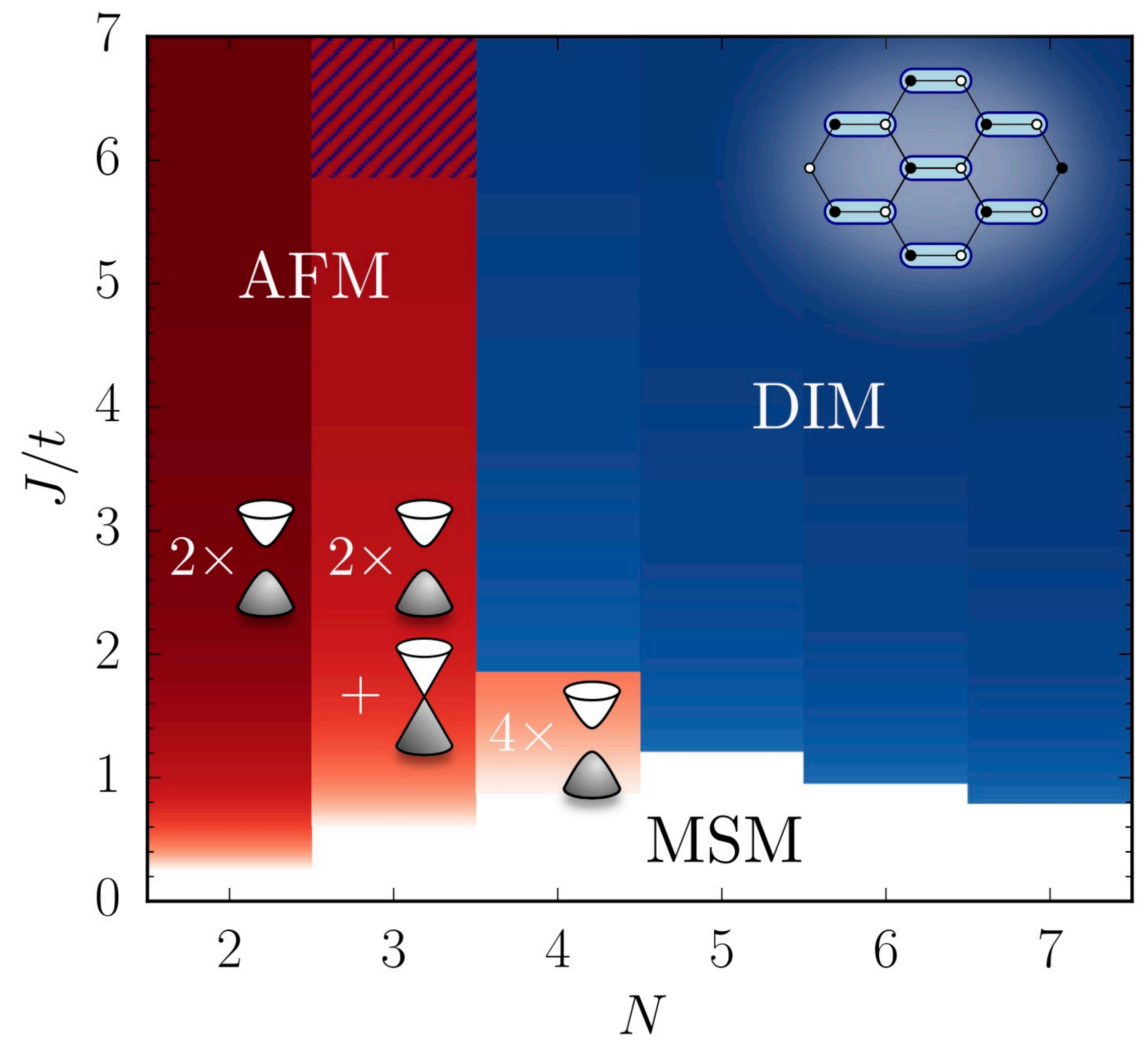
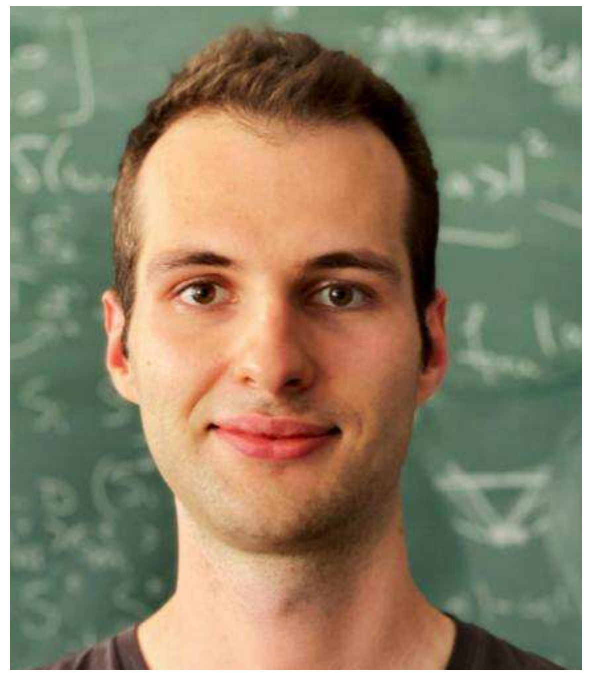
Interacting Majorana fermions



Lukas Janssen

TU Dresden



Urban Seifert (UCSB)

Sreejith Chulliparambil (TUD)
Hong-Hao Tu (TUD)
Matthias Vojta (TUD)
Xiao-Yu Dong (Ghent)

Shouryya Ray (TUD)
John Gracey (Liverpool)
Bernhard Ihrig (Cologne)
Daniel Kruti (Cologne)
Michael Scherer (Bochum)

Quantum Critical Matter JA 2306/4-1
Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

ct.qmat
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence

SFB 1143

Outline

(1) Introduction

(2) SO(N) Majoranas in frustrated magnets

(3) SO(N) Majorana-Hubbard models

(4) Conclusions

Slides available on https://tu-dresden.de/physik/qcm/vortraege

Outline

(1) Introduction

(2) SO(N) Majoranas in frustrated magnets

(3) SO(N) Majorana-Hubbard models

(4) Conclusions

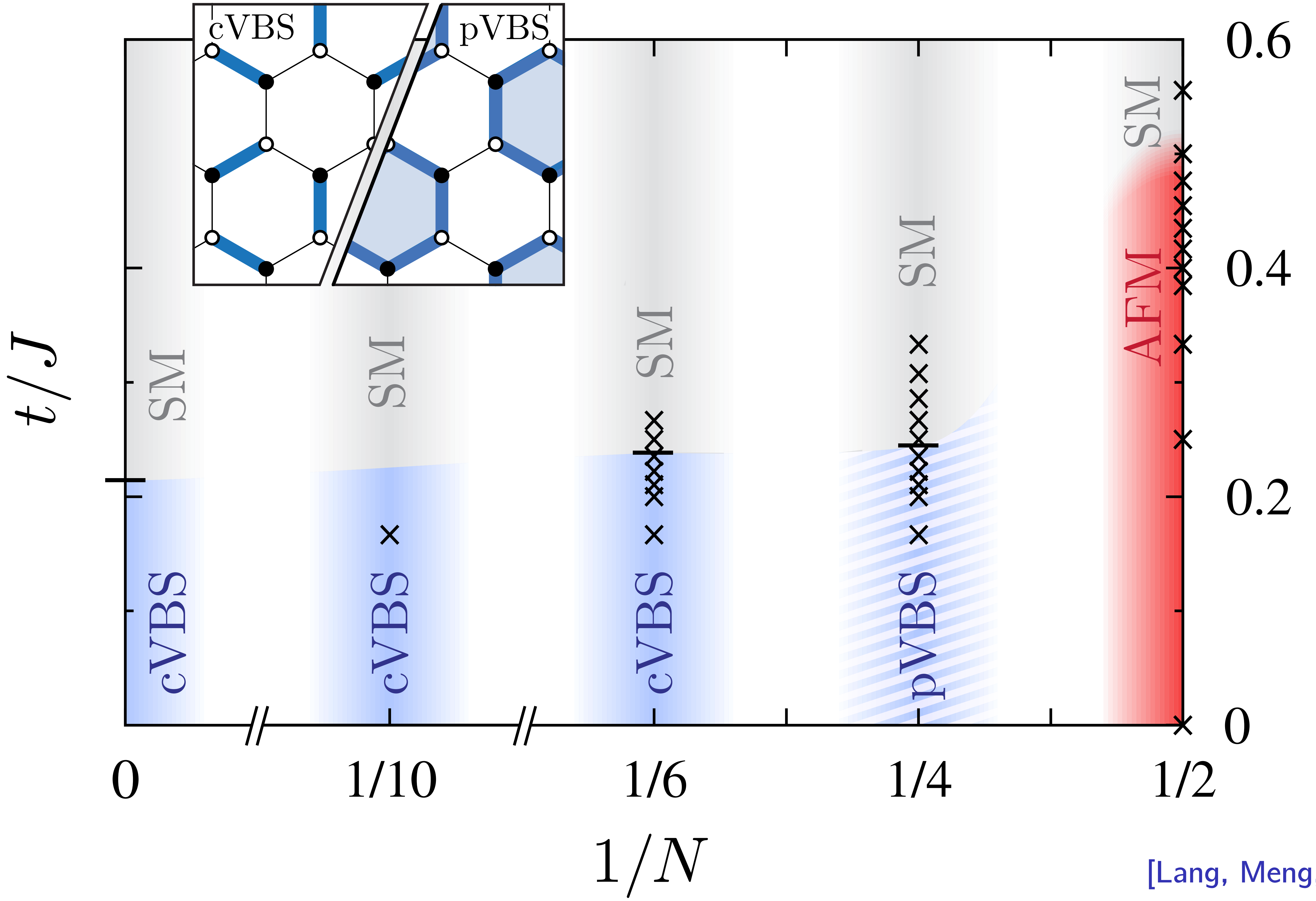
Slides available on https://tu-dresden.de/physik/qcm/vortraege

Motivation: SU(N) Hubbard-Heisenberg models

Hamiltonian: $$H = -t \sum_{\langle ij \rangle, \alpha} (c^\dagger_{i\alpha} c_{j\alpha} + \text{H.c.}) - \frac{J}{2N} \sum_{\langle ij \rangle, \alpha, \beta} (c^\dagger_{i\alpha} c_{j\alpha} c^\dagger_{j\beta} c_{i\beta} + c^\dagger_{j\alpha} c_{i\alpha} c^\dagger_{i\beta} c_{j\beta})$$ $\alpha, \beta = 1, \ldots, N$

[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
...

Phase diagram:



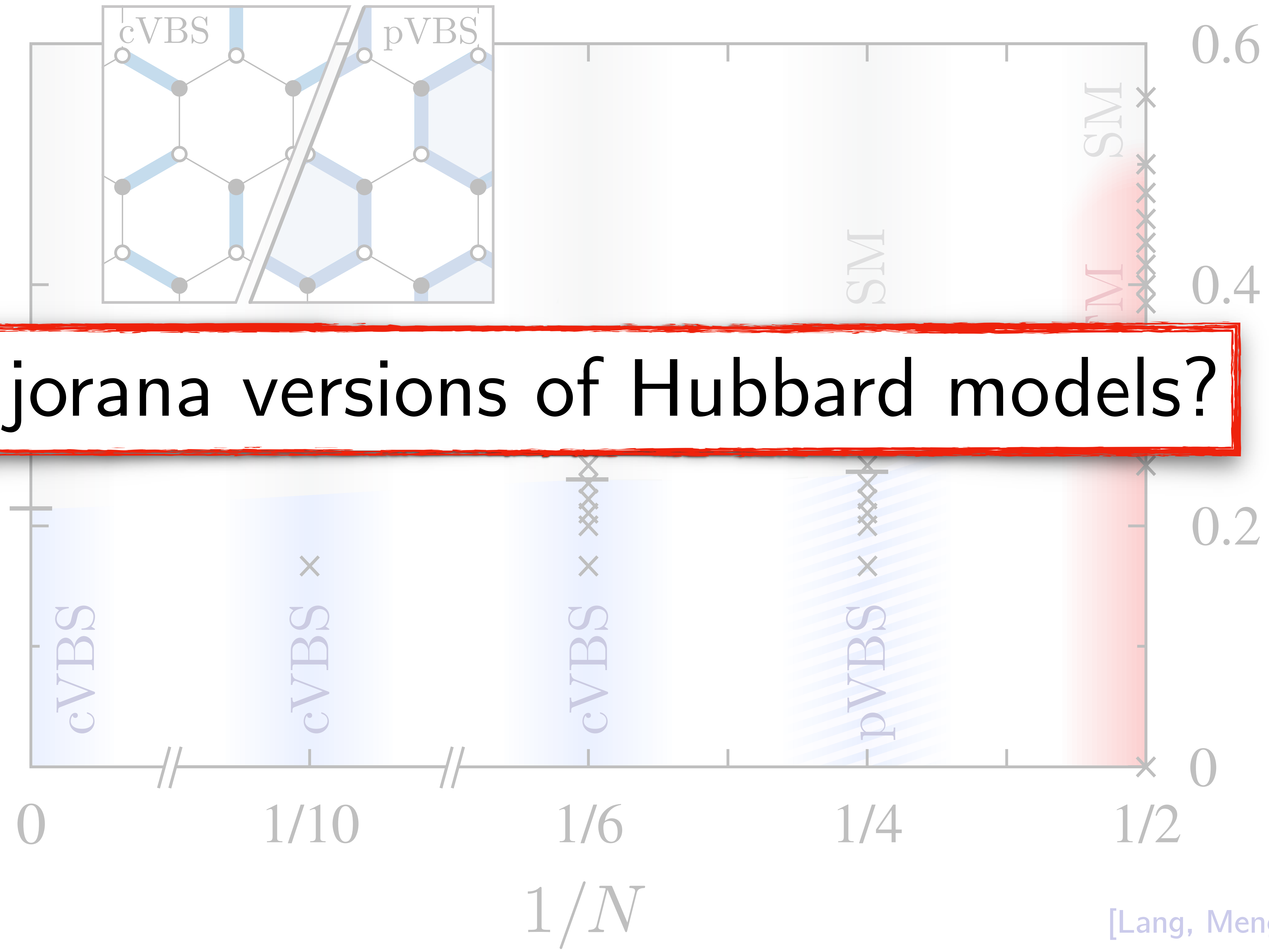
... on honeycomb lattice
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]

Motivation: SU(N) Hubbard-Heisenberg models

Hamiltonian: $$H = -t \sum_{\langle ij \rangle, \alpha} (c^\dagger_{i\alpha} c_{j\alpha} + \text{H.c.}) - \frac{J}{2N} \sum_{\langle ij \rangle, \alpha, \beta} (c^\dagger_{i\alpha} c_{j\alpha} c^\dagger_{j\beta} c_{i\beta} + c^\dagger_{j\alpha} c_{i\alpha} c^\dagger_{i\beta} c_{j\beta})$$ $\alpha, \beta = 1, \ldots, N$

[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
...

Phase diagram:

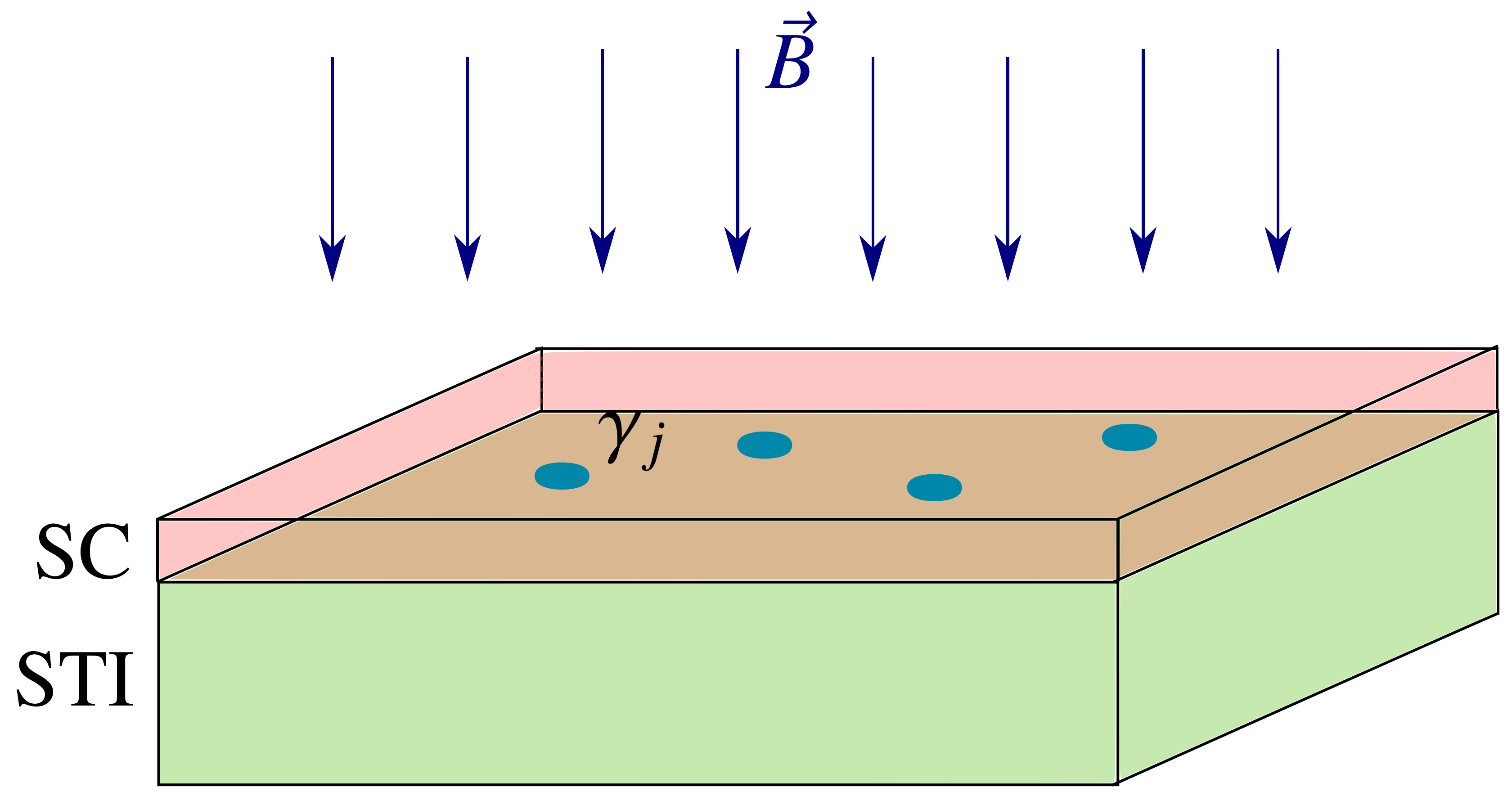


Majorana versions of Hubbard models?

... on honeycomb lattice

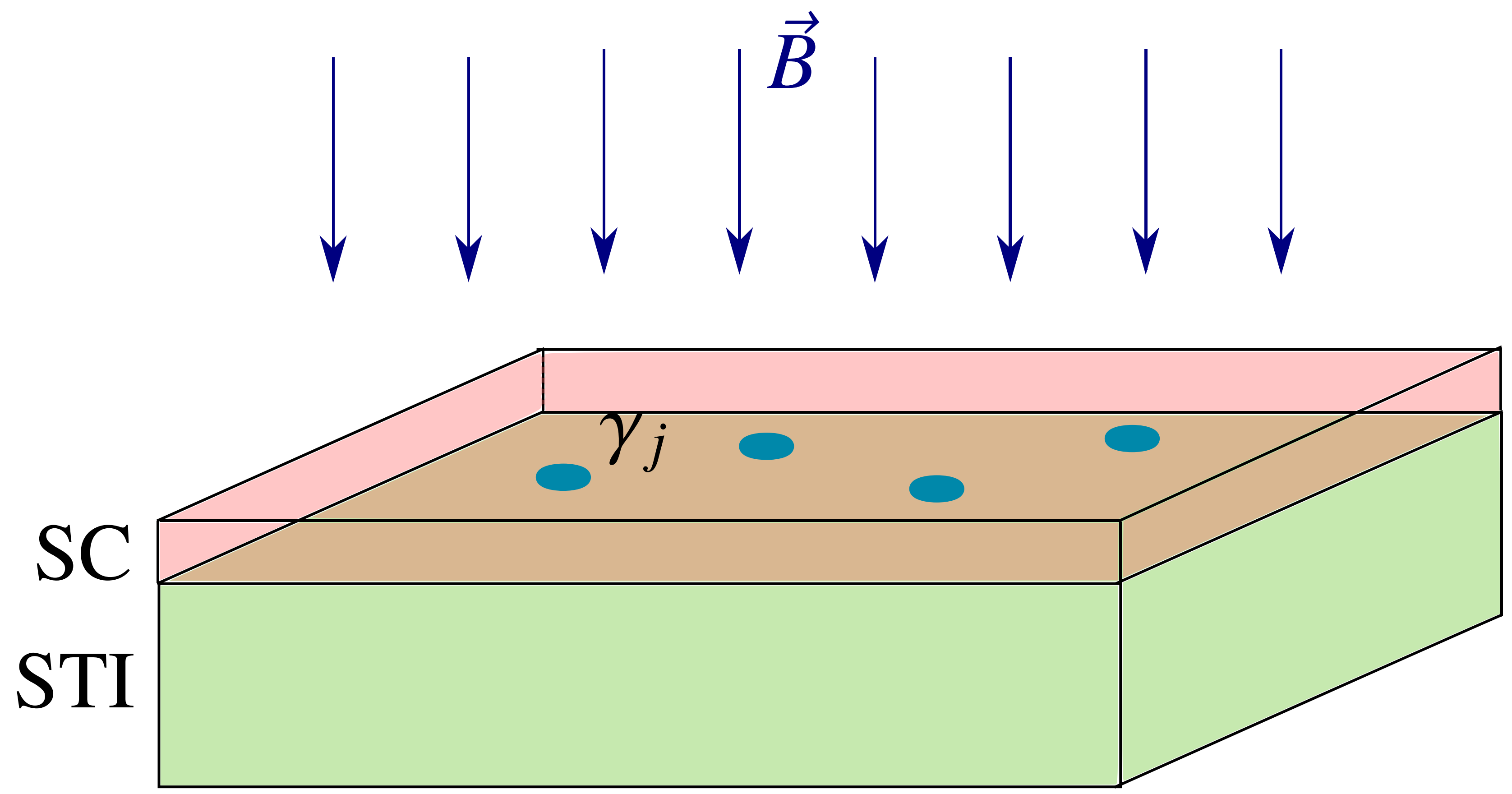
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]

Realization #1: Superconductor / topological insulator heterostructures



[Fu & Kane, PRL '08]
[Rahmani & Franz, Rep. Progr. Phys. '19]

Realization #1: Superconductor / topological insulator heterostructures



[Fu & Kane, PRL '08]
[Rahmani & Franz, Rep. Progr. Phys. '19]

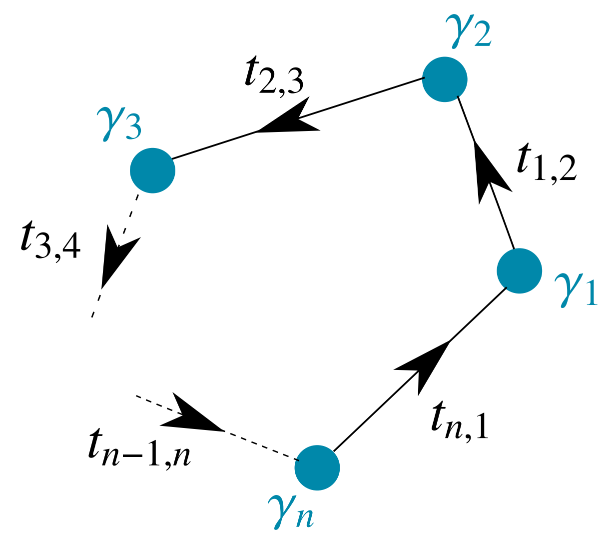
Effective model:

$$H = \sum_{ij} \mathrm{i} t_{ij} \gamma_i \gamma_j + \sum_{ijkl} g_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l + \dots$$

"Majorana-Hubbard models"

Grosfeld-Stern rule:

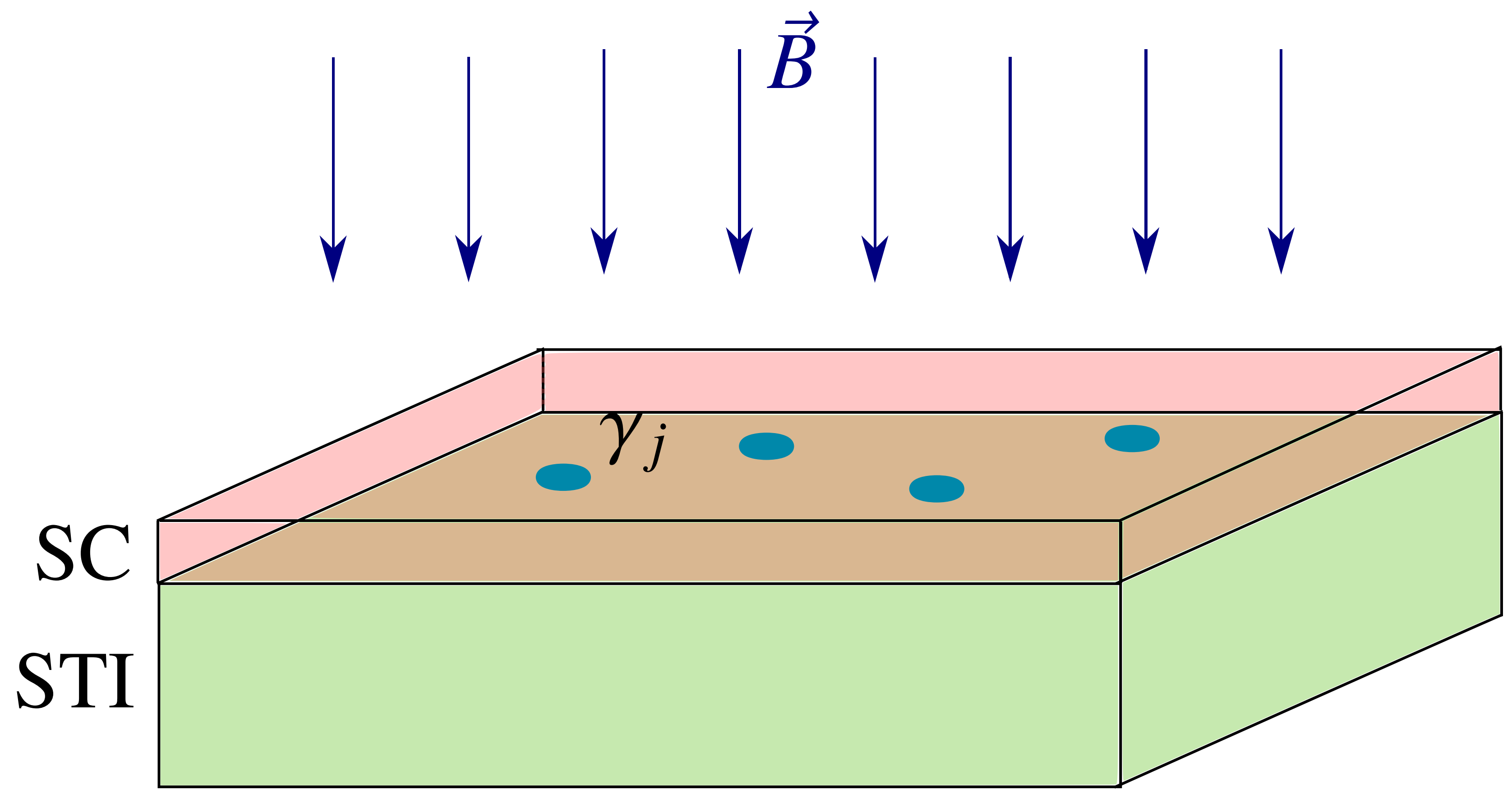
$$\arg\left(i^n t_{1,2} \dots t_{n-1,n} t_{n,1}\right) = \frac{\pi}{2}(n-2)$$



Square lattice: π flux
Honeycomb lattice: 0 flux

[Grosfeld & Stern, PRB '06]
[Liu & Franz, PRB '15]
[Wamer & Affleck, PRB '18]
[Li & Franz, PRB '18]
[Tummuru, Nocera, Affleck, PRB '21]
...

