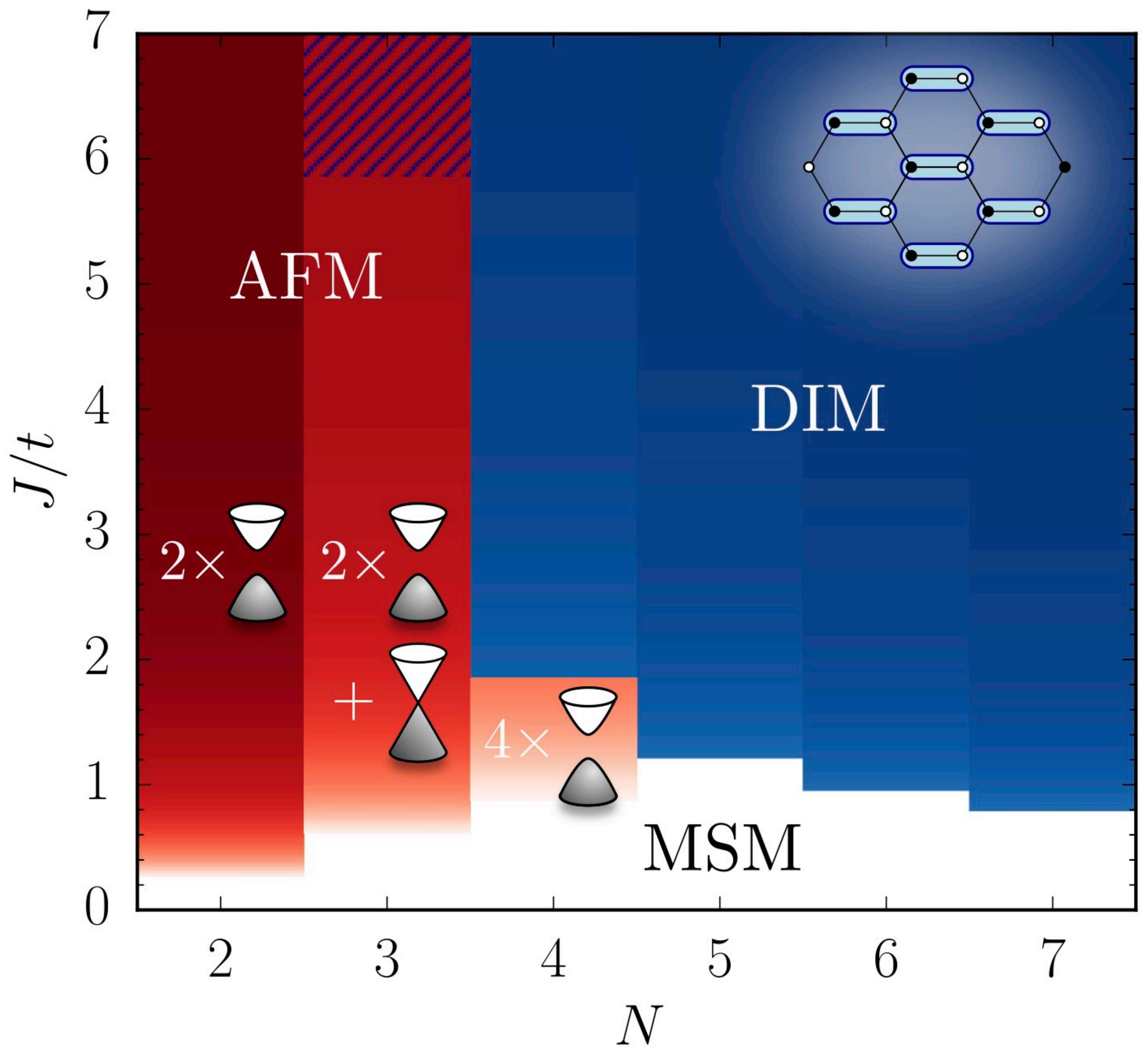
Interacting Majorana fermions

Lukas Janssen (TUD) & Urban Seifert (UCSB)



