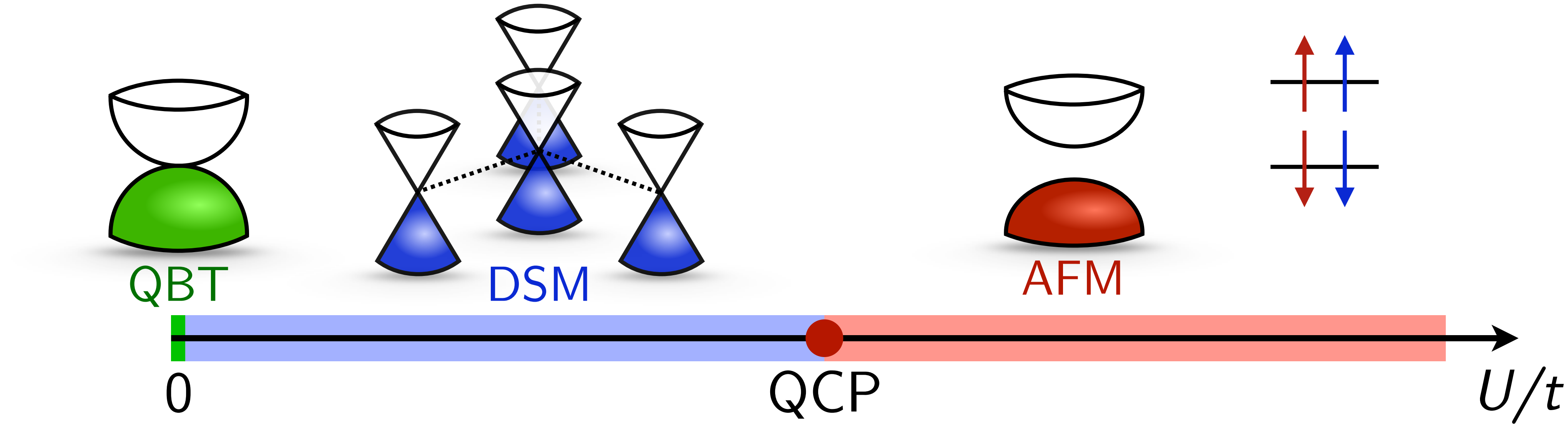
$$\langle \Psi(\tau, \mathbf{x}) \Psi^\dagger(0,0) \rangle \propto \frac{1}{(\tau^2 + \mathbf{x}^2)^{(1+z+\eta_\Psi)/2}} \quad \text{with} \quad \eta_\Psi \approx 0.026 \quad \text{and} \quad z = 1$$

emergent Lorentz symmetry!

. . . agrees with $\mathcal{O}(1/N^2)$ estimates:
$\nu = 0.98(9), \quad \eta_\Psi = 0.020(1)$

[Gracey, IJMP '94]

Spin-1/2 fermions



Gross-Neveu-Heisenberg universality:

... with 16 two-component fermions

$$\nu \approx 1.06(4), \qquad \eta_\phi \approx 1.01(1), \qquad \eta_\Psi \approx 0.026(1), \qquad z = 1$$

[LJ, Herbut, PRB ’14]
[Zerf *et al.*, PRD ’17]
[Gracey, PRD ’18]

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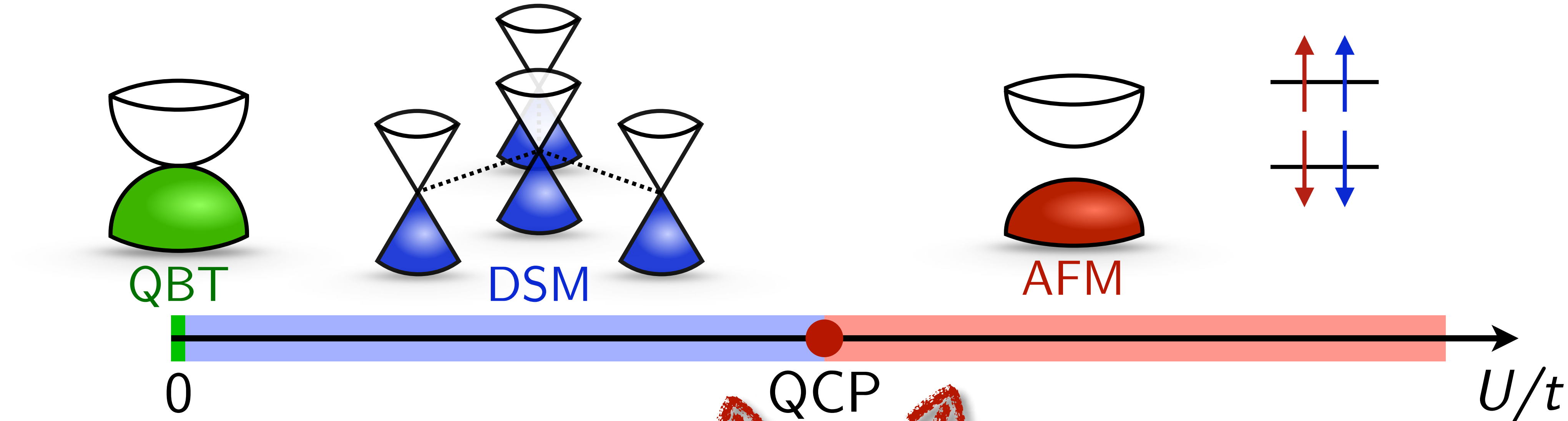