Realization #1: Superconductor / topological insulator heterostructures

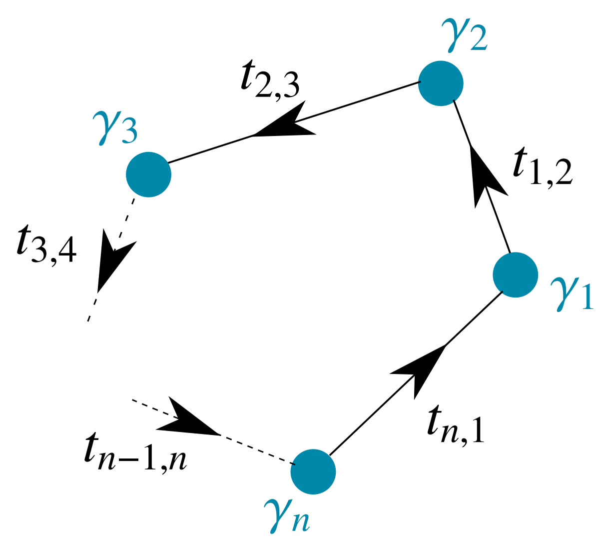


[Fu & Kane, PRL '08]
[Rahmani & Franz, Rep. Progr. Phys. '19]

tunable by gate voltage

Effective model: $$H = \sum_{ij} \mathrm{i} t_{ij} \gamma_i \gamma_j + \sum_{ijkl} g_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l + \dots$$ *"Majorana-Hubbard models"*

Grosfeld-Stern rule: $$\arg\left(i^n t_{1,2} \dots t_{n-1,n} t_{n,1}\right) = \frac{\pi}{2}(n-2)$$

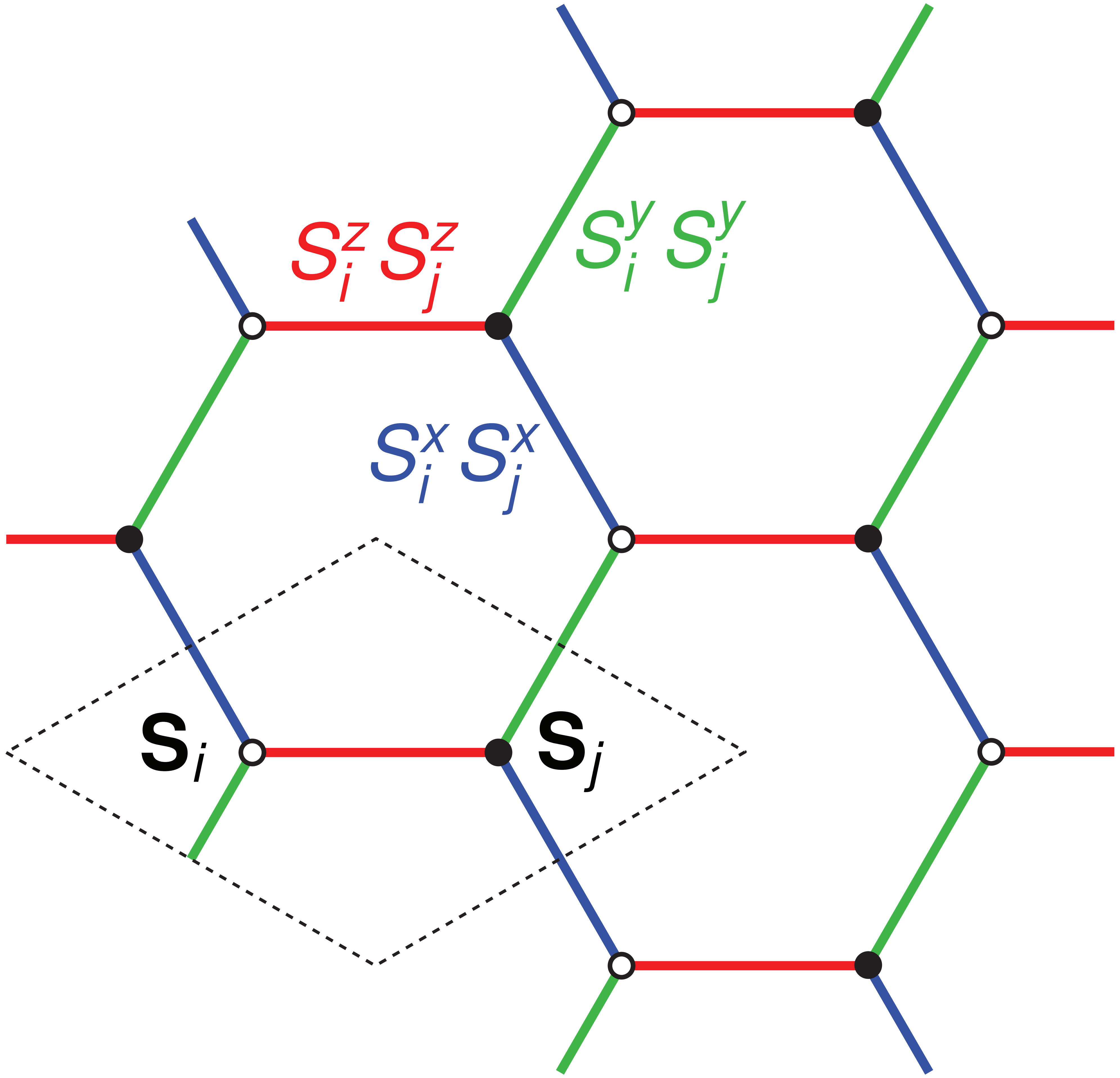


Square lattice: π flux
Honeycomb lattice: 0 flux

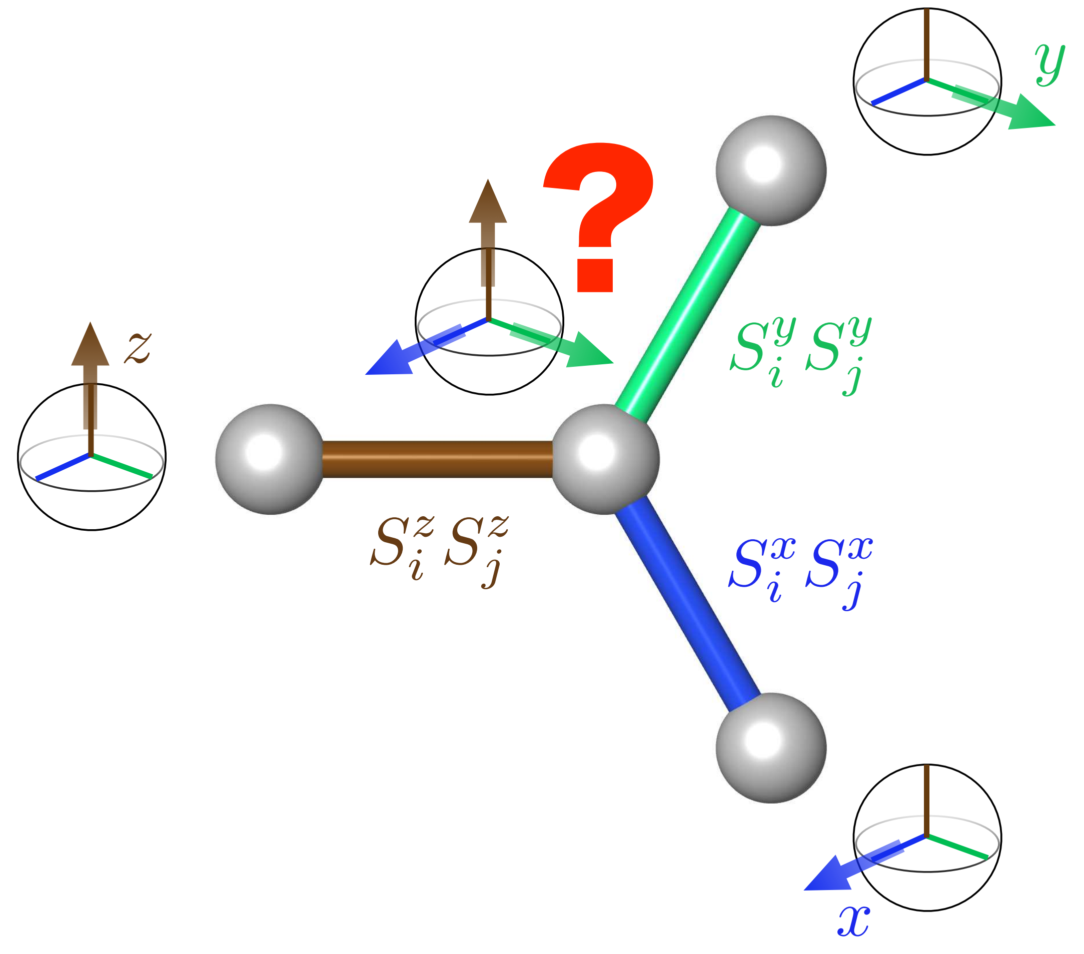
[Grosfeld & Stern, PRB '06]
[Liu & Franz, PRB '15]
[Wamer & Affleck, PRB '18]
[Li & Franz, PRB '18]
[Tummuru, Nocera, Affleck, PRB '21]
...

Realization #2: Kitaev honeycomb model

Spin-1/2 on honeycomb lattice:



Alexei Kitaev



Exchange frustration

Hamiltonian:

$$H = -K_x \sum_{\text{blue links}} \sigma_i^x \sigma_j^x - K_y \sum_{\text{green links}} \sigma_i^y \sigma_j^y - K_z \sum_{\text{red links}} \sigma_i^z \sigma_j^z$$

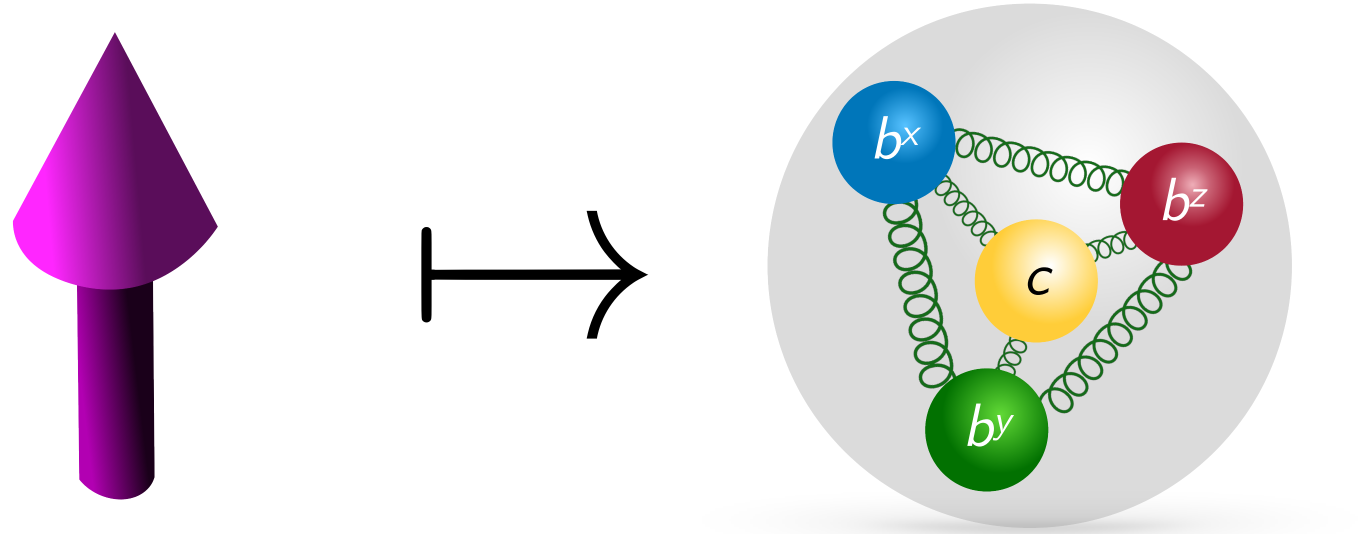
[Kitaev, Ann. Phys. '06]

Review: [Trebst, arXiv:1701.07056]

Parton construction

Majorana representation:

$$\sigma^x \mapsto \tilde{\sigma}^x = i b^x c$$
$$\sigma^y \mapsto \tilde{\sigma}^y = i b^y c$$
$$\sigma^z \mapsto \tilde{\sigma}^z = i b^z c$$

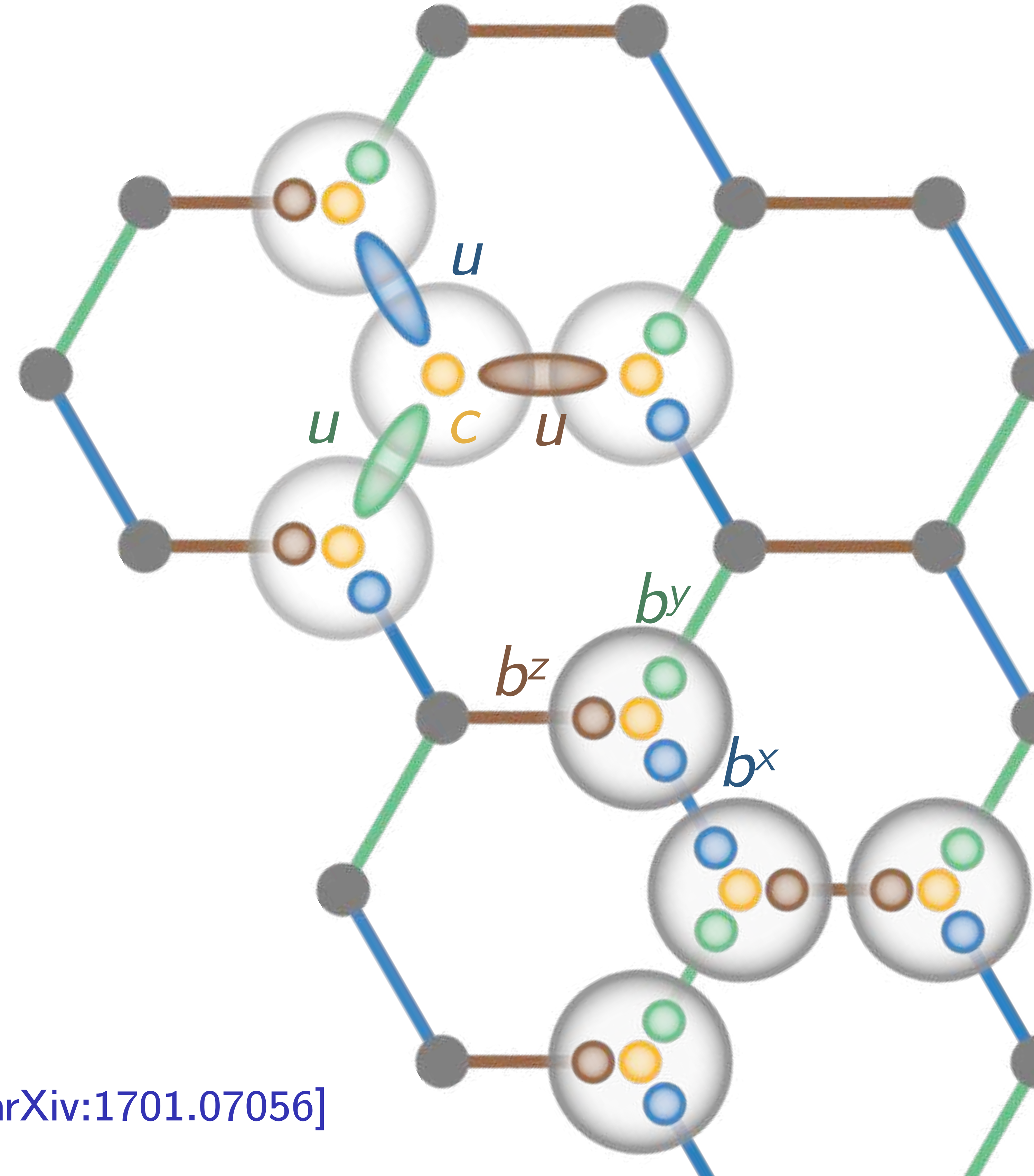


Fractionalization:

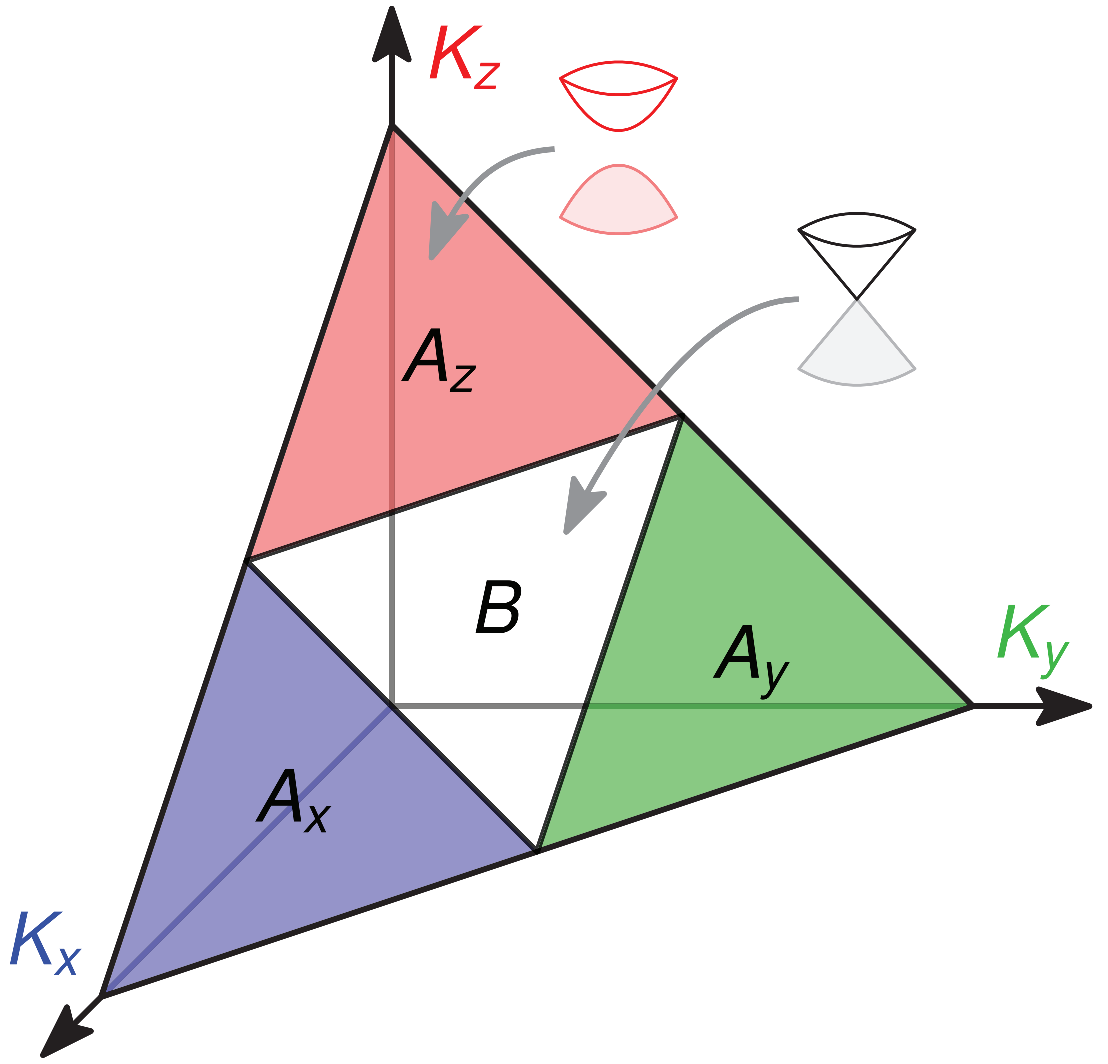
$$H \mapsto \tilde{H} = -i \sum_{\langle ij \rangle_\gamma} K_\gamma \underbrace{(i b_i^\gamma b_j^\gamma)}_{\equiv \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j$$

static!

Ground-state flux pattern: $u \equiv 1$
[Lieb, PRL '94]



Fermion spectrum:

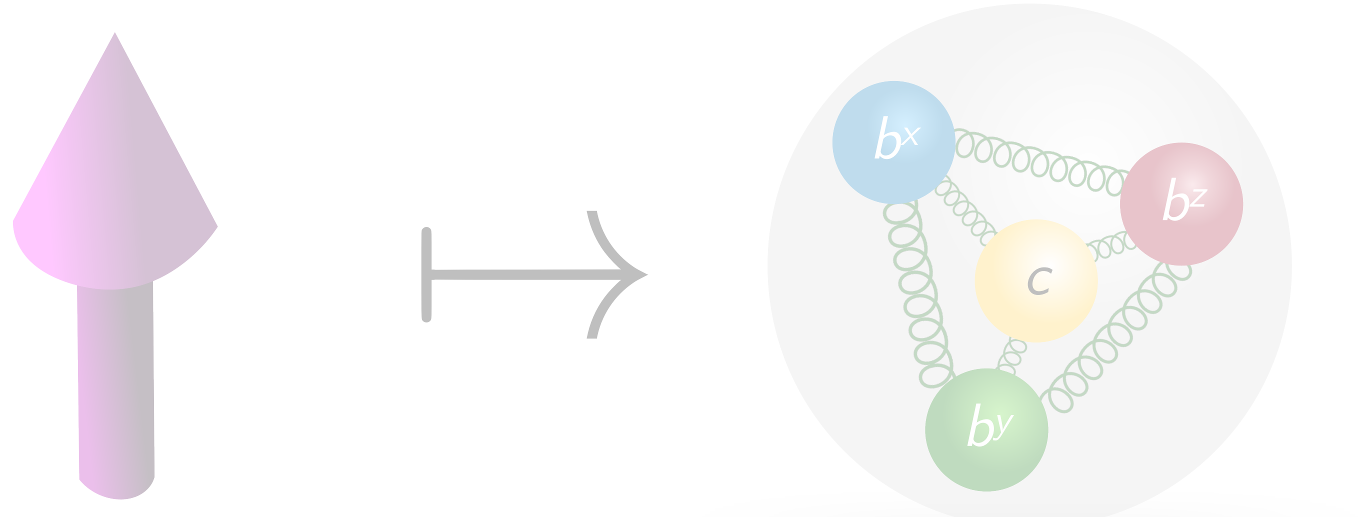


Review: [Trebst, arXiv:1701.07056]

Parton construction

Majorana representation:

$$\sigma^x \mapsto \tilde{\sigma}^x = i b^x c$$
$$\sigma^y \mapsto \tilde{\sigma}^y = i b^y c$$
$$\sigma^z \mapsto \tilde{\sigma}^z = i b^z c$$



Fractionalization:

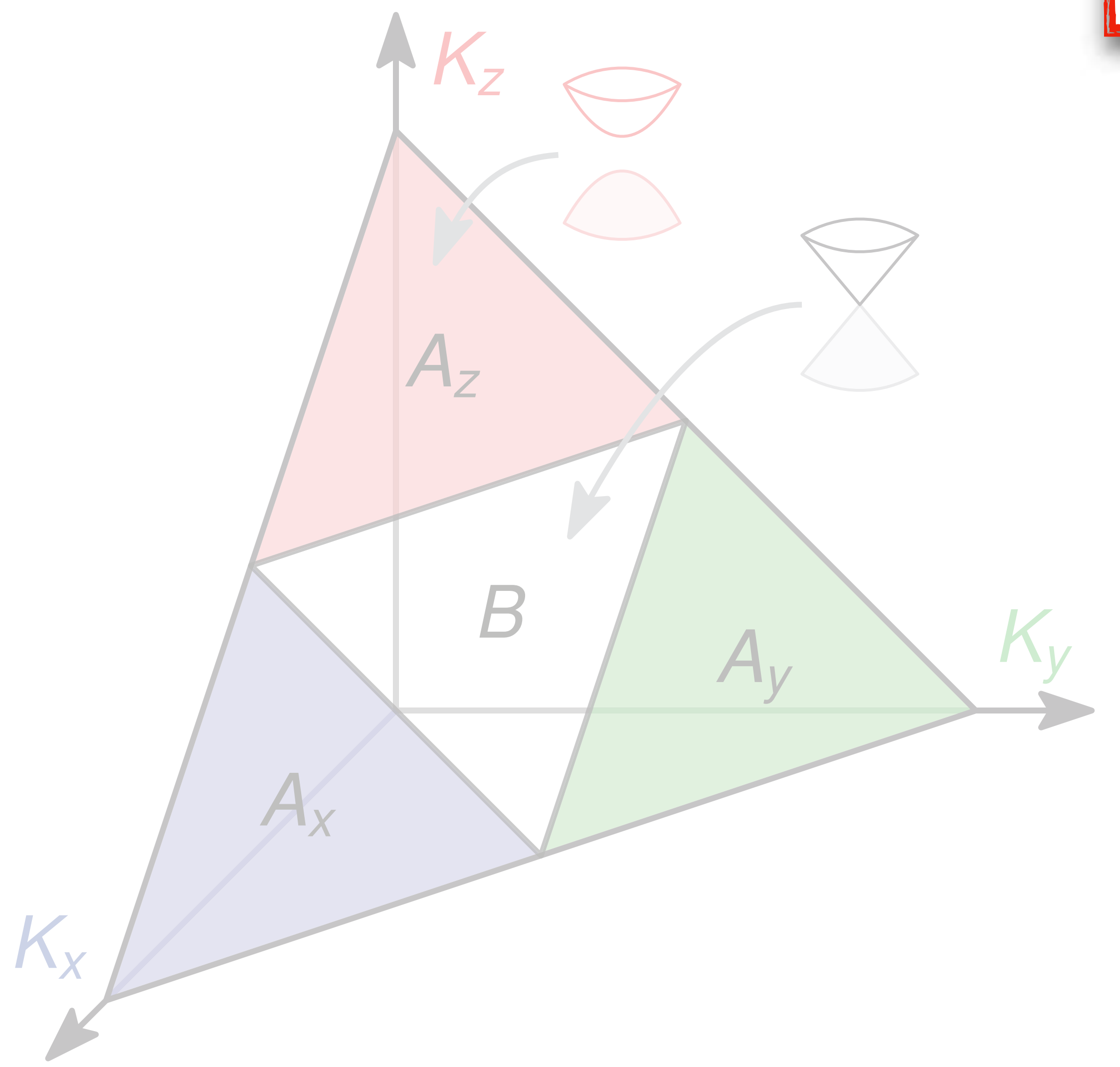
$$H \mapsto \tilde{H} = -i \sum_{\langle ij \rangle_\gamma} K_\gamma \underbrace{(i b_i^\gamma b_j^\gamma)}_{\equiv \hat{u}_{ij} =} c_i c_j$$

Interactions?

: $u \equiv 1$

RL '94]

Fermion spectrum:



Review: [Trebst, arXiv:1701.07056]

Outline

(1) Introduction

(2) SO(N) Majoranas in frustrated magnets

(3) SO(N) Majorana-Hubbard models

(4) Conclusions

Generalizations of Kitaev model: Spin-orbital liquids

Spin + orbital + ... degrees of freedom:

...

$\sigma^\alpha \quad 2 \times 2$ | $\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$ | $\Gamma^\mu \quad 8 \times 8$

$\mathcal{C} = 0, 1$ | $\mathcal{C} = 2, 3$ | $\mathcal{C} = 4, 5$

... can realize all 16 topological superconductors

[Chulliparambil, ..., LJ, Tu, PRB '20]

Generalizations of Kitaev model: Spin-orbital liquids

Spin + orbital + ... degrees of freedom:

$\sigma^\alpha \quad 2 \times 2$ | $\sigma^\alpha \otimes \tau^\beta = \gamma^i \quad 4 \times 4$ | ... $\Gamma^\mu \quad 8 \times 8$

$\mathcal{C} = 0, 1$ | $\mathcal{C} = 2, 3$ | $\mathcal{C} = 4, 5$

... can realize all 16 topological superconductors
[Chulliparambil, ..., LJ, Tu, PRB '20]

Example #1 (square lattice):

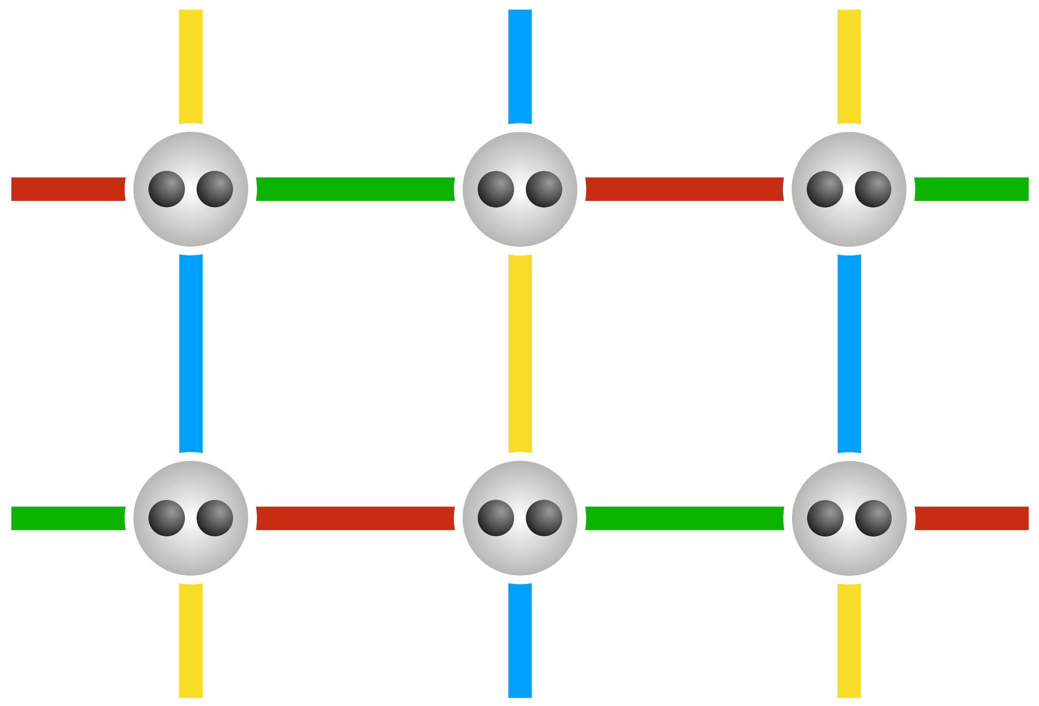
XY spin — Kitaev orbital

$$H_K = -K \sum_{\langle ij \rangle_\gamma} (\sigma_i^x \sigma_j^x + \sigma_i^y \sigma_j^y) \otimes \tau_i^\gamma \tau_j^\gamma$$

Majorana representation:

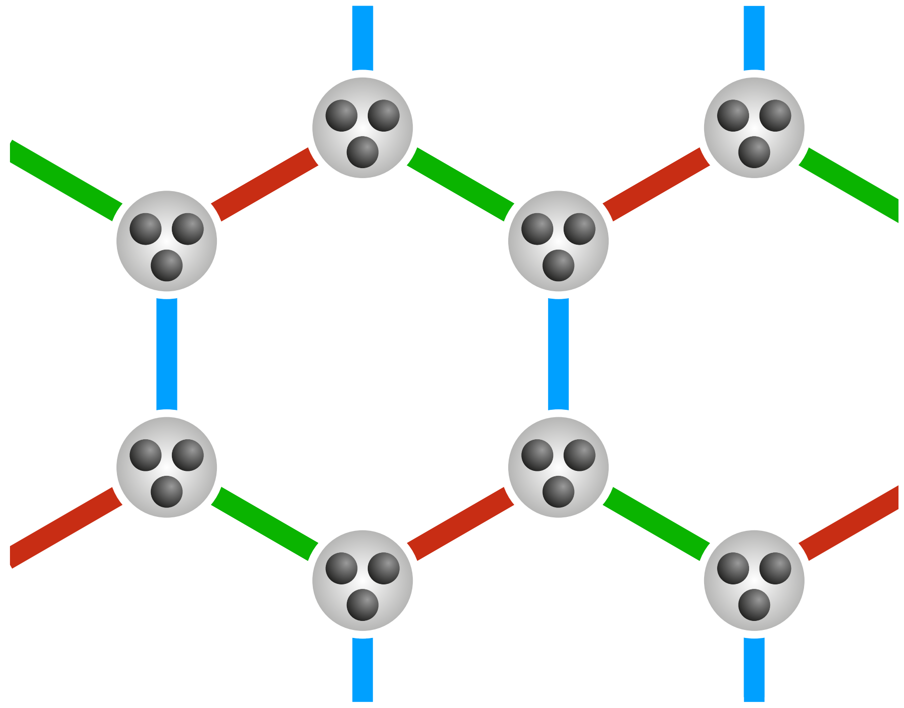
$$\sigma^y \otimes \tau^x = i b^1 c^x$$
$$\sigma^y \otimes \tau^y = i b^2 c^x$$
$$\sigma^y \otimes \tau^z = i b^3 c^x$$
$$\sigma^x \otimes \mathbb{1} = i b^4 c^x$$
$$\sigma^z \otimes \mathbb{1} = i c^y c^x$$

... recover known model for $j = 3/2$ square-lattice spin liquid:
[Yao, Zhang, Kivelson, PRL '09]
[Nakai, Ryu, Furusaki, PRB '12]



2 itinerant fermions
$\mathcal{C} = 2$

Kitaev honeycomb spin-orbital model



3 itinerant fermions

$\mathcal{C} = 3$

Example #2 (honeycomb lattice):

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma$$

Majorana representation:

$$H \mapsto K \sum_{\langle ij \rangle} i u_{ij} c_i^\top c_j$$

$c \equiv \begin{pmatrix} c^x \\ c^y \\ c^z \end{pmatrix}$

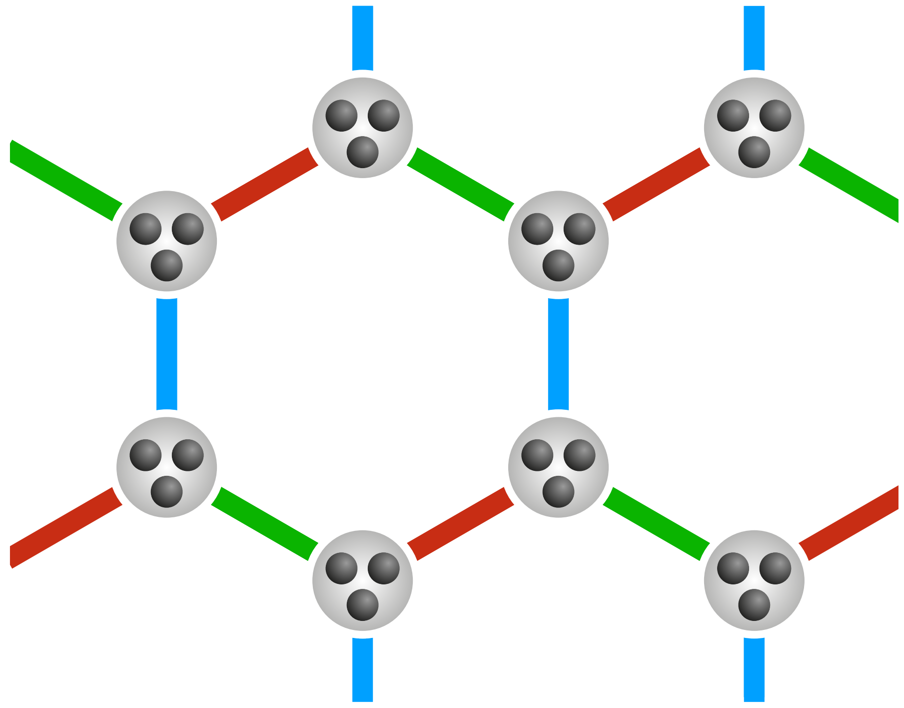
0 flux

... recover known model for $j = 3/2$ honeycomb spin liquid:

[Yao & Lee, PRL '11]

[Natori & Knolle, PRL '20]

Kitaev honeycomb spin-orbital model



3 itinerant fermions
$\mathcal{C} = 3$

Example #2 (honeycomb lattice):

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma$$

Majorana representation:

$$H \mapsto K \sum_{\langle ij \rangle} i u_{ij} c_i^\top c_j$$

$c \equiv \begin{pmatrix} c^x \\ c^y \\ c^z \end{pmatrix}$

0 flux

... recover known model for $j = 3/2$ honeycomb spin liquid:

[Yao & Lee, PRL '11]

[Natori & Knolle, PRL '20]

Can find perturbations that leave gauge field static!

... crucial for controlled analytical approximations

... allows sign-problem-free QMC

Flux-preserving perturbations: Heisenberg spin exchange

Kitaev-Heisenberg spin-orbital model:

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j$$

spin-1 matrices

Majorana representation:

$$H \mapsto \sum_{\langle ij \rangle} \left[K i u_{ij} c_i^\top c_j + \tfrac{J}{4} (c_i^\top \vec{L} c_i) \cdot (c_j^\top \vec{L} c_j) \right]$$

"SO(3) Majorana-Hubbard model"

Flux-preserving perturbations: Heisenberg spin exchange

Kitaev-Heisenberg spin-orbital model:

$$H = -K \sum_{\langle ij \rangle_\gamma} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} \vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j$$

spin-1 matrices

Majorana representation:

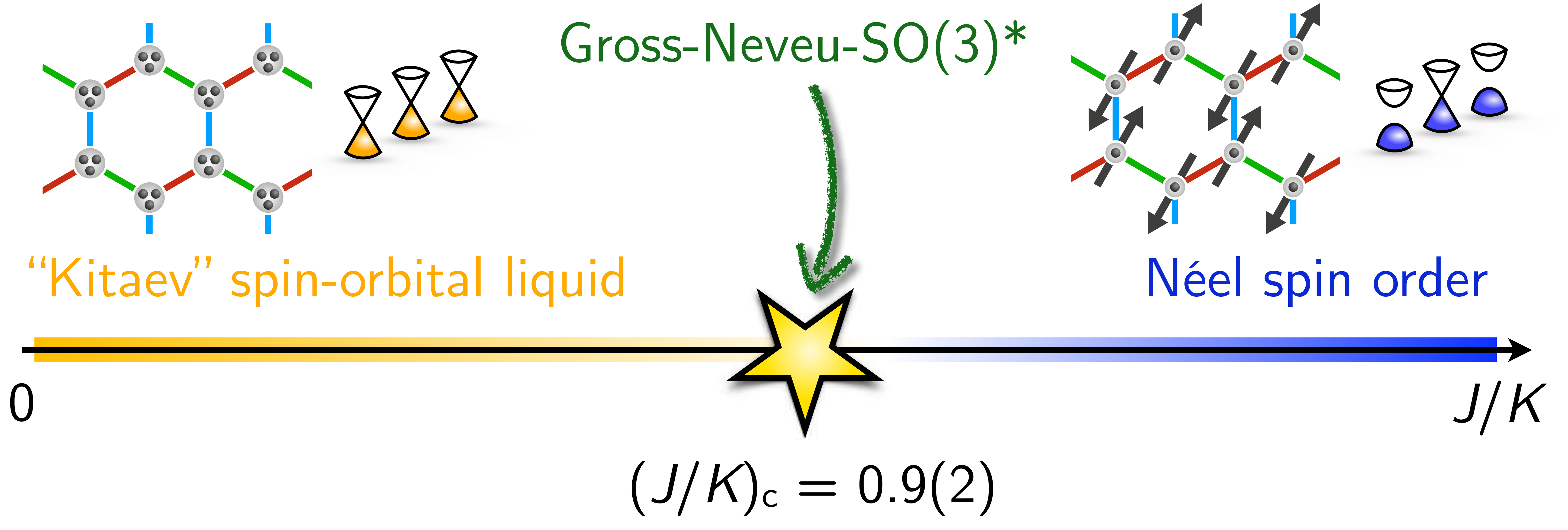
$$H \mapsto \sum_{\langle ij \rangle} \left[K i u_{ij} c_i^\top c_j + \frac{J}{4} (c_i^\top \vec{L} c_i) \cdot (c_j^\top \vec{L} c_j) \right]$$

"SO(3) Majorana-Hubbard model"

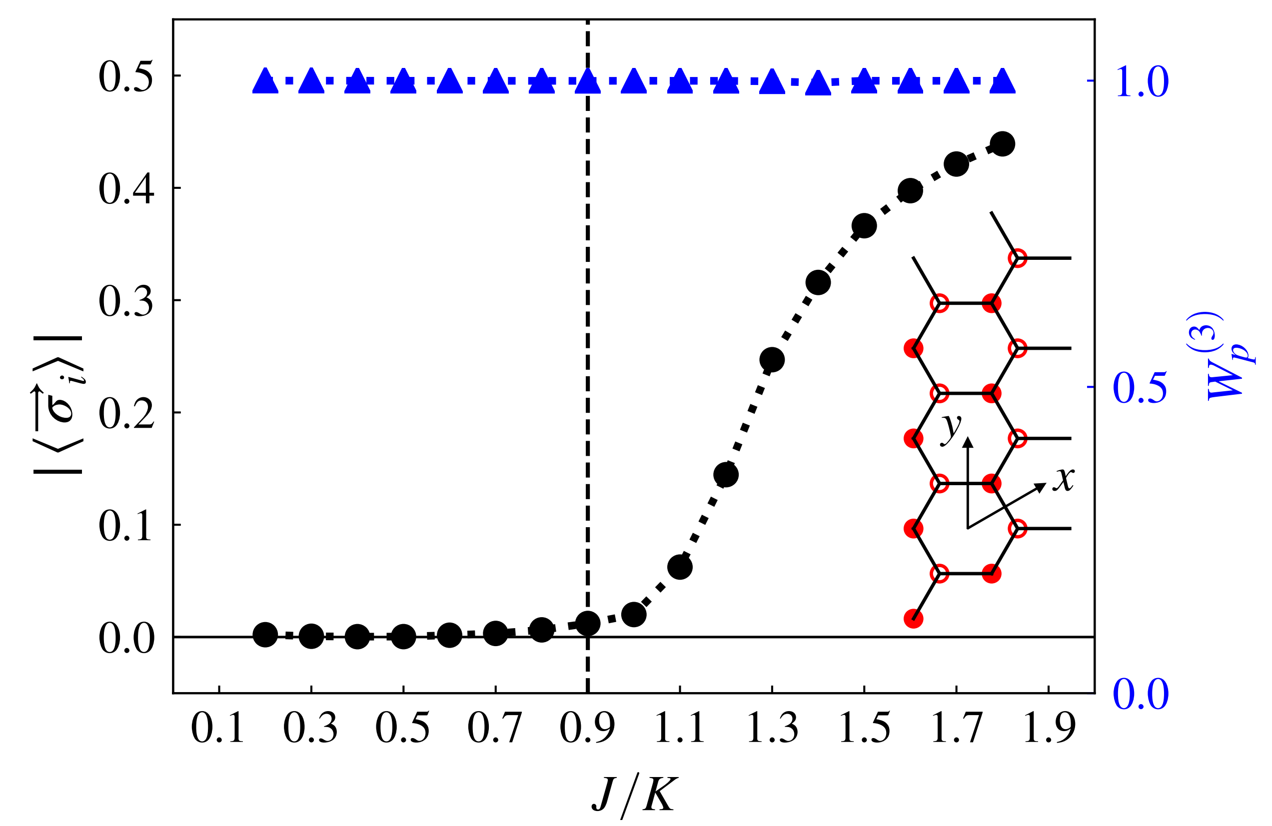
Phase diagram:

Gross-Neveu-SO(3)* quantum criticality

Phase diagram:



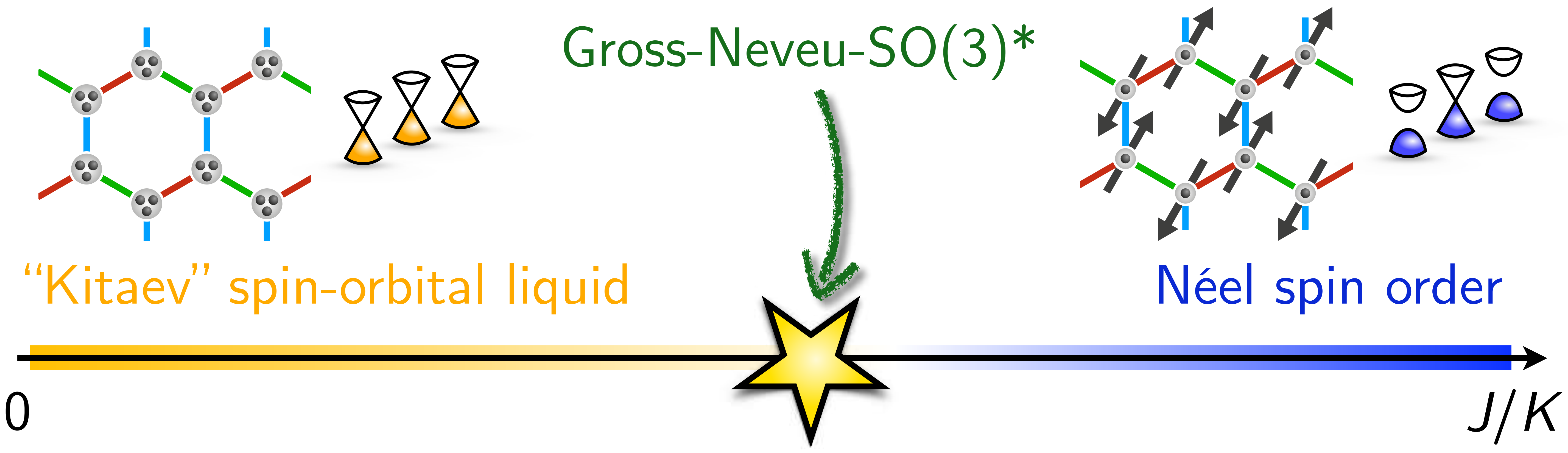
iDMRG:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]