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[LJ, Herbut, PRB '14]
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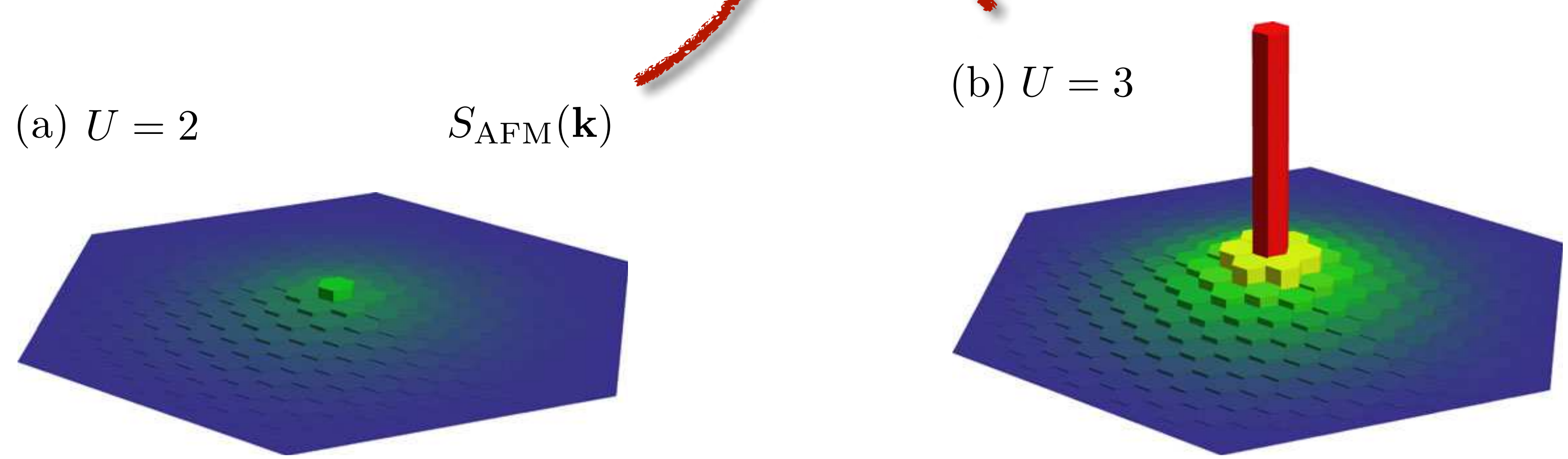
[Gracey, PRD '18]

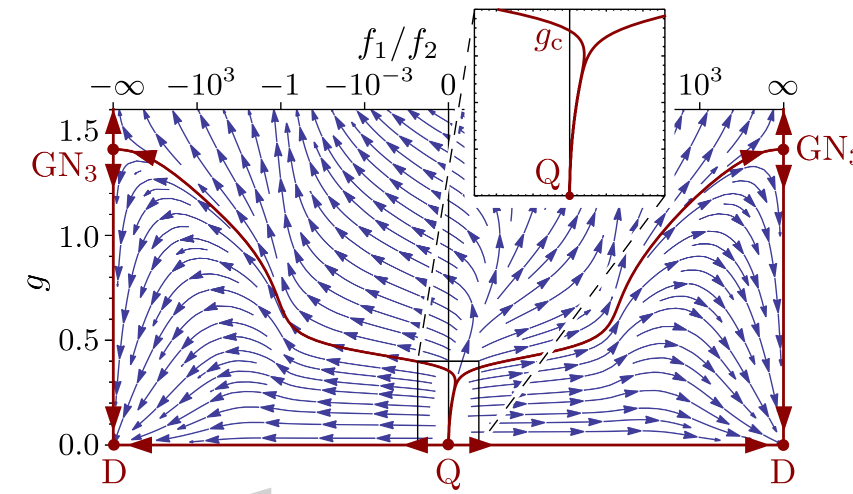
QMC:

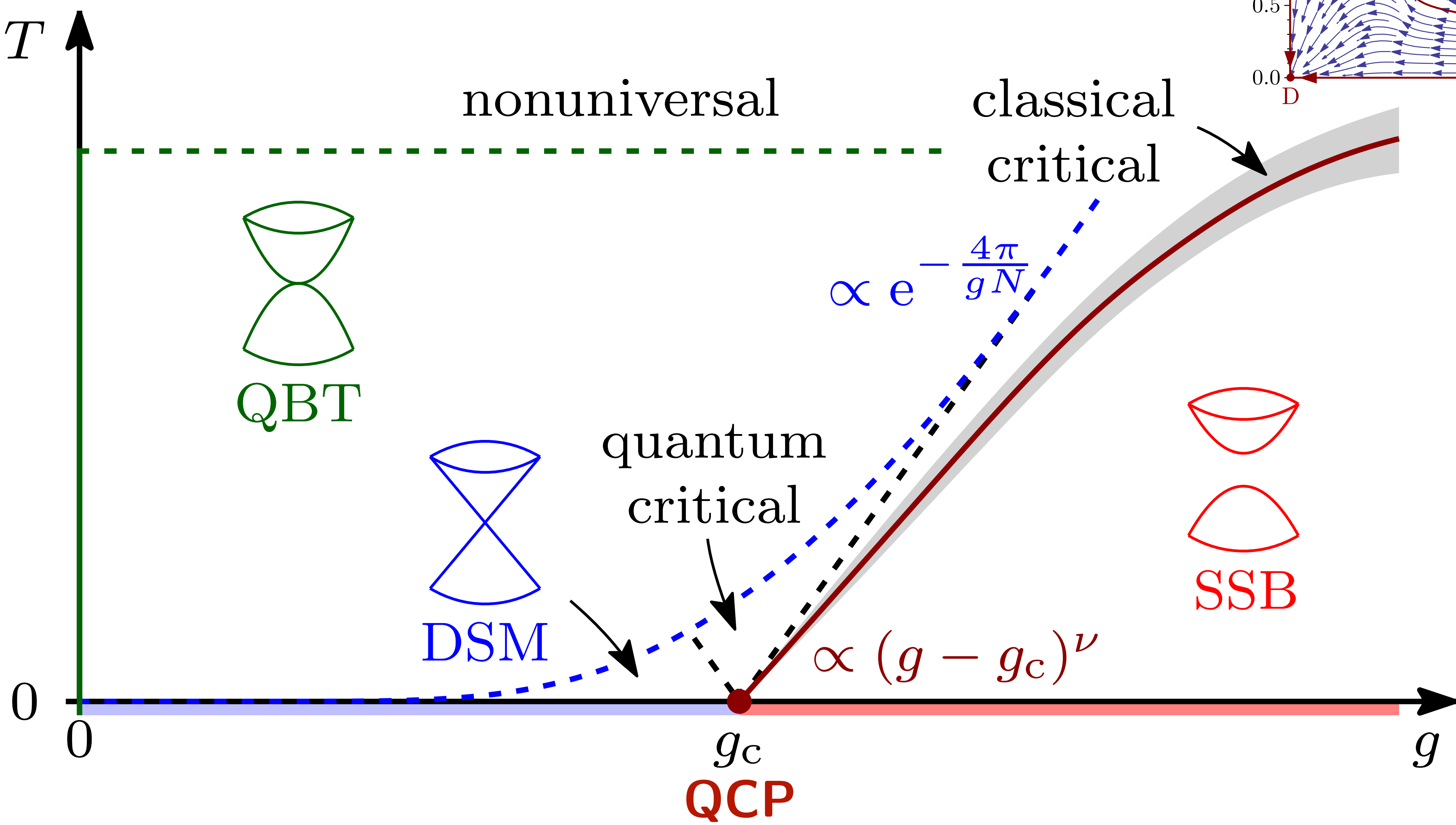


[Pujari *et al.*, PRL '16]