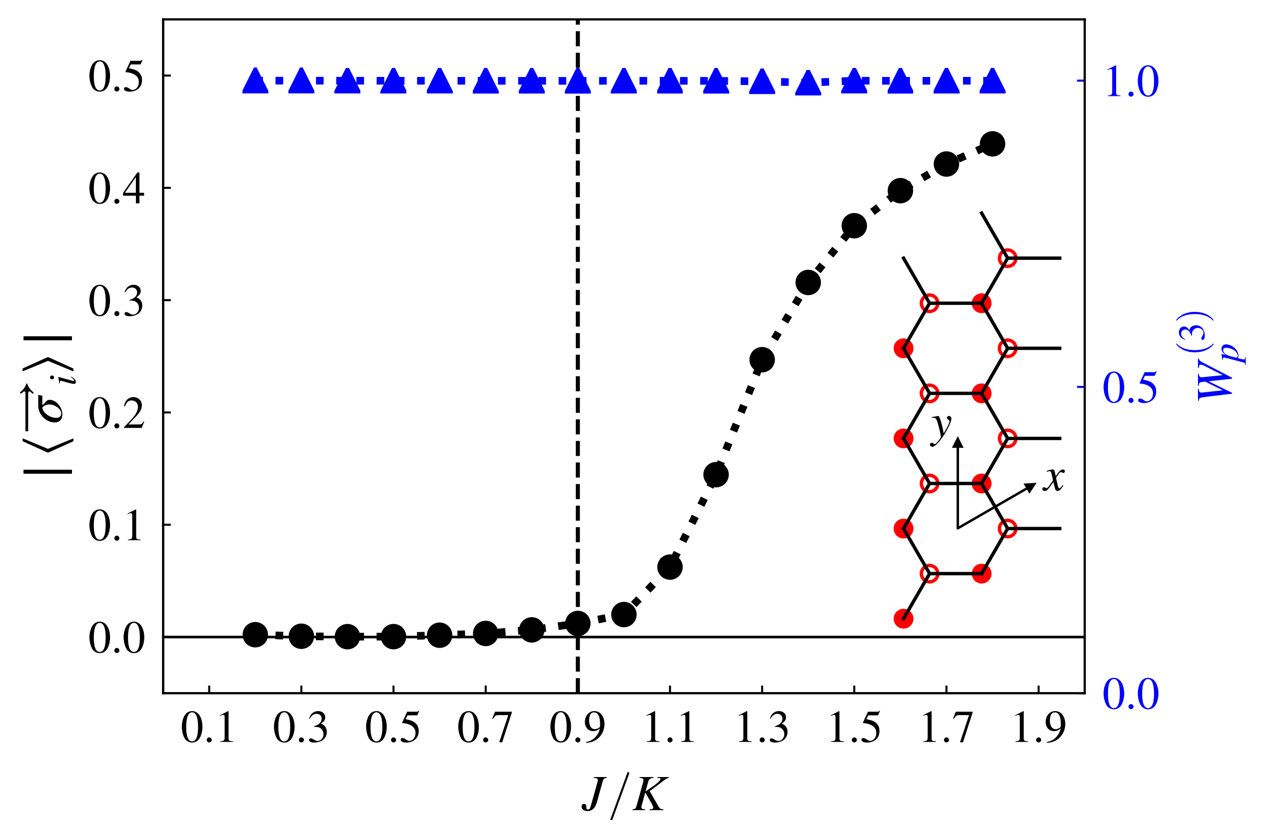
Gross-Neveu-SO(3)* quantum criticality

Phase diagram:



$(J/K)_c = 0.9(2)$

iDMRG:



[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL ’20]

Effective field theory:

$$\mathcal{S} = \int d^2\vec{x}d\tau \left[\bar{\psi}\gamma^\mu\partial_\mu\psi + g\vec{\varphi}\cdot\bar{\psi}(\mathbb{1}_2 \otimes \vec{L})\psi\right]$$

“Gross-Neveu-SO(3)”

Critical exponents:

... from:

- large-N expansion @ $O(1/N^2)$
- 4-ε expansion @ 3-loop
- functional RG @ LPA’

$$1/\nu = 1.03(15)$$

$$\eta_\phi = 0.42(7)$$

[Ray, Ihrig, Gracey, Scherer, LJ, PRB ’21]

Sign-problem-free QMC: [Liu, Vojta, Assaad, LJ, PRL ’22 (in press)]

Outline

(1) Introduction

(2) SO(N) Majoranas in frustrated magnets

(3) SO(N) Majorana-Hubbard models

(4) Conclusions

SO(N) Majorana-Hubbard models

Generators of SO(N)

Hamiltonian:

$$\mathcal{H} = \sum_{\langle ij \rangle} \mathrm{i} t_{ij} c_i^\top c_j + J \sum_{a<b} \sum_{\langle ij \rangle} \left(\frac{1}{2} c_i^\top L^{ab} c_i \right) \left(\frac{1}{2} c_j^\top L^{ab} c_j \right)$$

... with $c_i \equiv (c_i^1, \ldots, c_i^N)^\top$

SO(N) generators:

$$(L^{ab})_{\alpha\beta} = -\mathrm{i}(\delta_a^\alpha \delta_b^\beta - \delta_a^\beta \delta_b^\alpha)$$

... N x N matrices for given $a < b$

SO(N) Majorana-Hubbard models

Generators of SO(N)

Hamiltonian:

$$\mathcal{H} = \sum_{\langle ij \rangle} \mathrm{i} t_{ij} c_i^\top c_j + J \sum_{a<b} \sum_{\langle ij \rangle} \left(\frac{1}{2} c_i^\top L^{ab} c_i \right) \left(\frac{1}{2} c_j^\top L^{ab} c_j \right)$$

... with $c_i \equiv (c_i^1, \ldots, c_i^N)^\top$

SO(N) generators:

$$(L^{ab})_{\alpha\beta} = -\mathrm{i}(\delta_a^\alpha \delta_b^\beta - \delta_a^\beta \delta_b^\alpha)$$

... N x N matrices for given $a < b$

Interaction term:

$$\mathcal{H}\Big|_{t=0} = \frac{J}{2} \sum_{\langle ij \rangle} (c_i^\top c_j)(c_i^\top c_j) + \text{const.}$$

"Majorana analog of SU(N) Hubbard-Heisenberg model"

SO(N) Majorana-Hubbard models

Generators of SO(N)

Hamiltonian:

$$\mathcal{H} = \sum_{\langle ij \rangle} \mathrm{i} t_{ij} c_i^\top c_j + J \sum_{a<b} \sum_{\langle ij \rangle} \left(\frac{1}{2} c_i^\top L^{ab} c_i \right) \left(\frac{1}{2} c_j^\top L^{ab} c_j \right)$$

... with $c_i \equiv (c_i^1, \ldots, c_i^N)^\top$

SO(N) generators:

$$(L^{ab})_{\alpha\beta} = -\mathrm{i}(\delta_a^\alpha \delta_b^\beta - \delta_a^\beta \delta_b^\alpha)$$

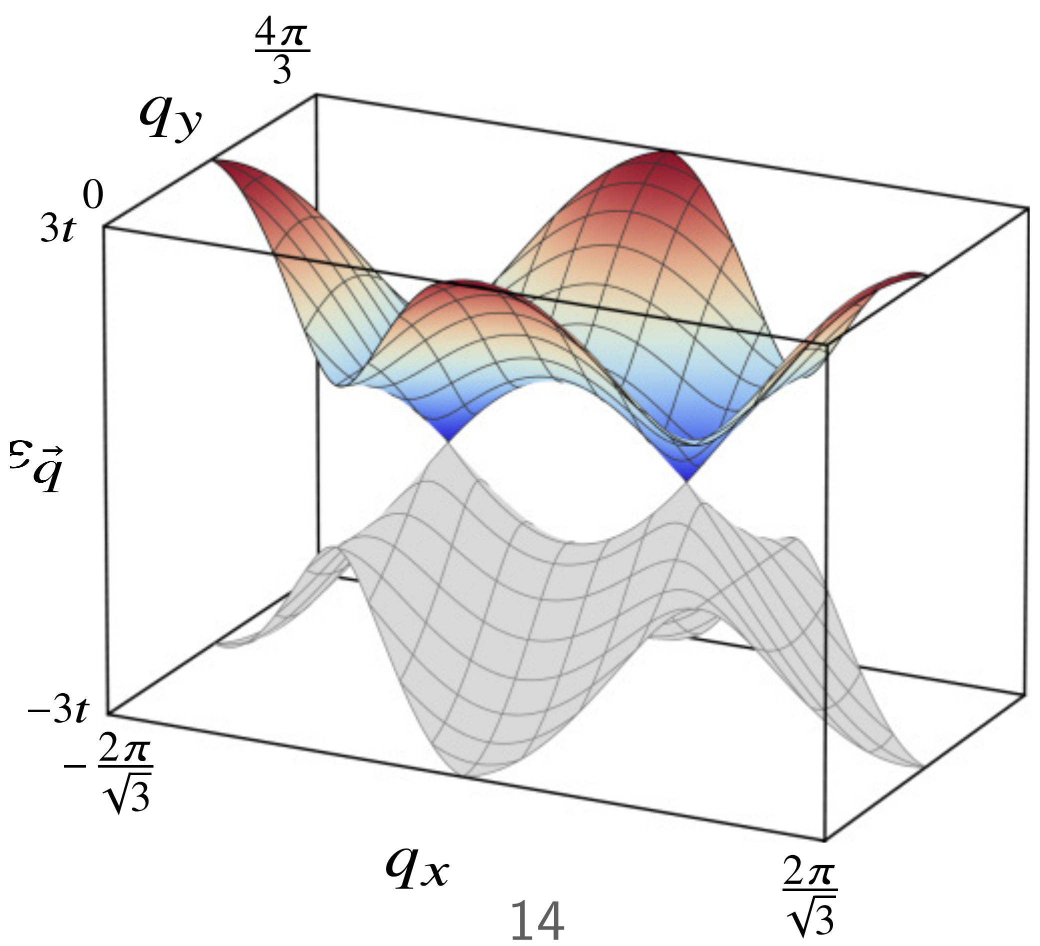
... N x N matrices for given $a < b$

Interaction term:

$$\mathcal{H}\Big|_{t=0} = \frac{J}{2} \sum_{\langle ij \rangle} (c_i^\top c_j)(c_i^\top c_j) + \text{const.}$$

"Majorana analog of SU(N) Hubbard-Heisenberg model"

Single-particle spectrum:



... on honeycomb lattice
... and similar on π-flux square lattice

Zero-temperature phase diagram

Mean-field decoupling:

$$\sum_{a<b} \left(\tfrac{1}{2} c_i^\top L^{ab} c_i\right) \left(\tfrac{1}{2} c_j^\top L^{ab} c_j\right) \mapsto \sum_{a<b} \left[\phi_i^{ab} \left(\tfrac{1}{2} c_i^\top L^{ab} c_i\right) + \left(\tfrac{1}{2} c_j^\top L^{ab} c_j\right) \phi_j^{ab} - \phi_i^{ab}\phi_j^{ab}\right]$$

$$- \mathrm{i}(N-1)\chi_{ij} c_i^\top c_j + \frac{N(N-1)}{2}\chi_{ij}^2$$

SO(N) order parameter: $\phi_i^{ab} = \langle \tfrac{1}{2} c_i^\top L^{ab} c_i \rangle$

Dimer order parameter: $\chi_{ij} = \langle \mathrm{i} c_i^\top c_j \rangle / N$

Zero-temperature phase diagram

Mean-field decoupling:

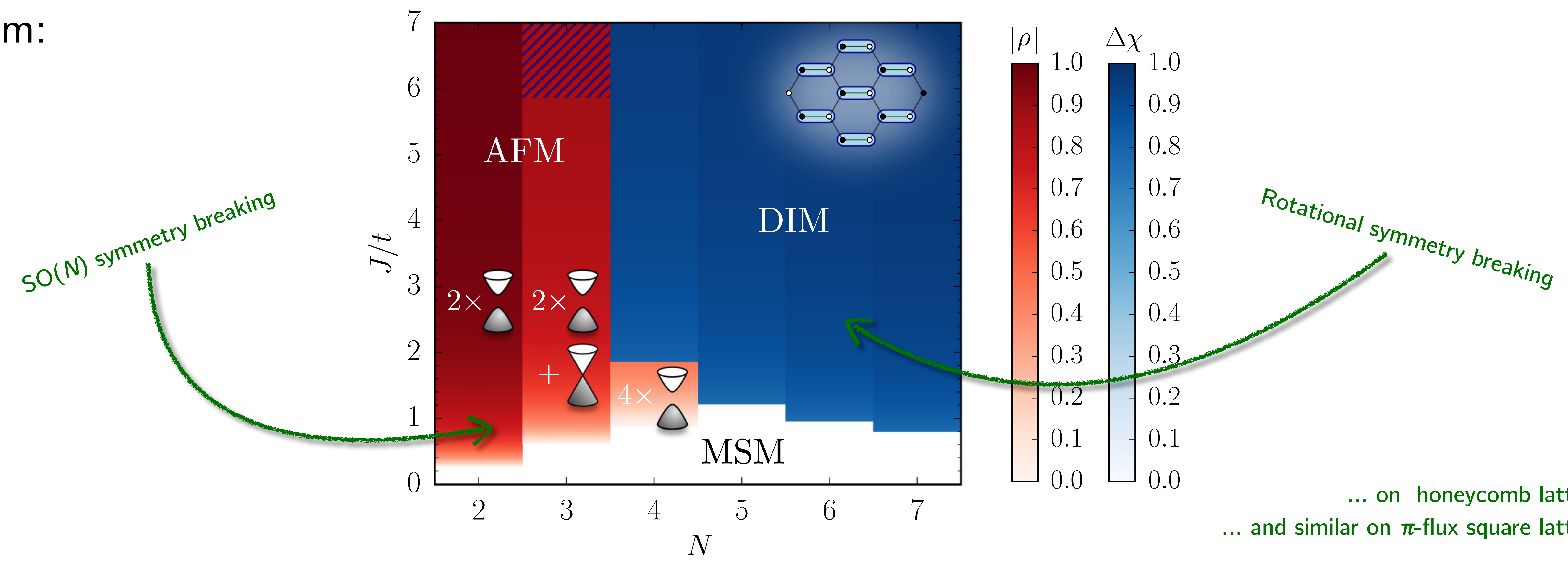
$$\sum_{a<b} \left(\tfrac{1}{2} c_i^\top L^{ab} c_i\right)\left(\tfrac{1}{2} c_j^\top L^{ab} c_j\right) \mapsto \sum_{a<b} \left[\phi_i^{ab}\left(\tfrac{1}{2} c_i^\top L^{ab} c_i\right) + \left(\tfrac{1}{2} c_j^\top L^{ab} c_j\right)\phi_j^{ab} - \phi_i^{ab}\phi_j^{ab}\right]$$

$$- \mathrm{i}(N-1)\chi_{ij} c_i^\top c_j + \frac{N(N-1)}{2}\chi_{ij}^2$$

SO(N) order parameter: $\phi_i^{ab} = \langle \tfrac{1}{2} c_i^\top L^{ab} c_i \rangle$

Dimer order parameter: $\chi_{ij} = \langle \mathrm{i} c_i^\top c_j \rangle / N$

Phase diagram:



... on honeycomb lattice
... and similar on π-flux square lattice

Quantum phase transitions

antisymmetric tensor order parameter

Effective model (semimetal-to-antiferromagnet transition):

$$\mathcal{L} = \bar{\psi}_\alpha \gamma_\mu \partial_\mu \psi_\alpha + \frac{1}{4}\phi^{ab}\left(r - \partial_\mu^2\right)\phi^{ab} + \frac{g}{2}\phi^{ab}\bar{\psi}_\alpha (L^{ab})_{\alpha\beta}\psi_\beta + \frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da}$$

"Gross-Neveu-SO(N)"

$N \leq 3$:

$$\frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da} = \frac{\lambda_1 + 2\lambda_2}{2}\left[\mathrm{Tr}\left(\phi^2\right)\right]^2$$

... but not for $N > 3$

Quantum phase transitions

antisymmetric tensor order parameter

Effective model (semimetal-to-antiferromagnet transition):

$$\mathcal{L} = \bar{\psi}_\alpha \gamma_\mu \partial_\mu \psi_\alpha + \frac{1}{4}\phi^{ab}\left(r - \partial_\mu^2\right)\phi^{ab} + \frac{g}{2}\phi^{ab}\bar{\psi}_\alpha (L^{ab})_{\alpha\beta}\psi_\beta + \frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da}$$

"Gross-Neveu-SO(N)"

$N \leq 3$:

$$\frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da} = \frac{\lambda_1 + 2\lambda_2}{2}\left[\mathrm{Tr}\left(\phi^2\right)\right]^2$$

... but not for $N > 3$

$N = 2$:

$$(\phi^{ab}) = \begin{pmatrix} 0 & \phi \\ -\phi & 0 \end{pmatrix}$$

"Gross-Neveu-Ising"

[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]
...

$N = 3$:

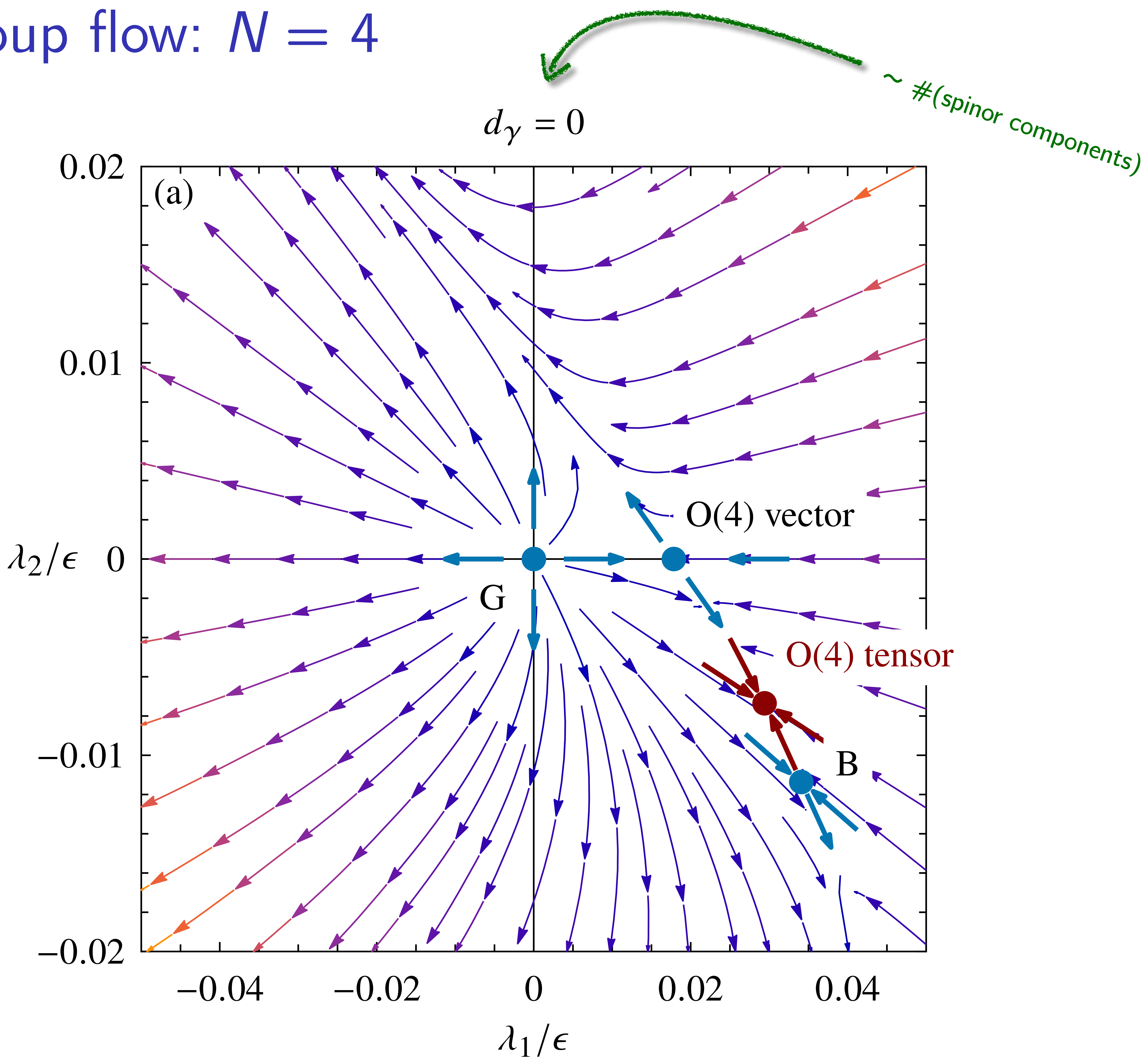
$$(\phi^{ab}) = \begin{pmatrix} 0 & \phi_3 & \phi_2 \\ -\phi_3 & 0 & \phi_1 \\ -\phi_2 & -\phi_1 & 0 \end{pmatrix}$$

"Gross-Neveu-SO(3)"

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]
[Ray, Ihrig, Gracey, Scherer, LJ, PRB '21]
[Liu, Vojta, Assaad, LJ, PRL '22 (in press)]

Renormalization group flow: $N = 4$

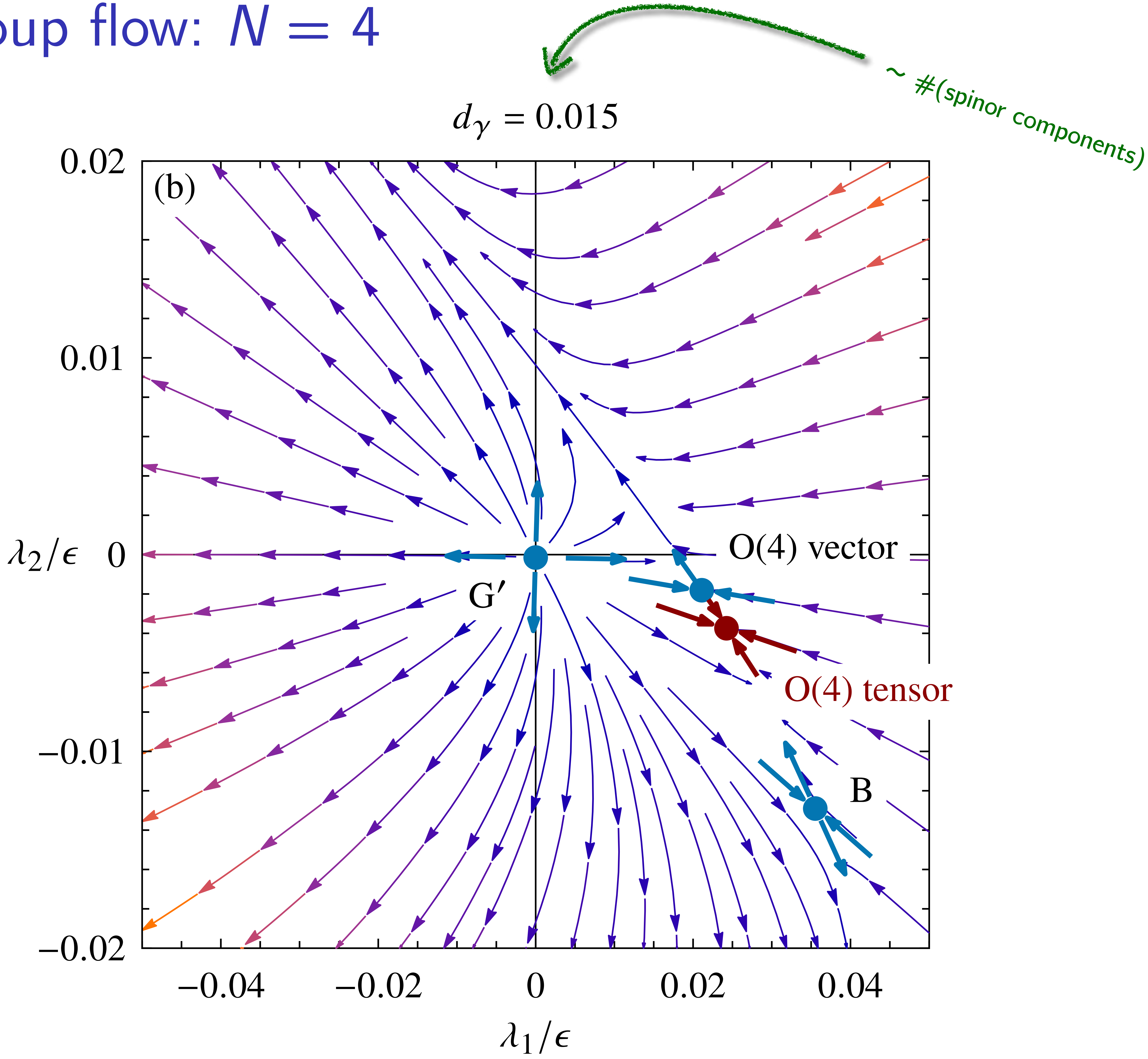
Bosons only:



[LJ & Seifert, PRB '22]

Renormalization group flow: $N = 4$

Bosons
+ "few" fermions:

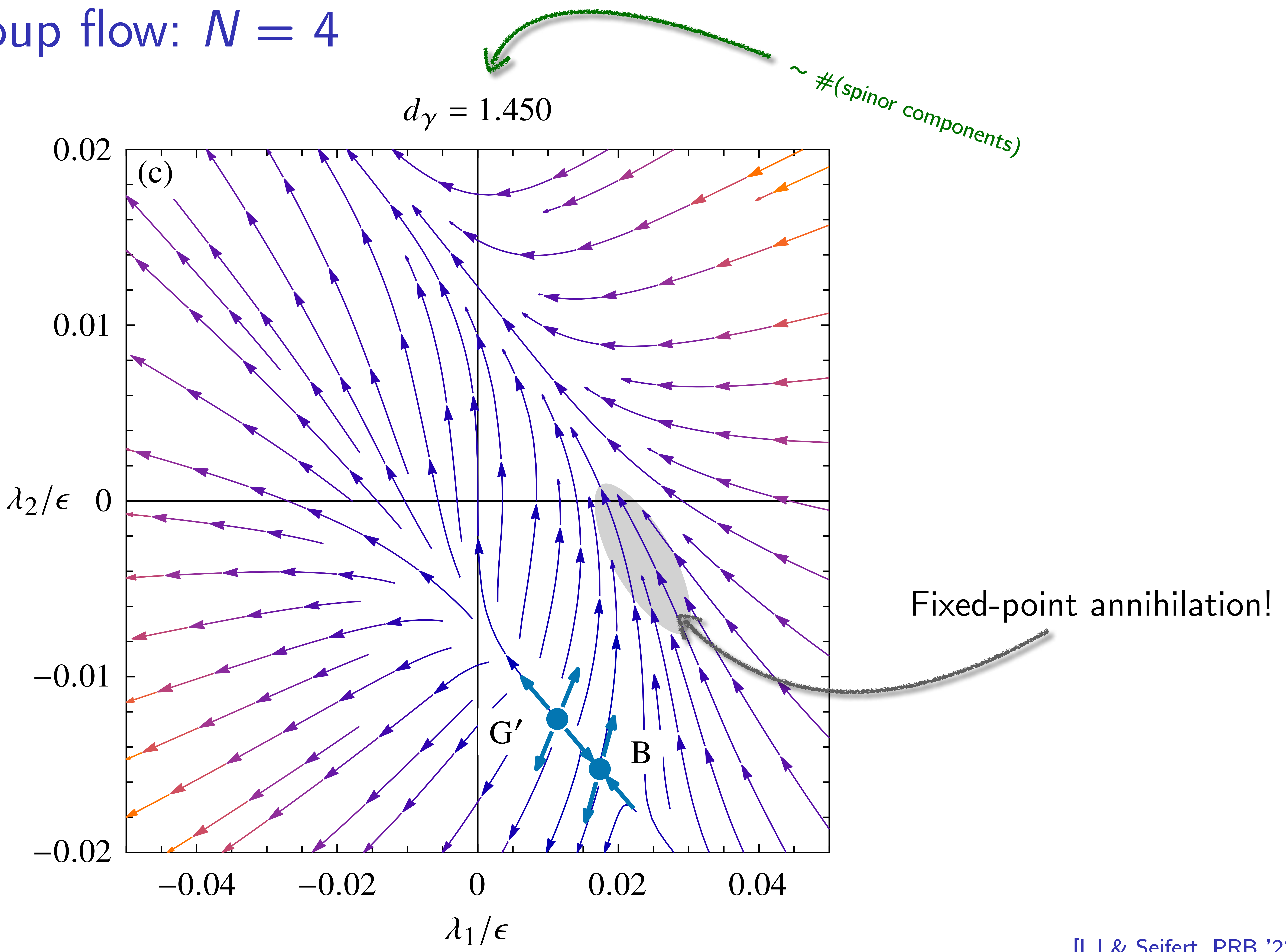


[LJ & Seifert, PRB '22]

Renormalization group flow: $N = 4$

Bosons

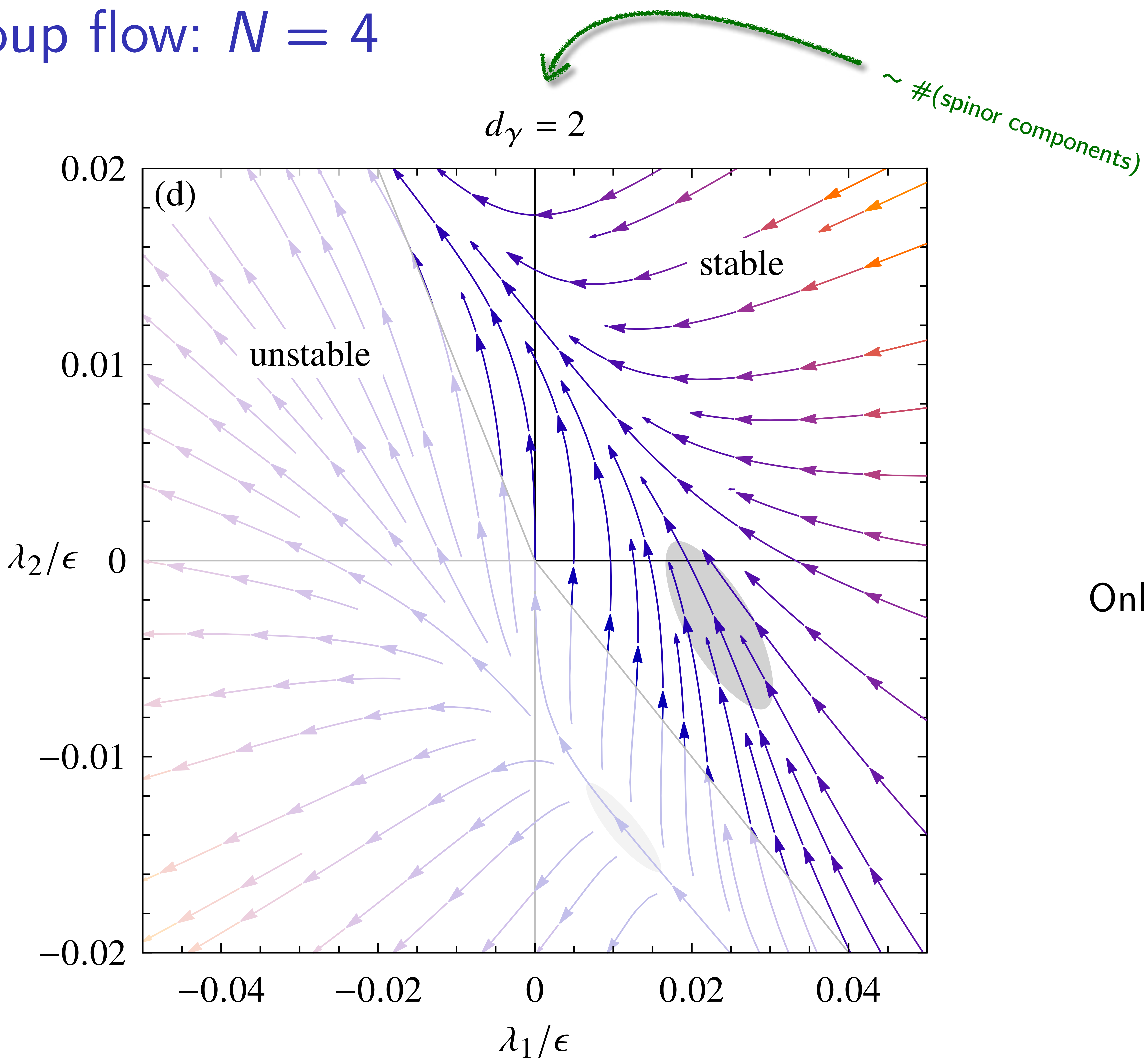
+ "few more" fermions:



[LJ & Seifert, PRB '22]

Renormalization group flow: $N = 4$

Bosons
+ "all" fermions:

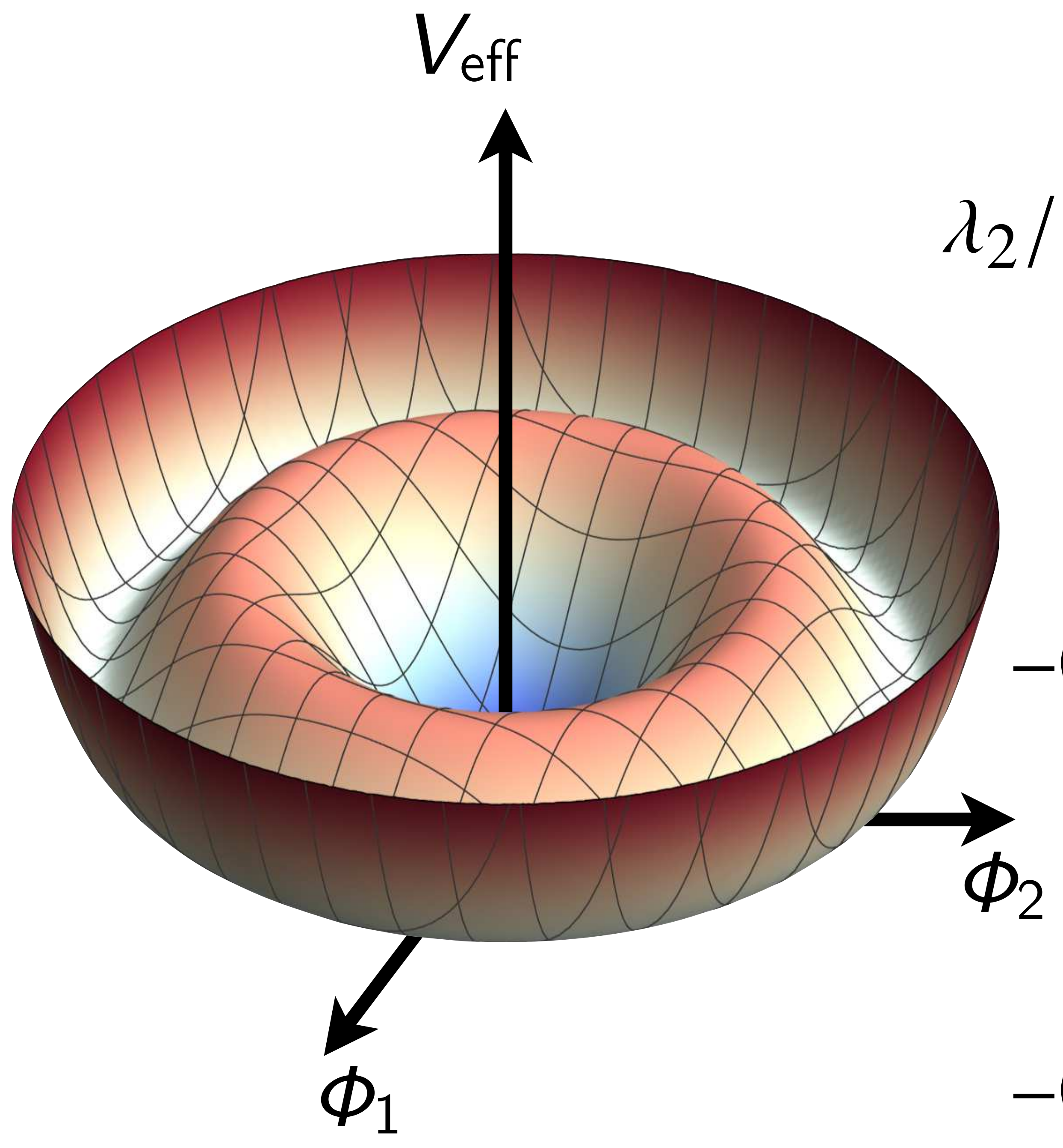


Only runaway flow!

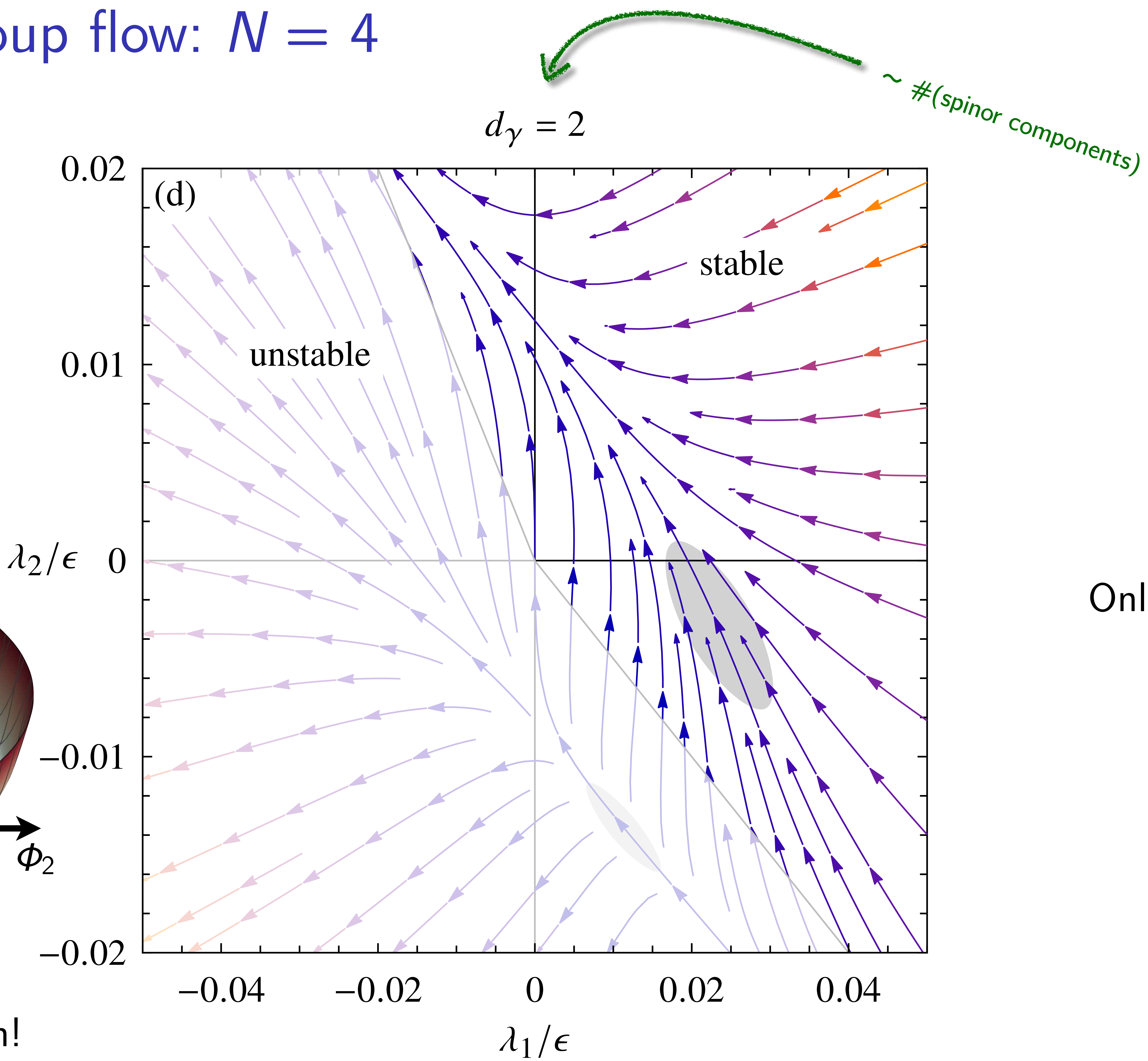
[LJ & Seifert, PRB '22]

Renormalization group flow: $N = 4$

Bosons
+ "all" fermions:



Weak first-order transition!



Only runaway flow!

[LJ & Seifert, PRB '22]

Outline

(1) Introduction

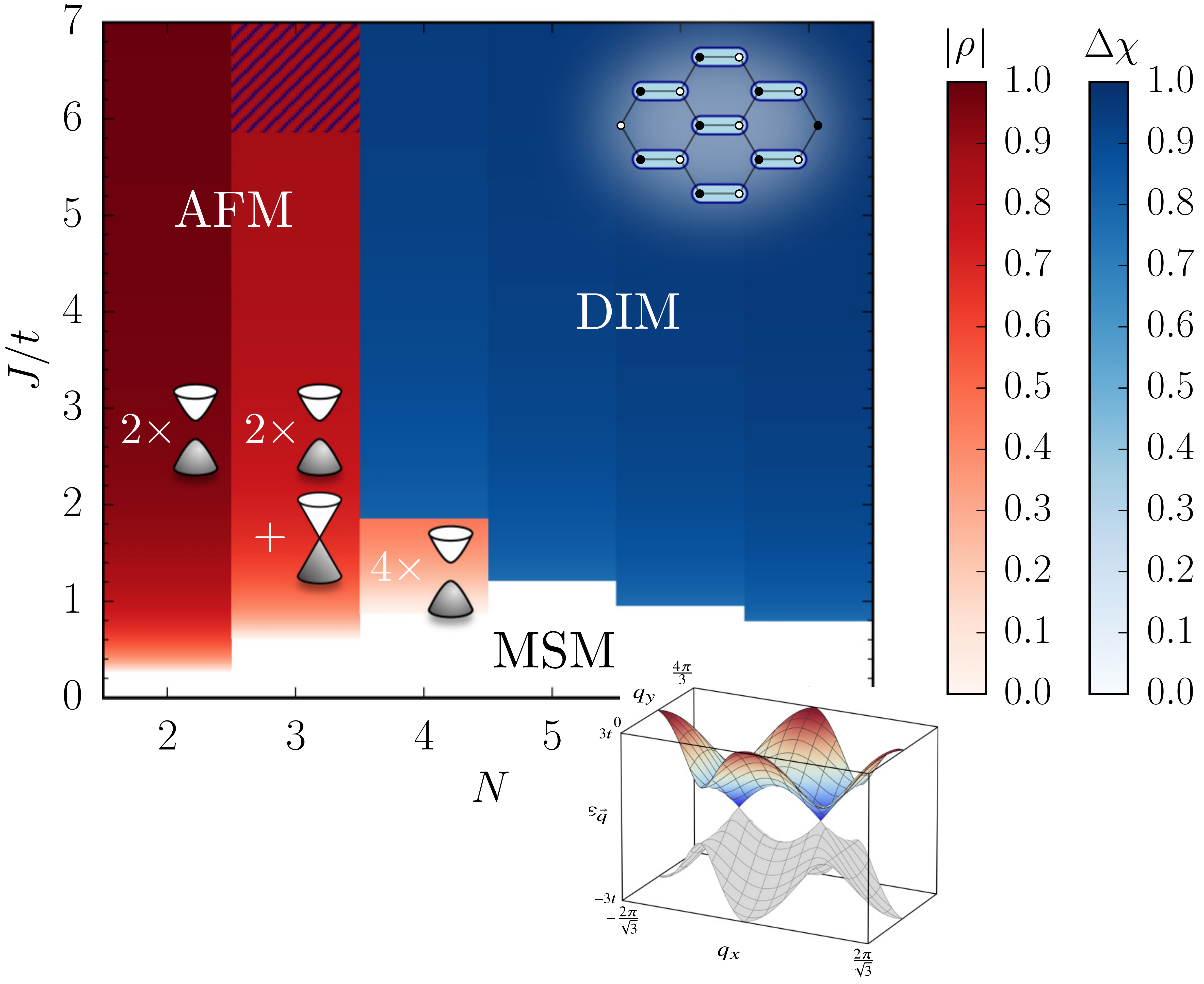
(2) SO(N) Majoranas in frustrated magnets

(3) SO(N) Majorana-Hubbard models

(4) Conclusions

Conclusions

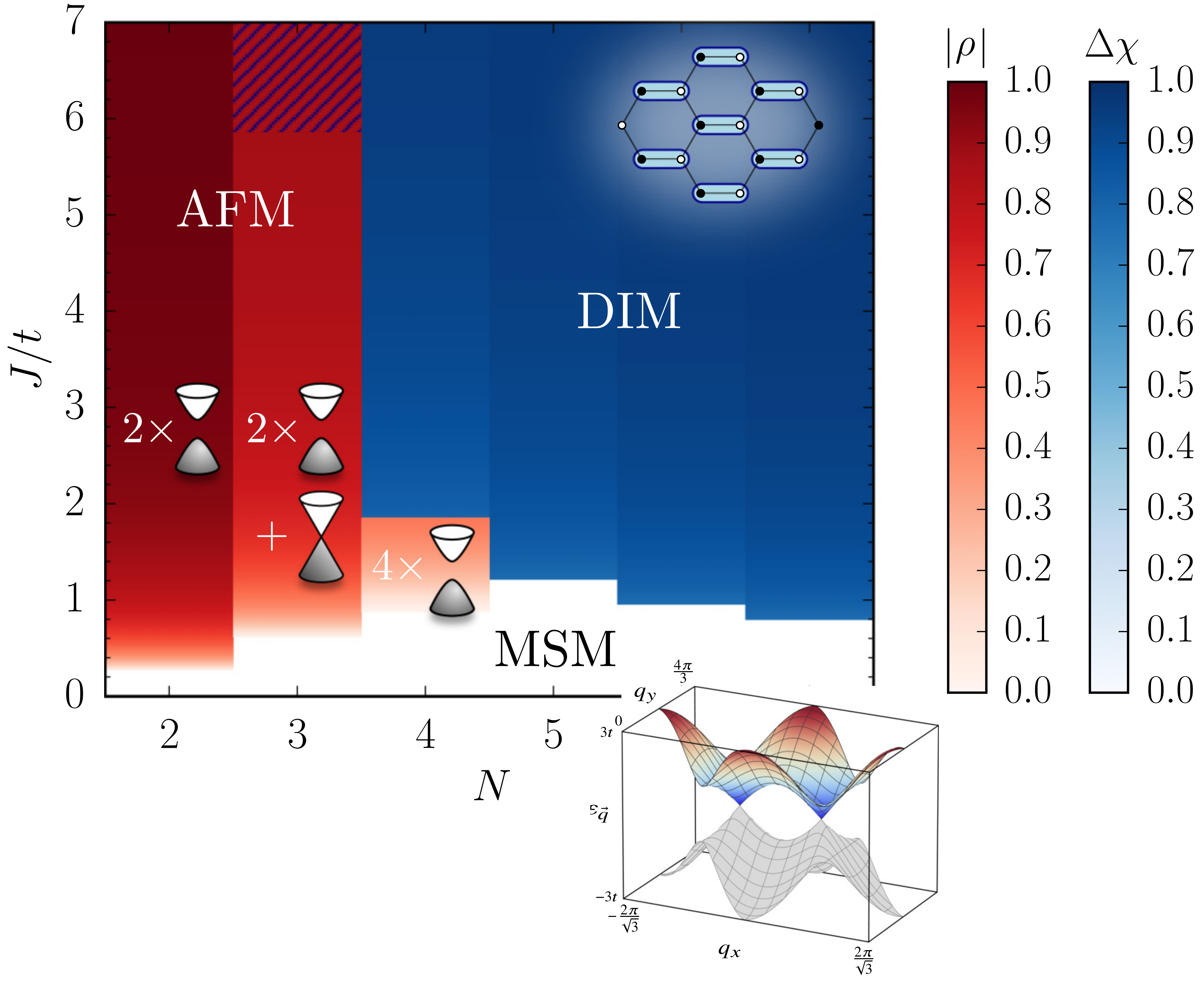
SO(N) Majorana-Hubbard models



[LJ & Seifert, PRB '22]

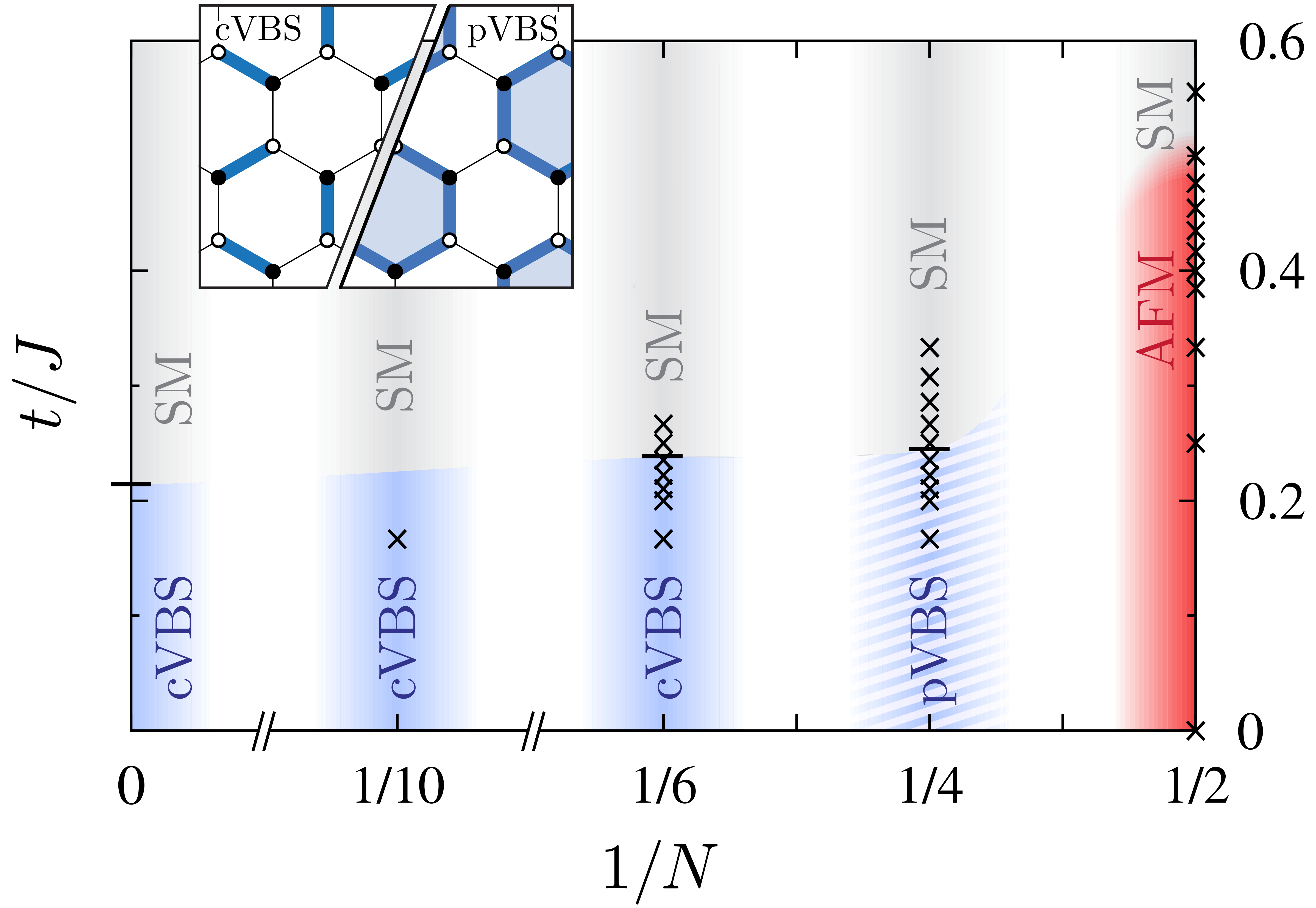
Conclusions

SO(N) Majorana-Hubbard models



[LJ & Seifert, PRB '22]