. . . with $\nu = 1.0(2)$ and $z = 0.9(2)$

Finite temperature





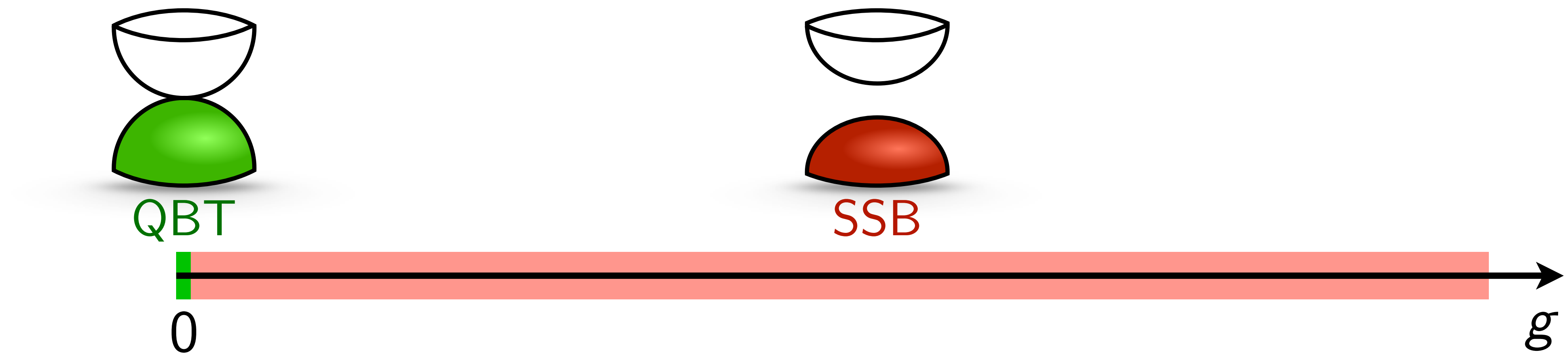
$\sim 5\,\mathrm{K}$

Bilayer graphene: $g \sim 2\pi/\ln(T_\star/T_c) \sim 0.6 \gtrsim g_c \sim 0.4$

$\sim t^2/t_\perp \sim 20\,\mathrm{eV}/k_B$

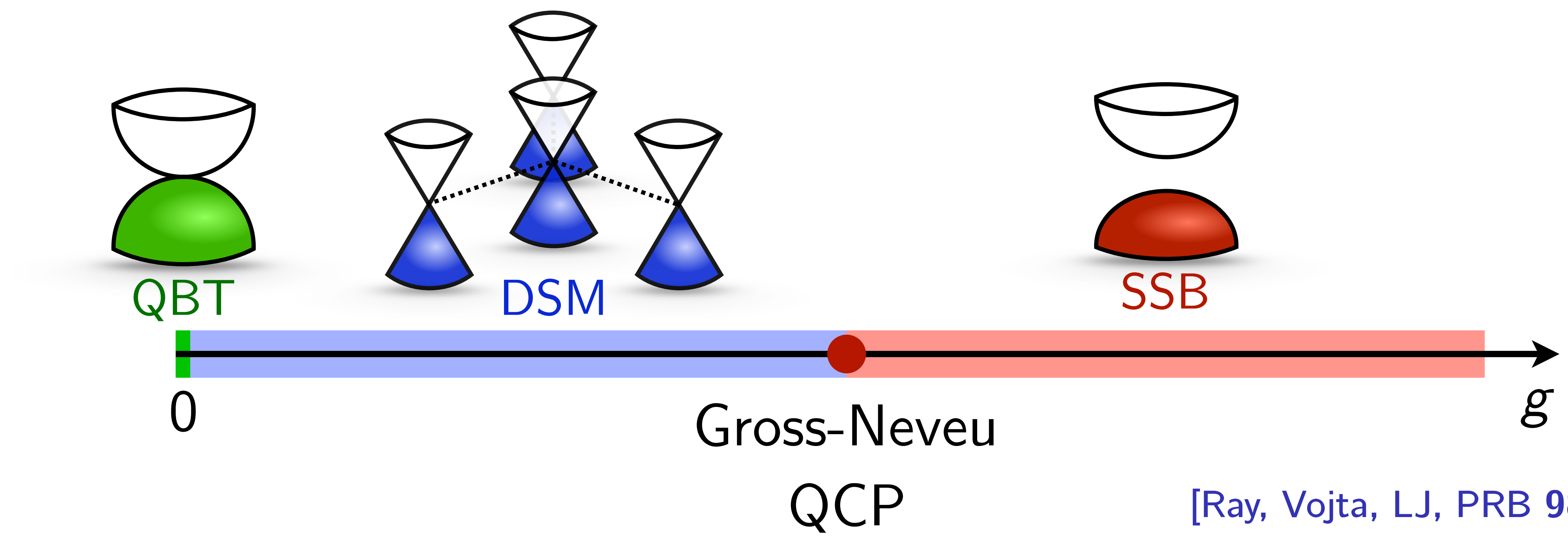
Conclusions

Isotropic quadratic band touching in 2D:



C_3-symmetric quadratic band touching in 2D:

... as in bilayer graphene



[Ray, Vojta, LJ, PRB **98**, 245128 (2018)]