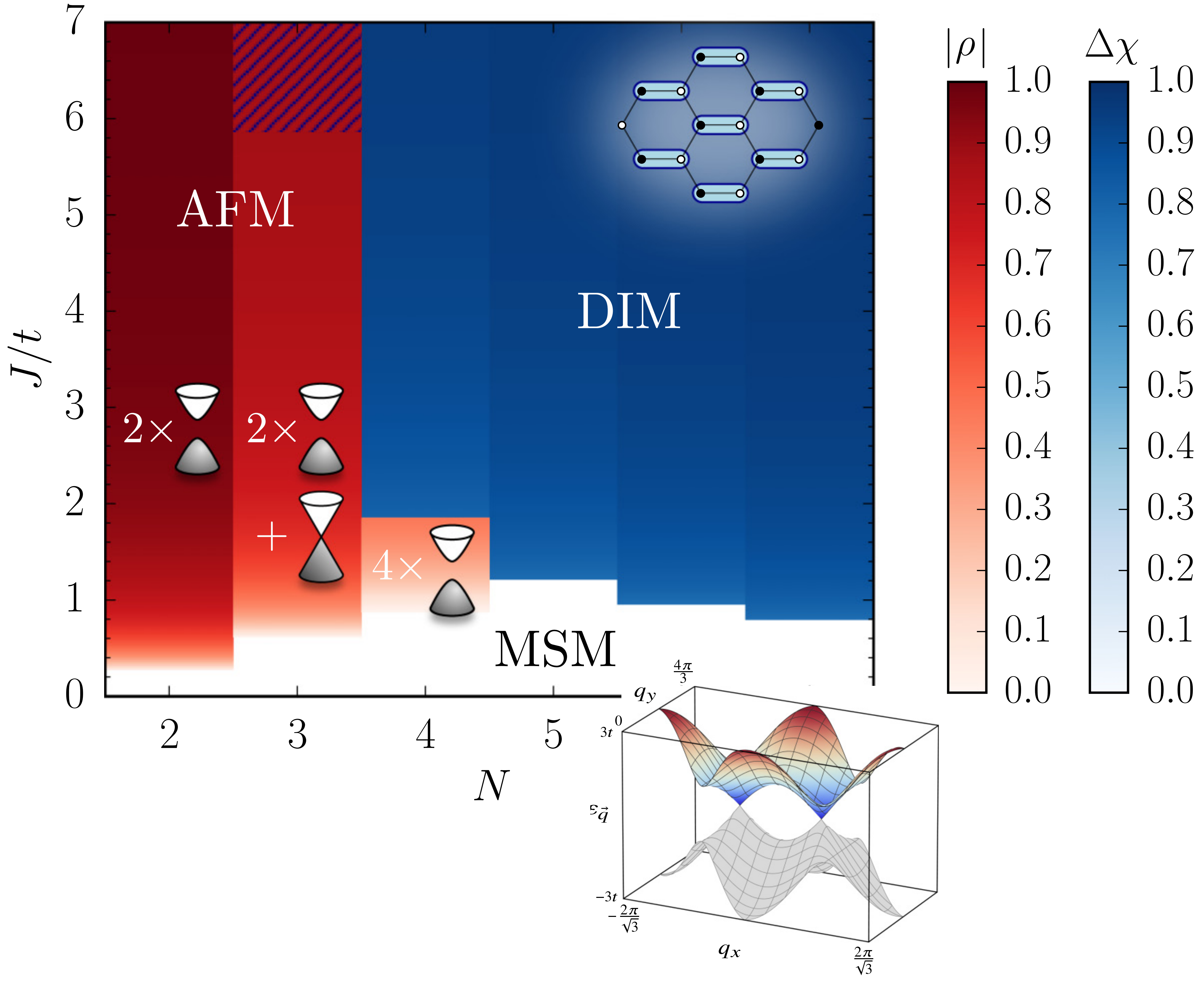
SU(N) Hubbard-Heisenberg models



[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]

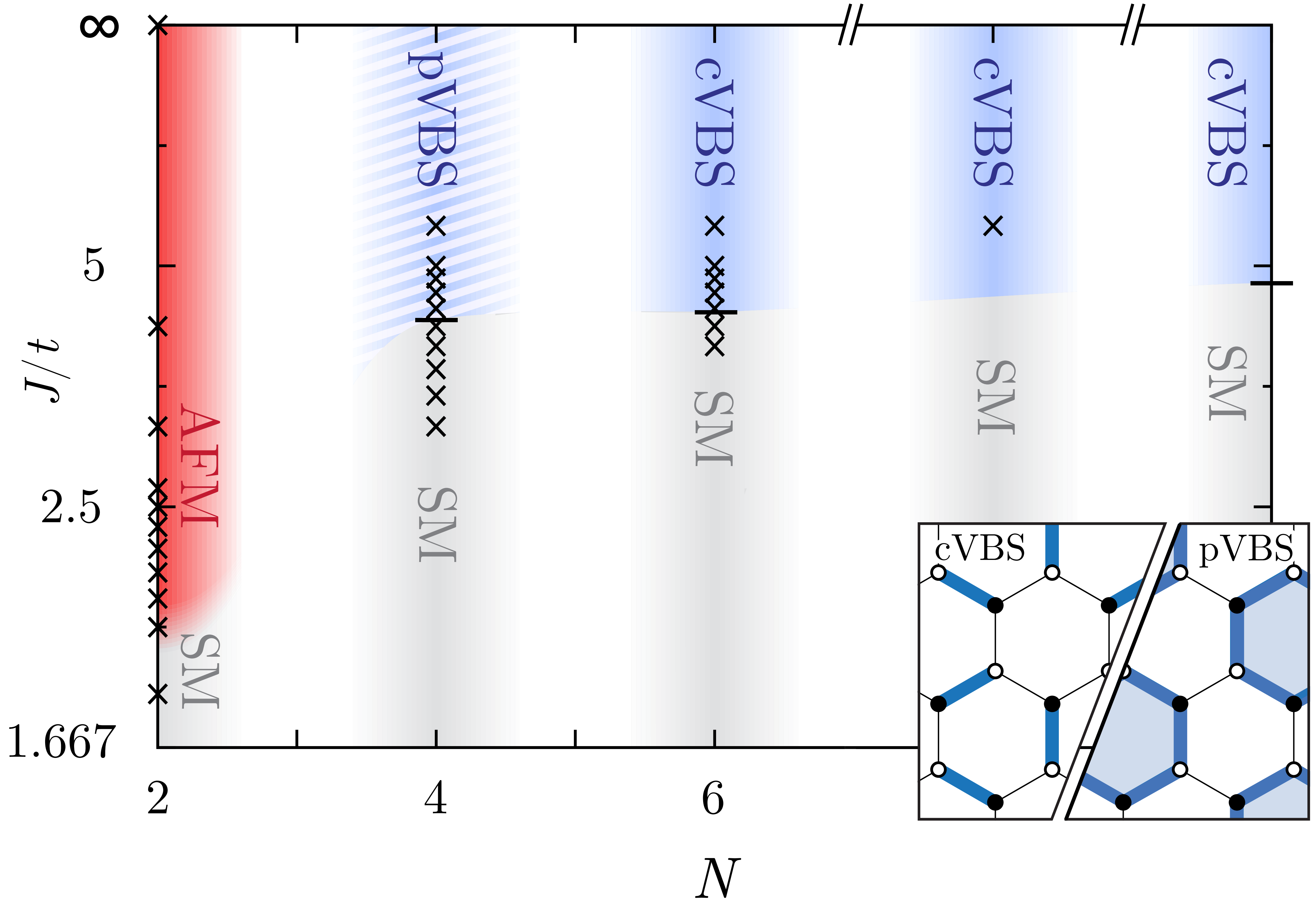
Conclusions

SO(N) Majorana-Hubbard models



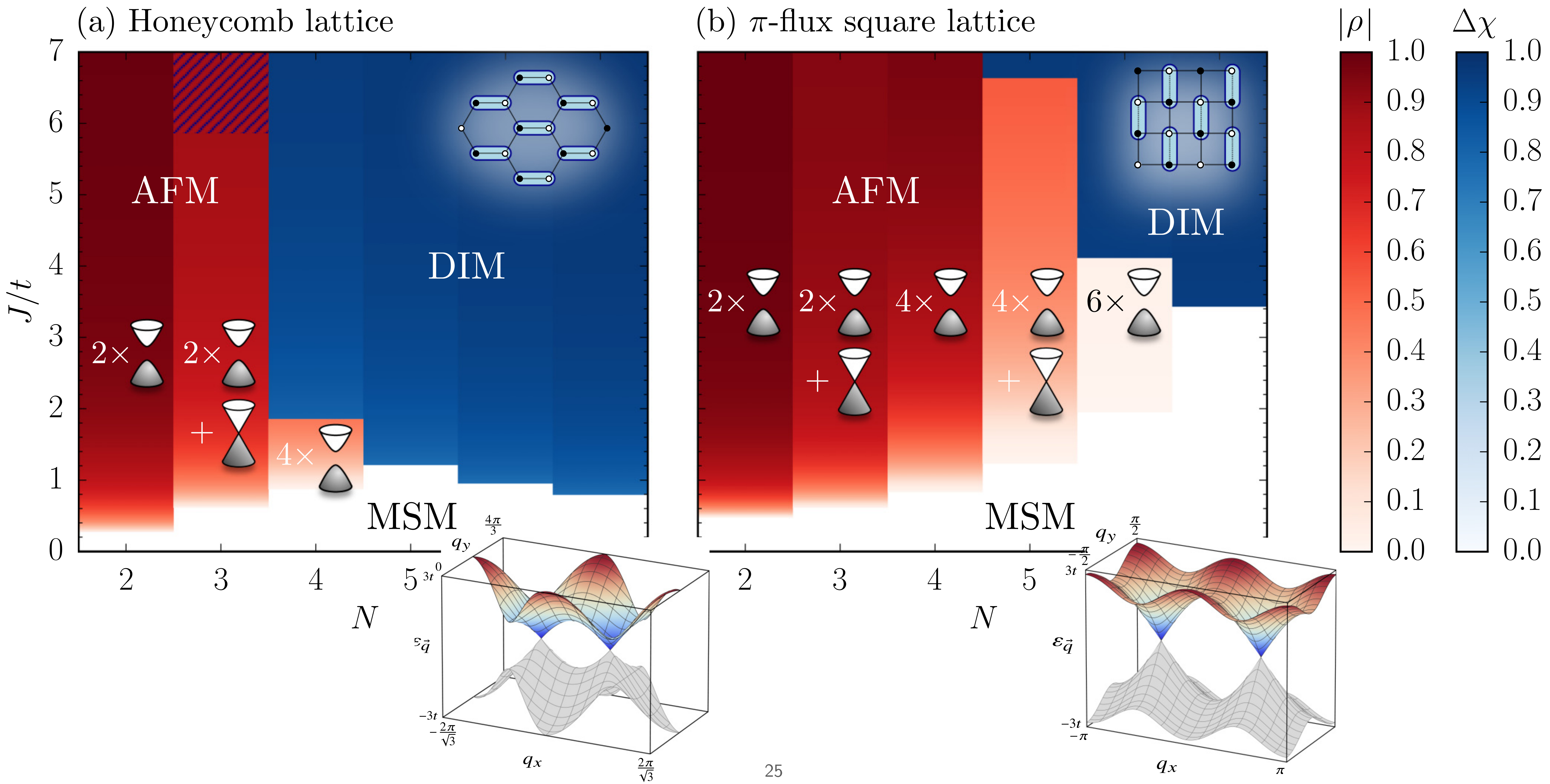
[LJ & Seifert, PRB '22]

SU(N) Hubbard-Heisenberg models



[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]

SO(N) Majorana-Hubbard models: Honeycomb vs. square lattices

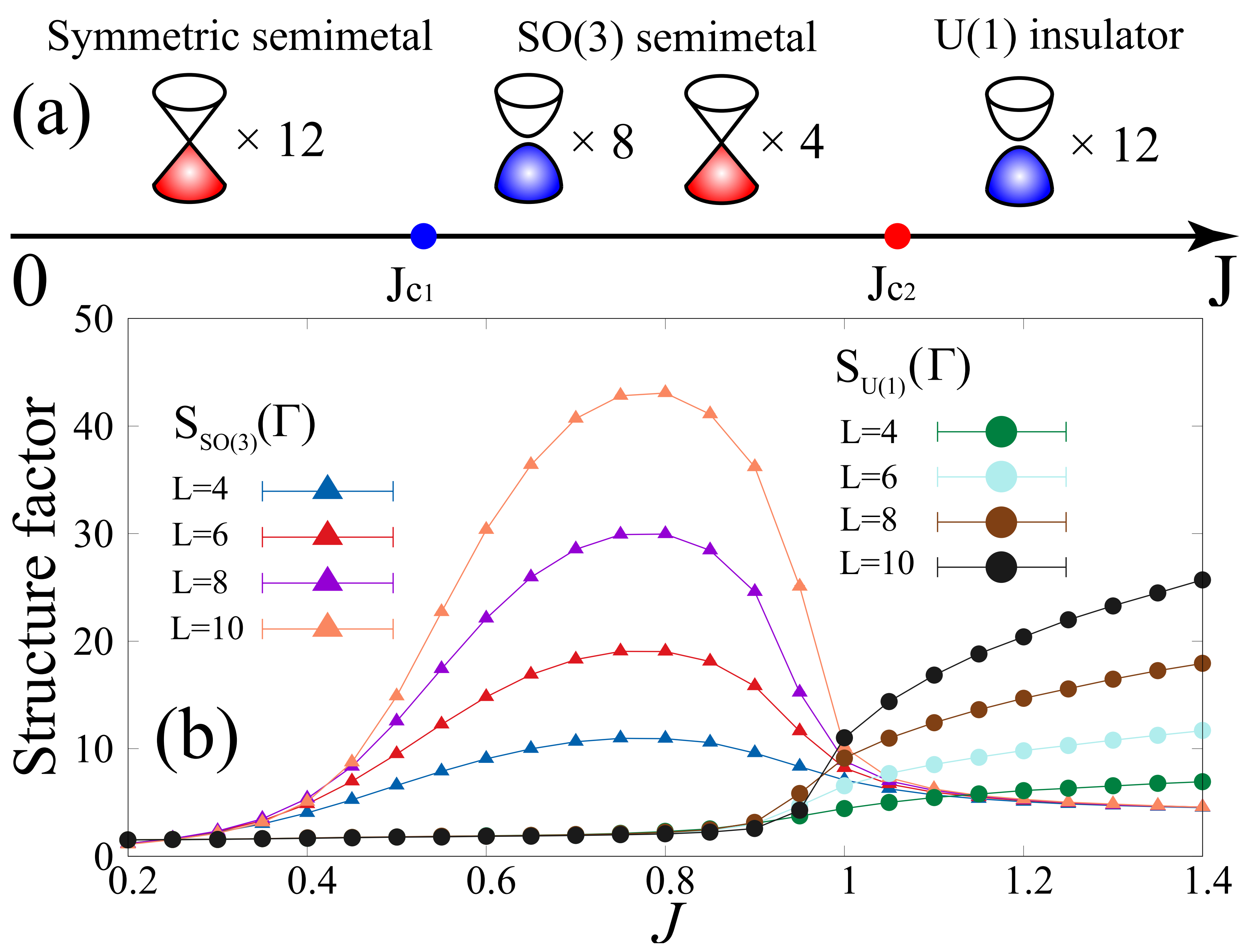


Gross-Neveu-SO(3): Sign-problem-free QMC

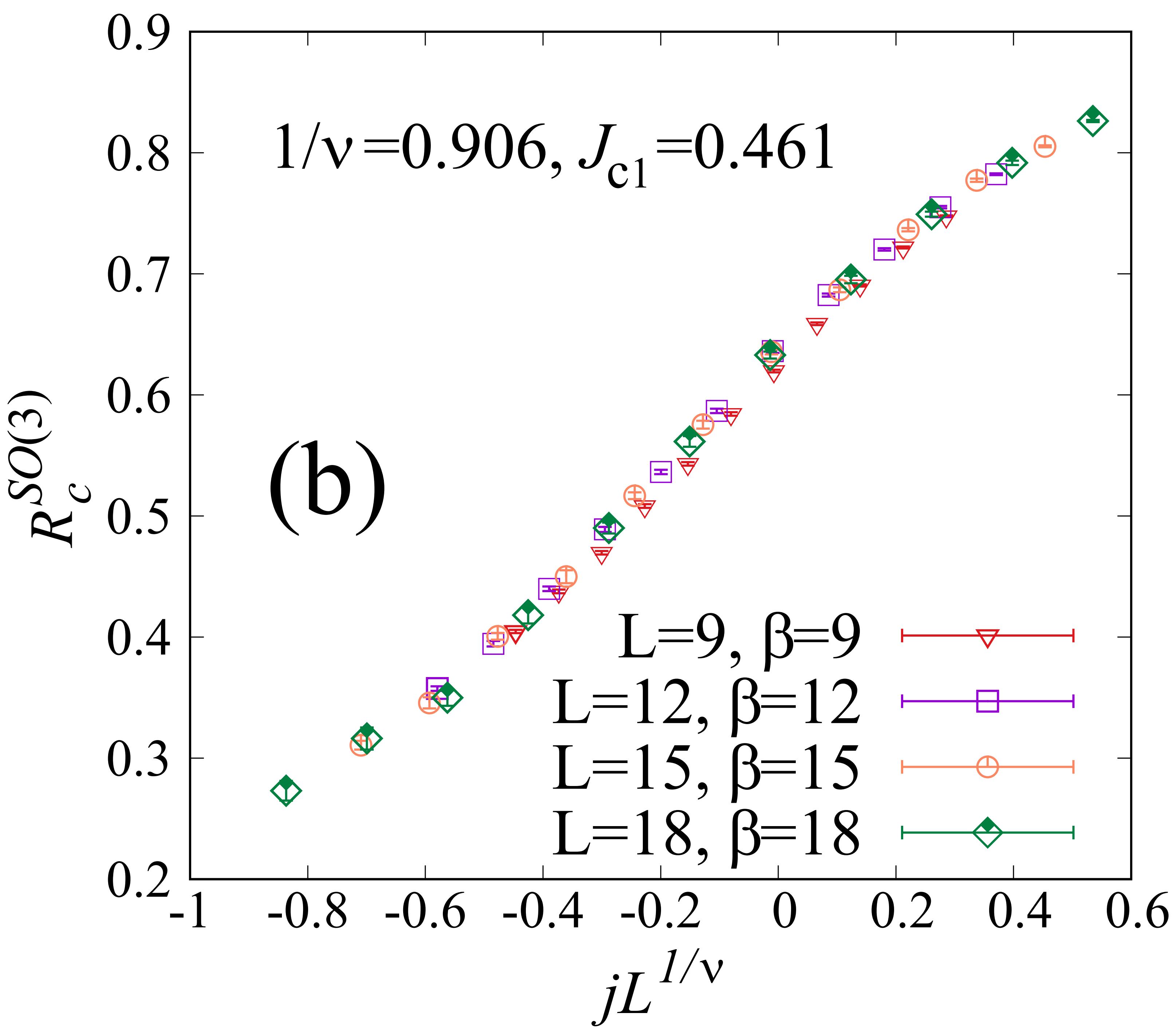
Hamiltonian:

$$H = -t \sum_{\langle \boldsymbol{i},\boldsymbol{j} \rangle} c^{\dagger}_{\boldsymbol{i}\sigma\lambda} c_{\boldsymbol{j}\sigma\lambda} - J \sum_{\boldsymbol{i}\alpha} \left(c^{\dagger}_{\boldsymbol{i}\sigma\lambda} K^{\alpha}_{\sigma\sigma'} \tau^{z}_{\lambda\lambda'} c_{\boldsymbol{i}\sigma'\lambda'} \right)^2$$

Phase diagram:



Finite-size scaling collapse:



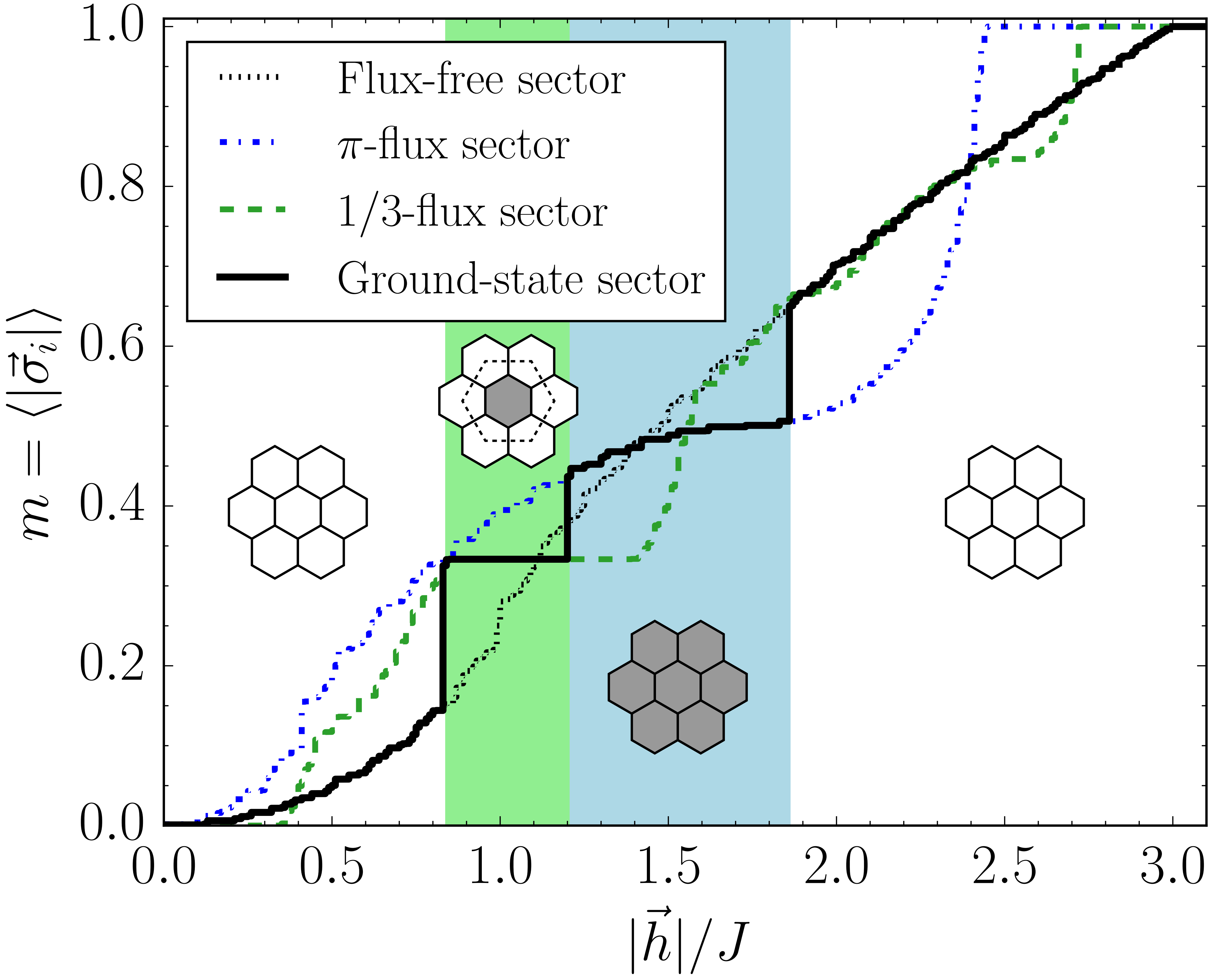
[Liu, Vojta, Assaad, LJ, arXiv:2108.06346]

Spin-orbital model in external magnetic field

Hamiltonian:

$$\mathcal{H} = -K \sum_{\langle ij \rangle_\gamma} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \otimes \tau_i^\gamma \tau_j^\gamma + J \sum_{\langle ij \rangle} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \otimes \mathbb{1}_i \mathbb{1}_j - \vec{h} \cdot \sum_i \vec{\sigma}_i \otimes \mathbb{1}$$

Magnetization:



Finite-size spectroscopy: Ising vs Ising*

Transverse-field Ising:

$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Transverse-field toric code:

$$H = -J \sum_s \prod_{i \in s} \sigma_i^x - J \sum_p \prod_{i \in p} \sigma_i^z - h \sum_i \sigma_i^x$$

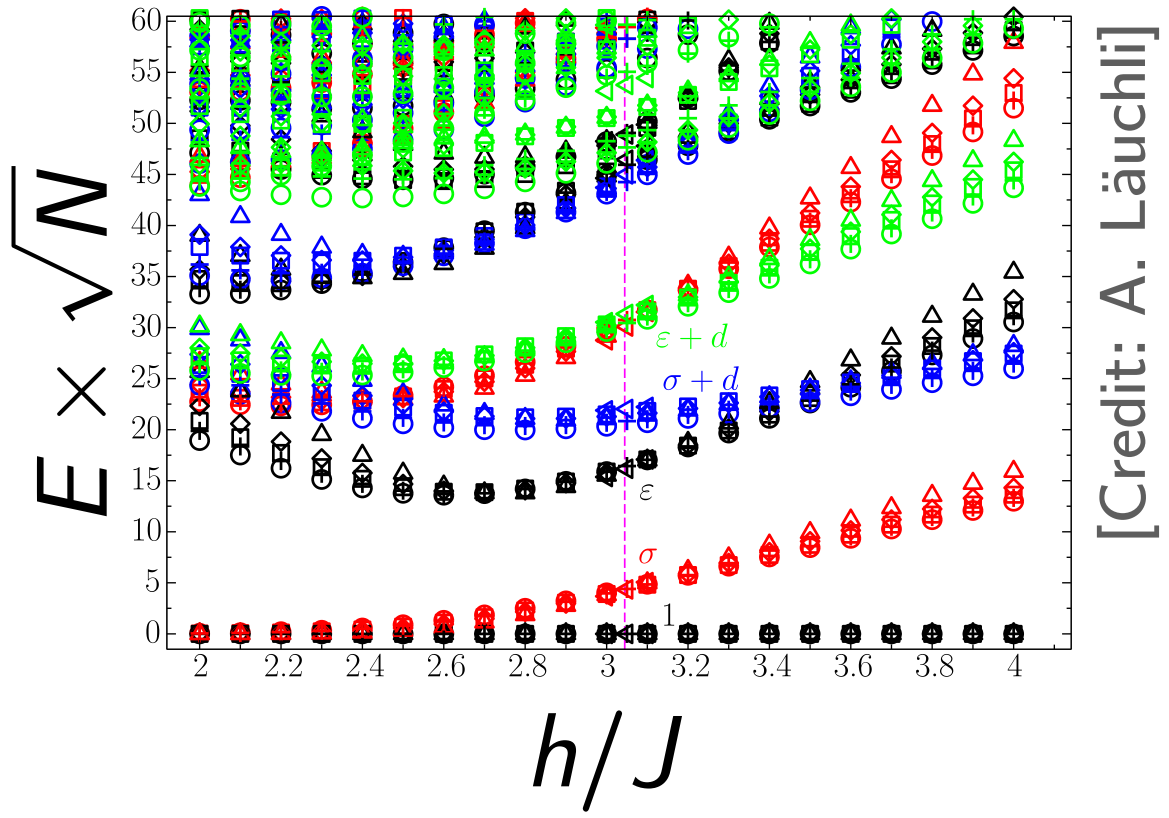
Finite-size spectroscopy: Ising vs Ising*

Transverse-field Ising:

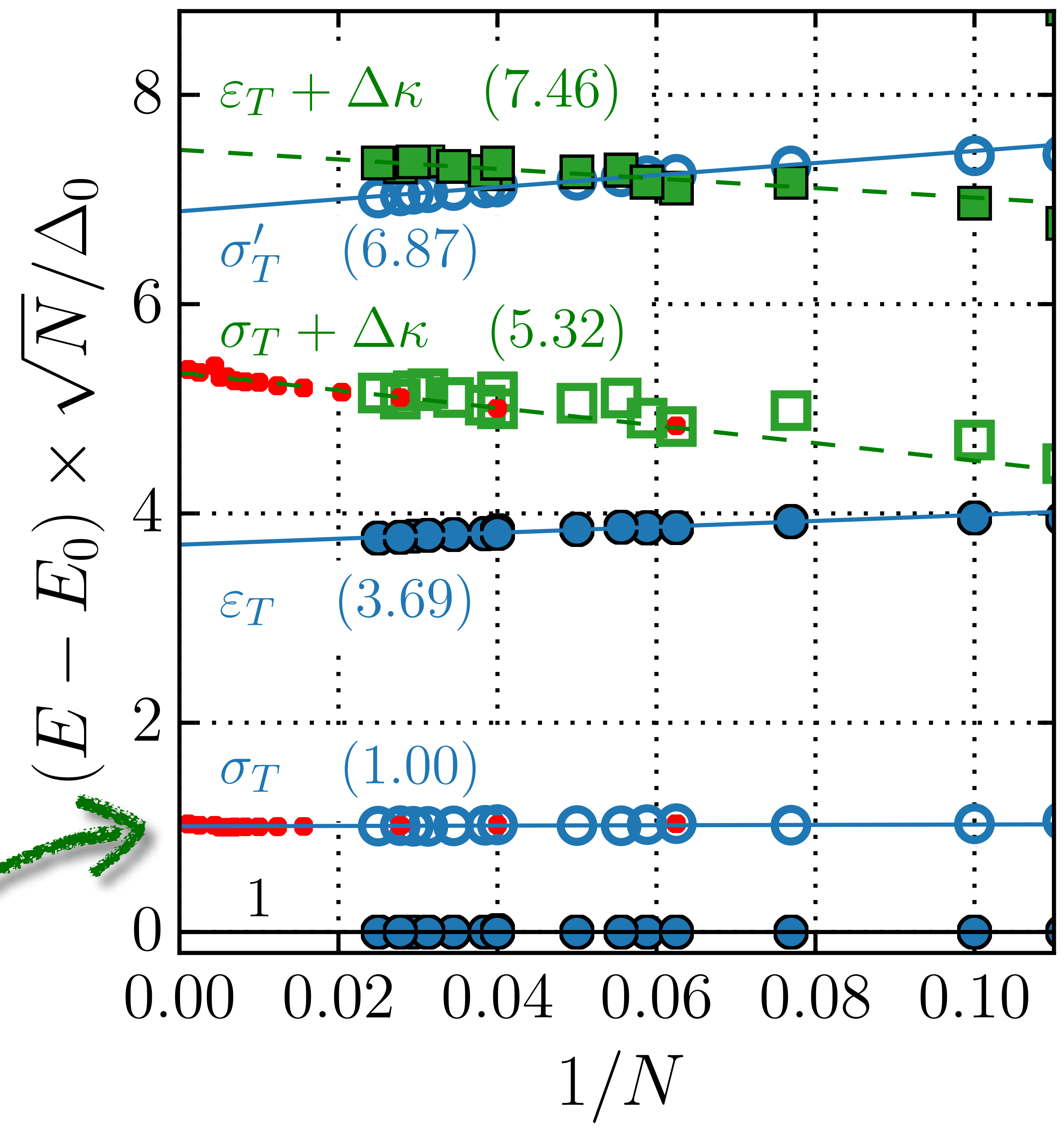
$$H = -J \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z - h \sum_i \sigma_i^x$$

Transverse-field toric code:

$$H = -J \sum_s \prod_{i \in s} \sigma_i^x - J \sum_p \prod_{i \in p} \sigma_i^z - h \sum_i \sigma_i^x$$

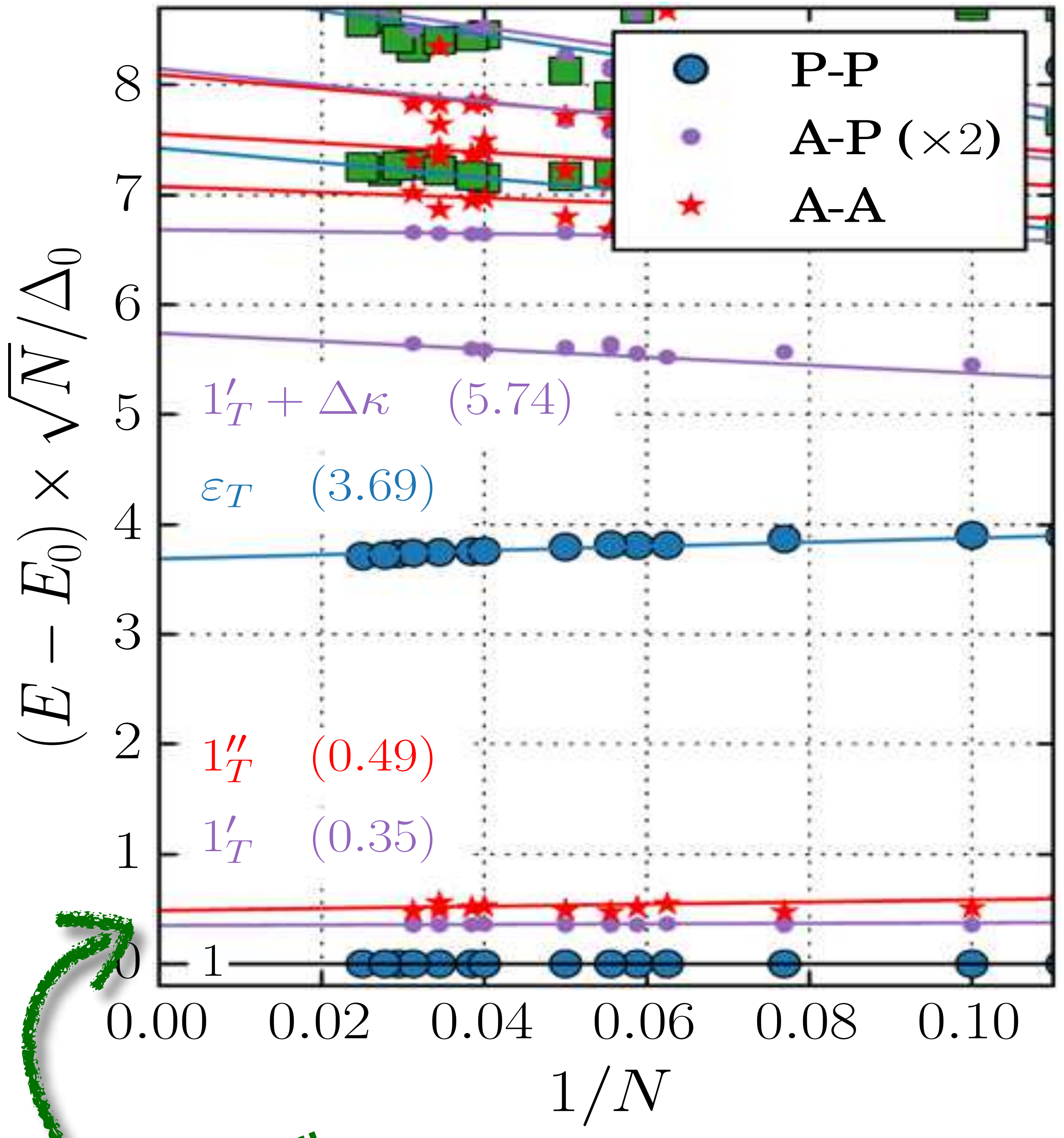


Ising
$h = h_c$

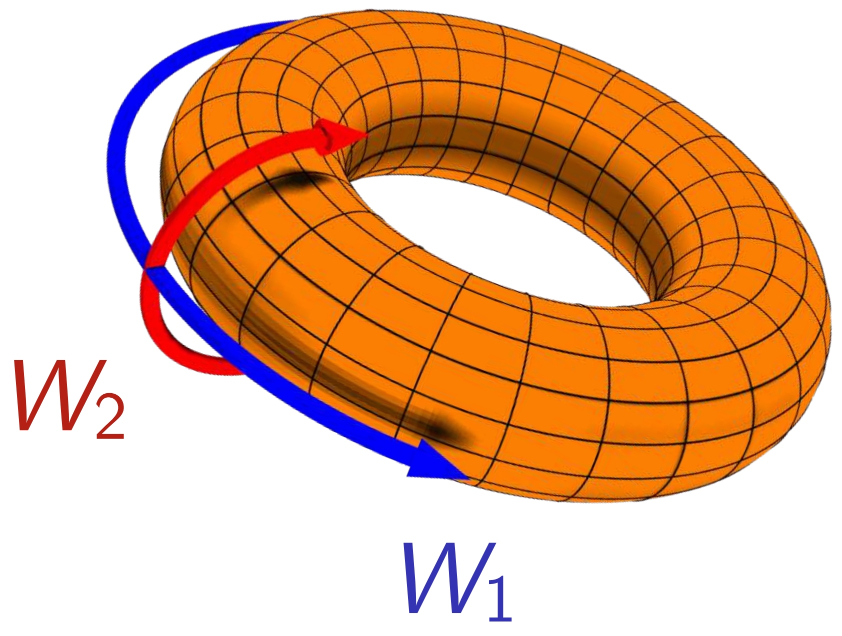


missing in Ising*

Ising*
$h = h_c$



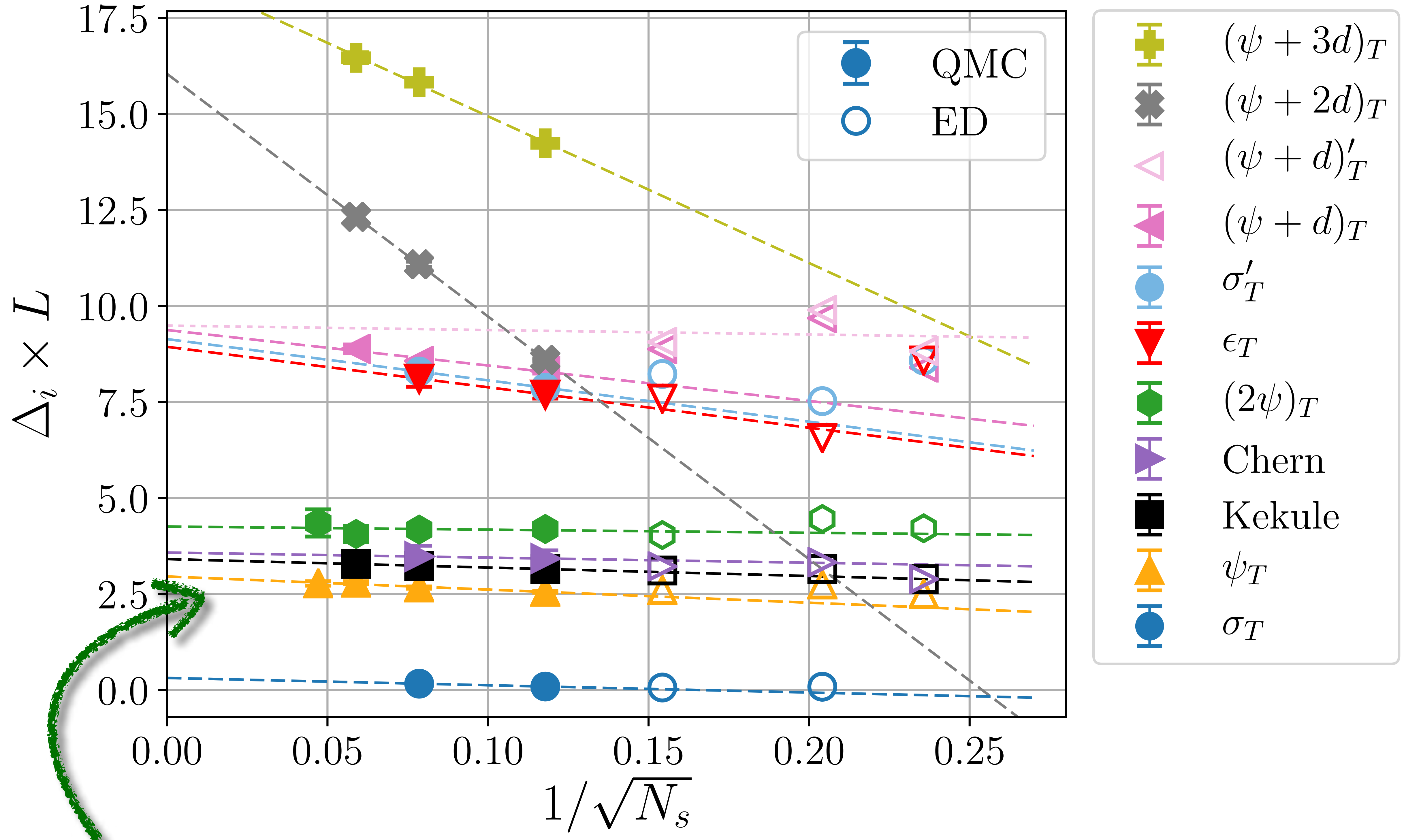
topological "copies"



[Schuler, Whitsitt, Henry, Sachdev, Läuchli, PRL '16]

Gross-Neveu vs Gross-Neveu*

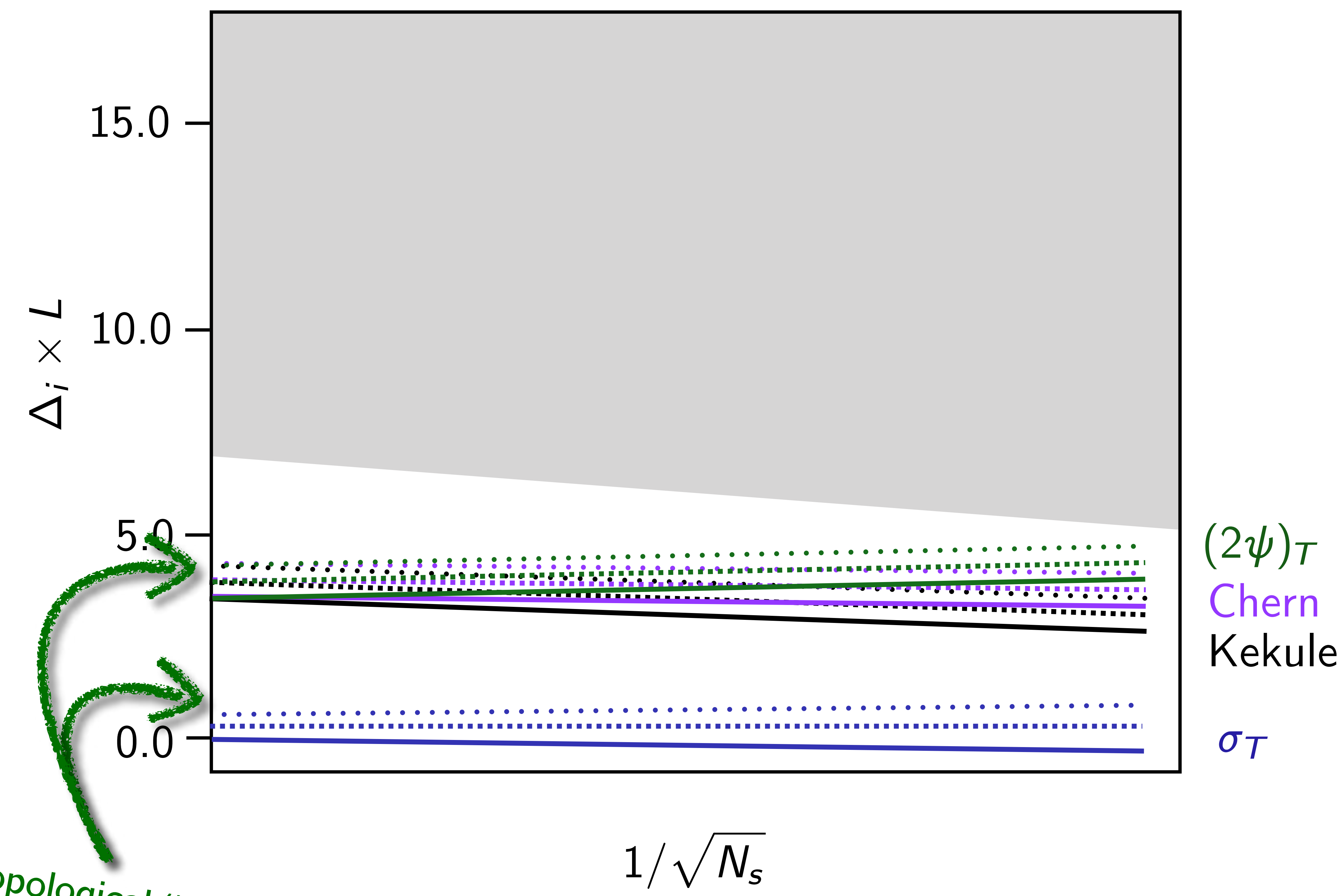
Gross-Neveu-$\mathbb{Z}_2$



missing in GN*

[Schuler *et al.*, PRB '21]

Gross-Neveu-$\mathbb{Z}_2$* (schematic)



topological "copies"

... testable in future simulations

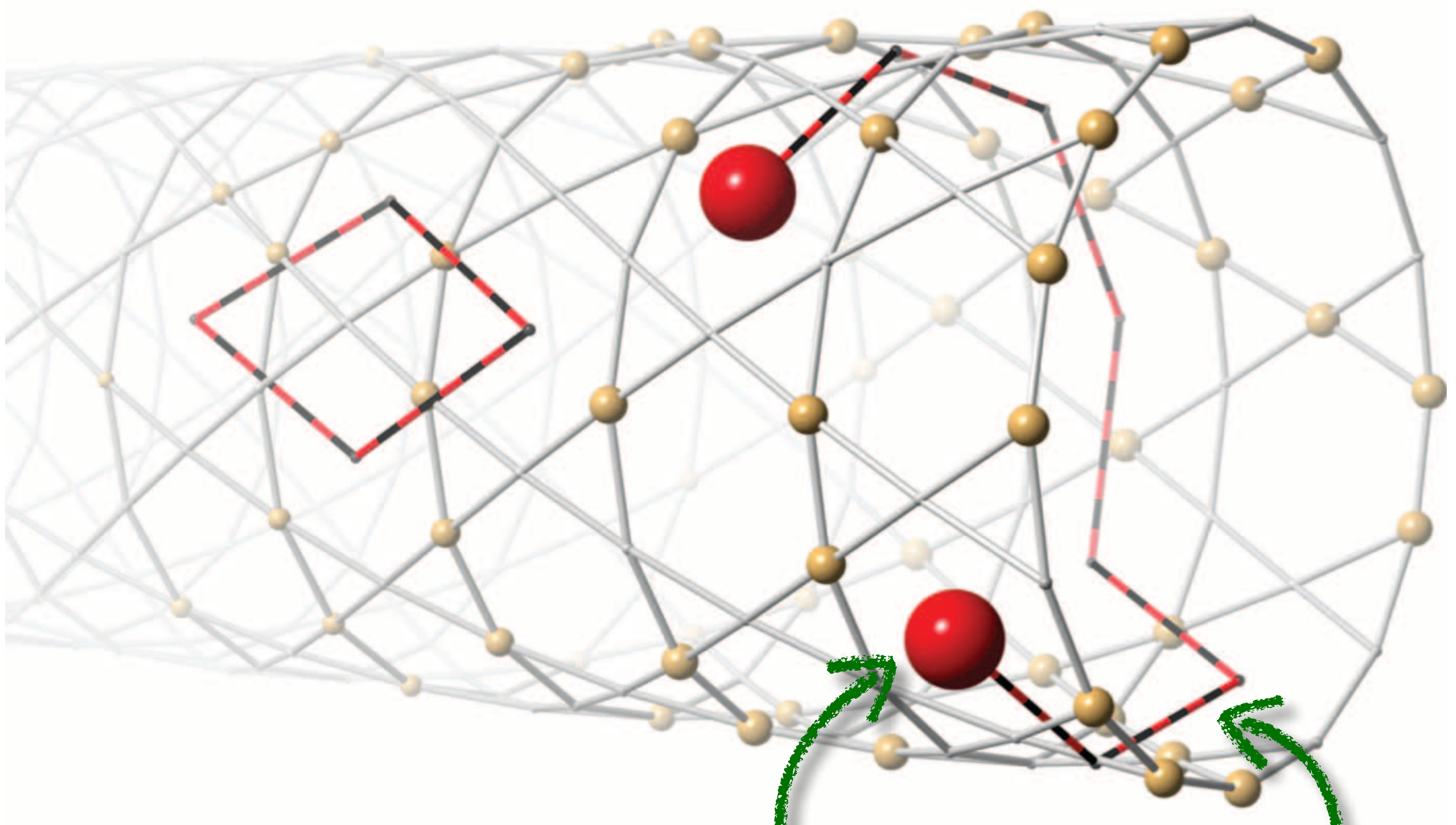
Fractionalized quantum criticality: XY*

Bose-Hubbard-like model (kagome lattice):

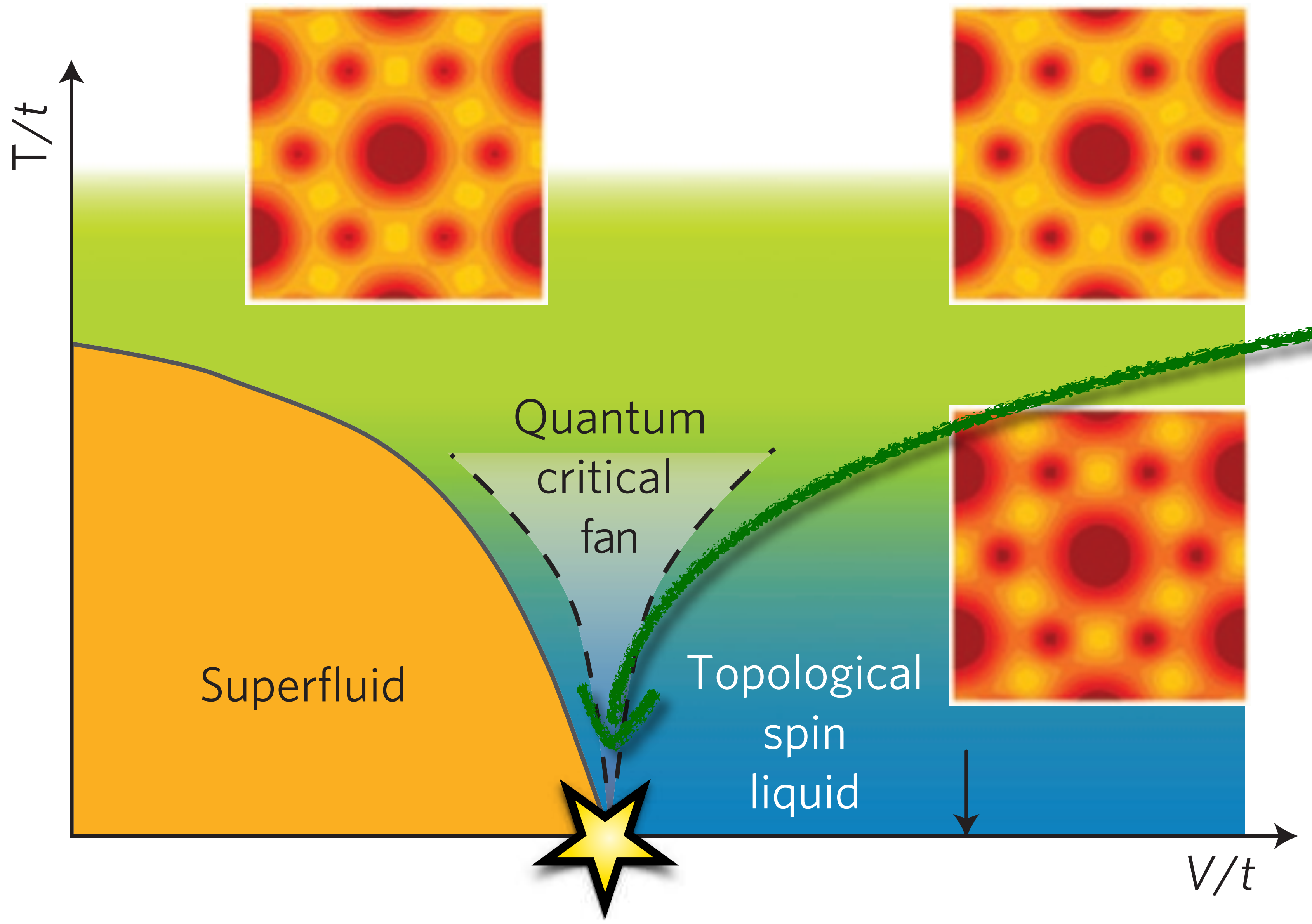
$$\mathcal{H} = -t \sum_{\langle ij \rangle} \left[b_i^\dagger b_j + b_i b_j^\dagger \right] + V \sum_{\hexagon} (n_{\hexagon})^2$$

Hopping bosons

Boson density in plaquette

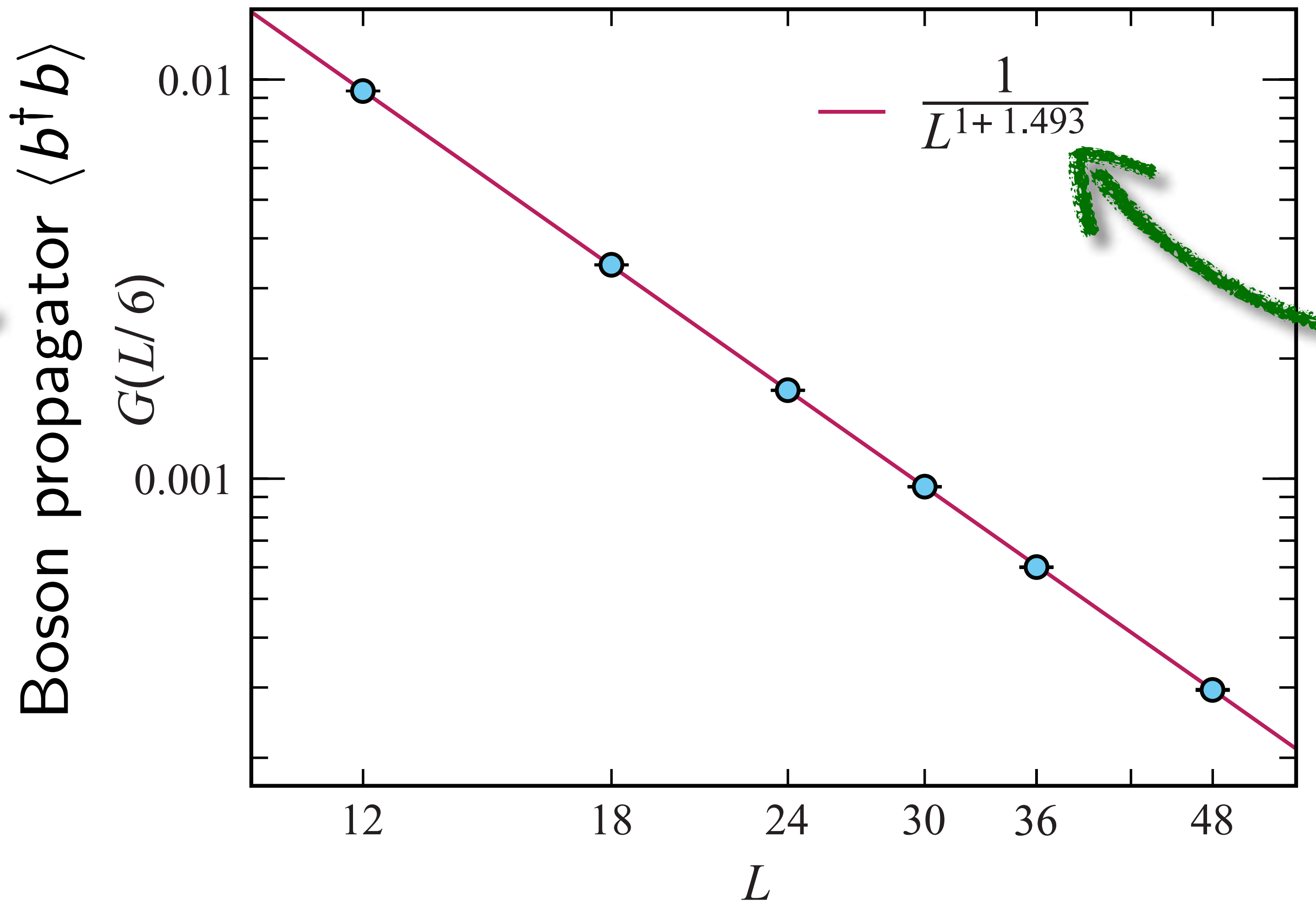


Phase diagram:



[Isakov, Hastings, Melko, Nat. Phys. '11]

$V/t = (V/t)_c$



System size

$\eta \approx 1.493$ (!)

... anomalous dimension of composite field

but $\nu \approx 0.67$

... as in conventional XY

[Isakov, Melko, Hastings, Science '12]
[Chubukov, Senthil, Sachdev, PRL '94]

Kitaev-Ising spin-orbital model

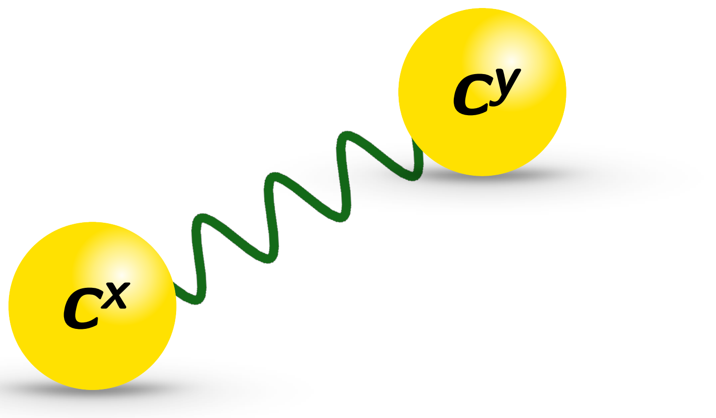
Ising perturbation:

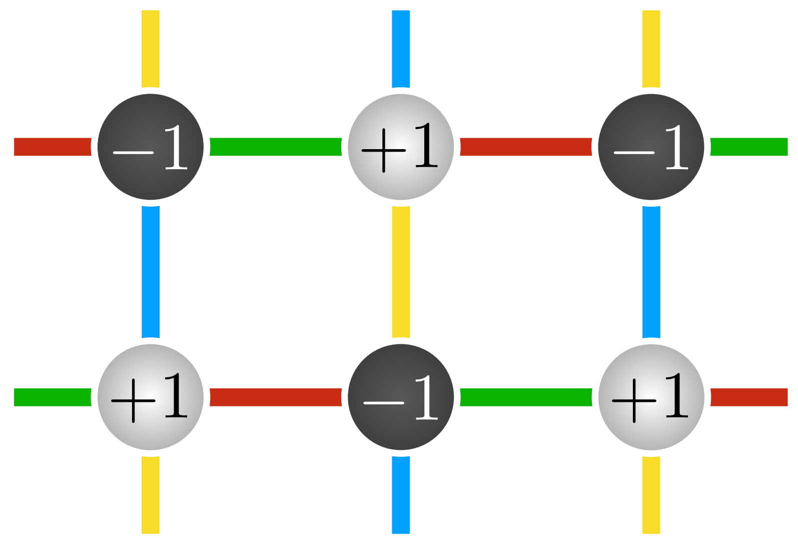
$$H = H_K + J^z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z \otimes \mathbb{1}_i \mathbb{1}_j$$

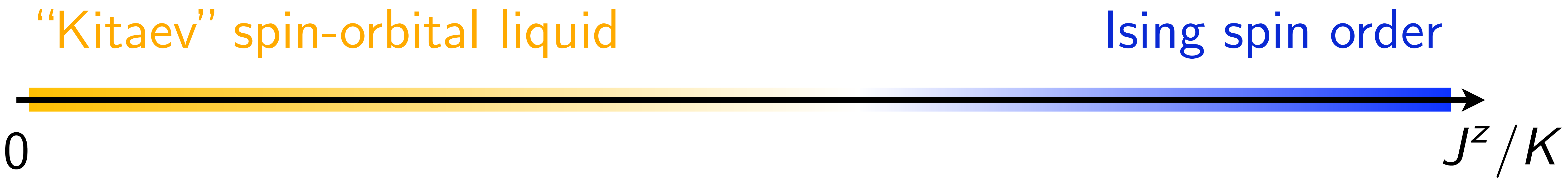
Kitaev-Ising spin-orbital model

Ising perturbation:

$$H = H_K + J^z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z \otimes \mathbb{1}_i \mathbb{1}_j$$







Parton representation:

hopping parameter $t = 2K$

π flux

nearest-neighbor repulsion $V = 4J^z$

$$H \mapsto \sum_{\langle ij \rangle} \left[2K\, u_{ij} (f_i^\dagger f_j + f_j^\dagger f_i) + 4J^z (n_i - \tfrac{1}{2})(n_j - \tfrac{1}{2}) \right]$$

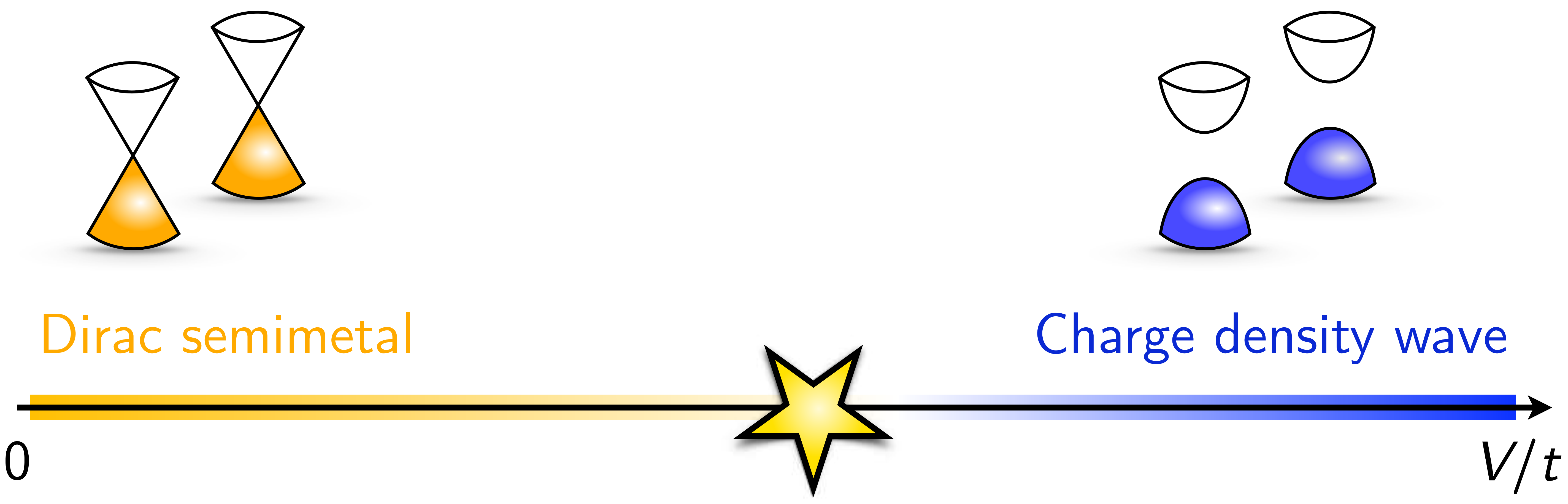
$f = \frac{1}{2}(c^x + ic^y)$

electron density $f^\dagger f$

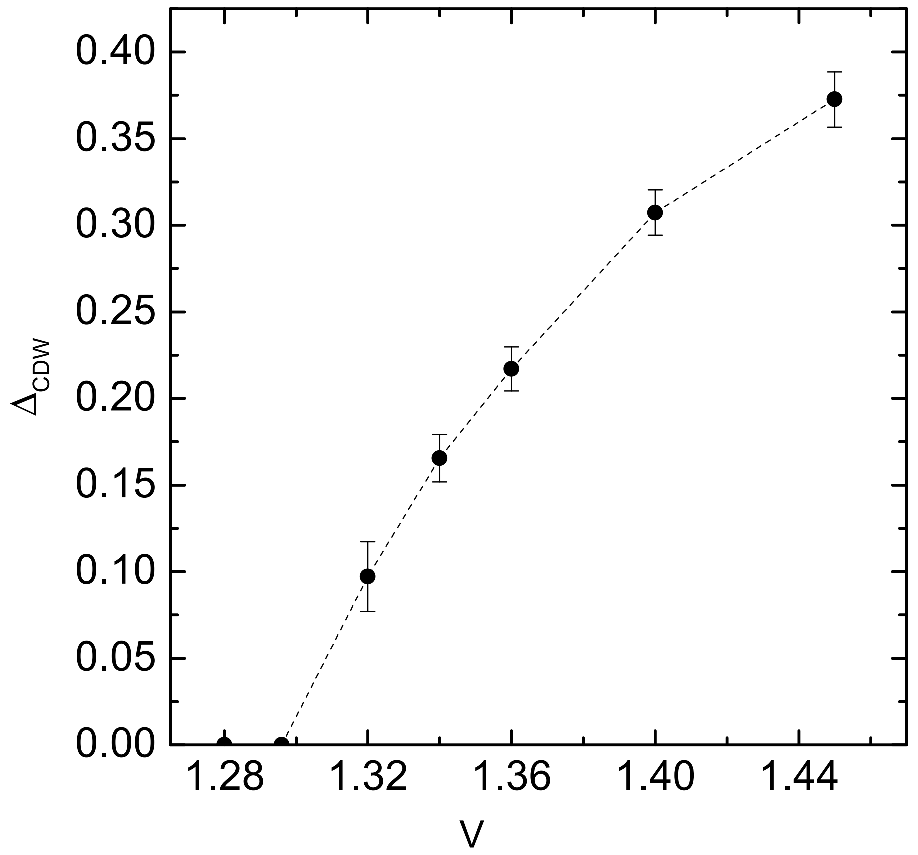
Ground-state flux pattern:
[Lieb, PRL '94]

Spin-orbital model $\mapsto$ interacting fermions on π-flux lattice

Spinless fermions on π-flux lattice: QMC



$(V/t)_c = 1.281(2)$

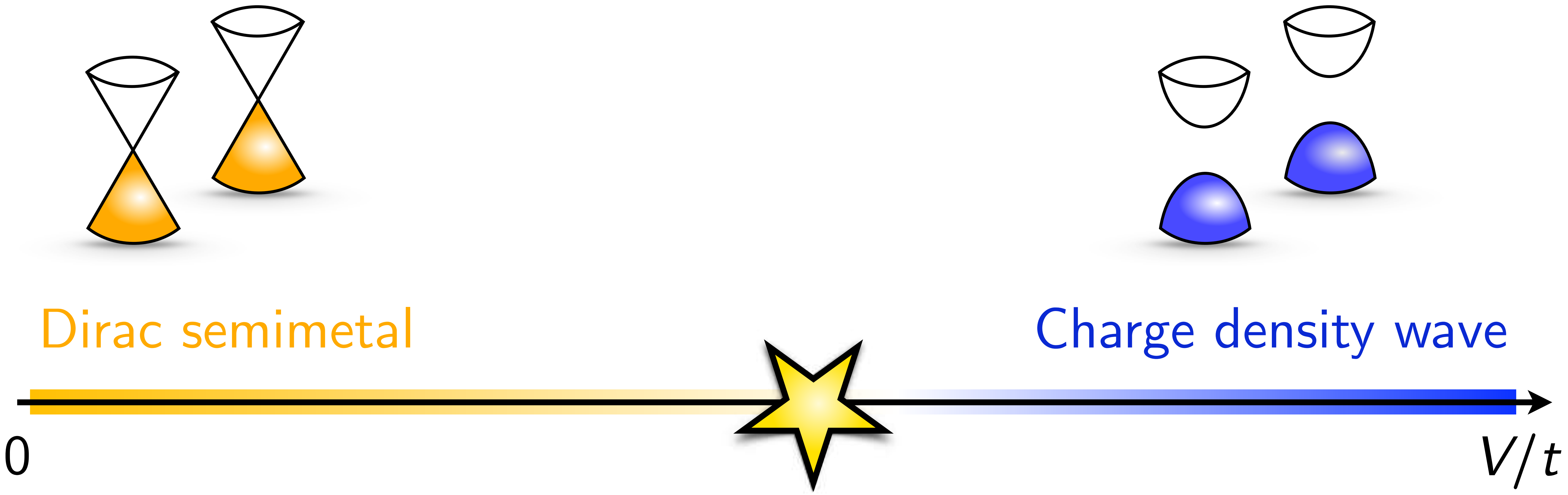


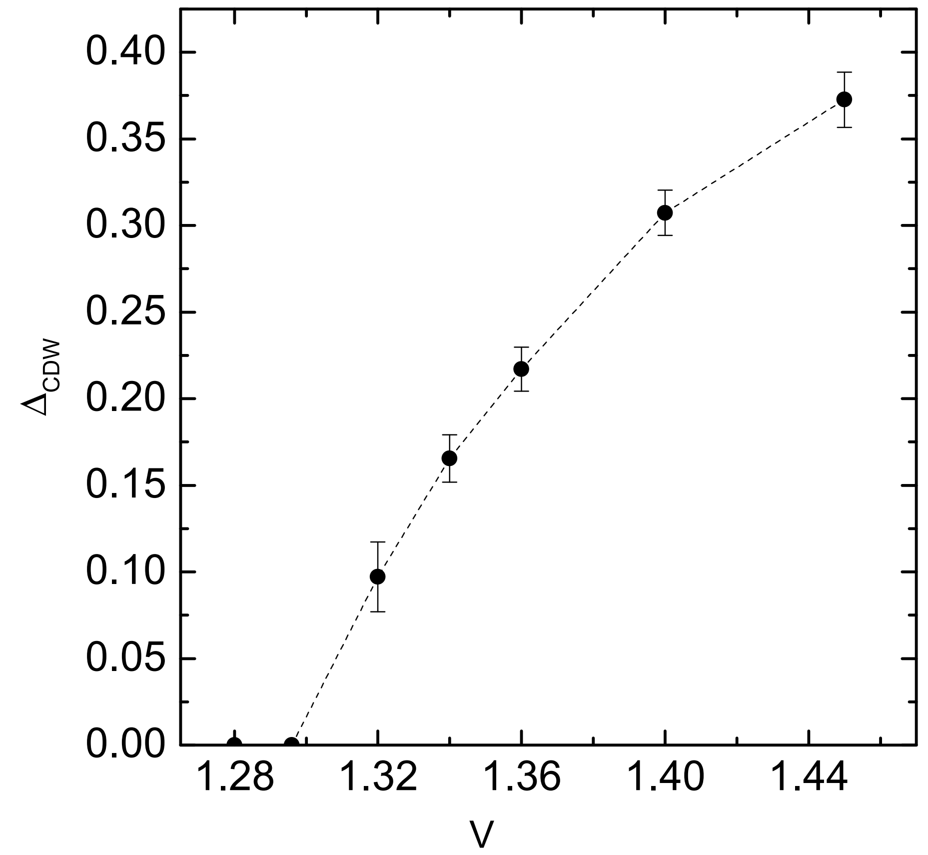
[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]

Gross-Neveu-$\mathbb{Z}_2$ universality: $1/\nu = 1.12(1), \quad \eta_\phi = 0.51(3)$

[Gracey, IJMP '94]
[LJ & Herbut, PRB '14]
[Iliesiu *et al.*, JHEP '18]
[Ihrig, Mihaila, Scherer, PRB '18]
...

Spinless fermions on π-flux lattice: QMC





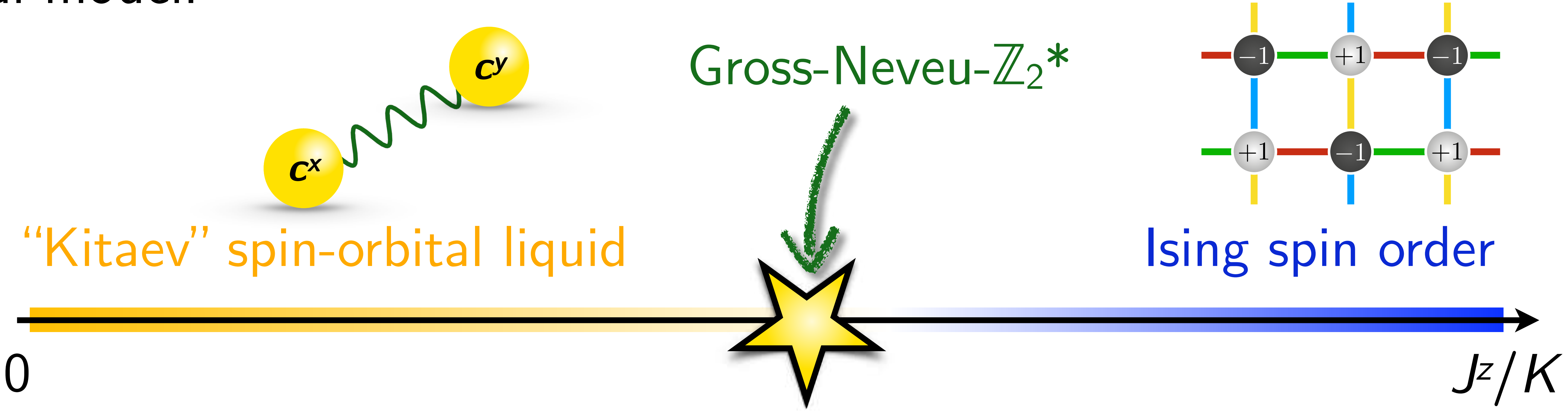
$(V/t)_c = 1.281(2)$

[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]

Gross-Neveu-$\mathbb{Z}_2$ universality: $1/\nu = 1.12(1), \quad \eta_\phi = 0.51(3)$

[Gracey, IJMP '94]
[LJ & Herbut, PRB '14]
[Iliesiu *et al.*, JHEP '18]
[Ihrig, Mihaila, Scherer, PRB '18]
...

Spin-orbital model:



$(J^z/K)_c = 0.642(2)$

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]