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DRESDEN concept
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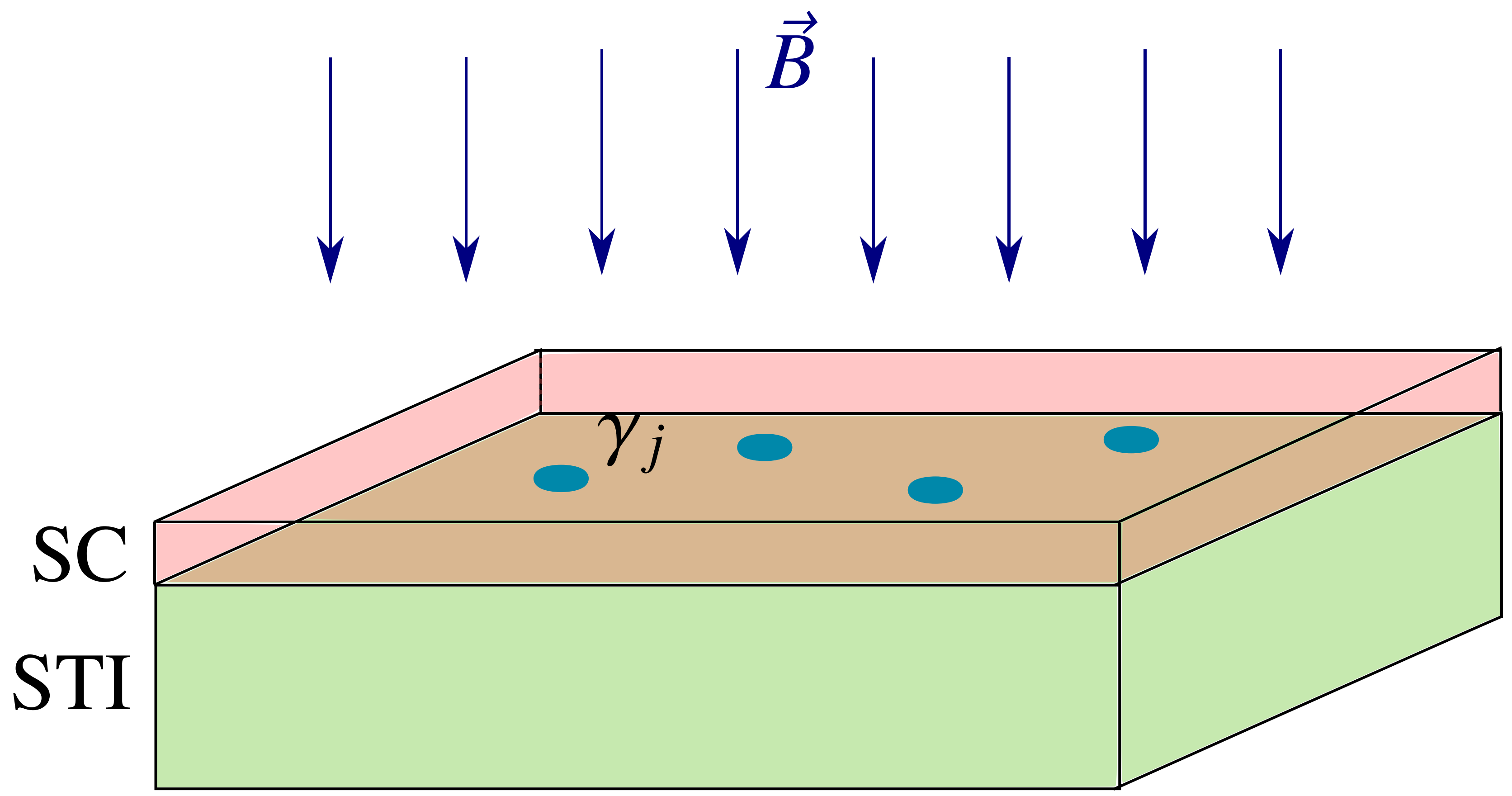
Complexity and Topology in Quantum Matter
Würzburg-Dresden Cluster of Excellence
SFB 1143

Motivation #1: Superconductor / topological insulator heterostructures



[Fu & Kane, PRL '08]
[Rahmani & Franz, Rep. Progr. Phys. '19]

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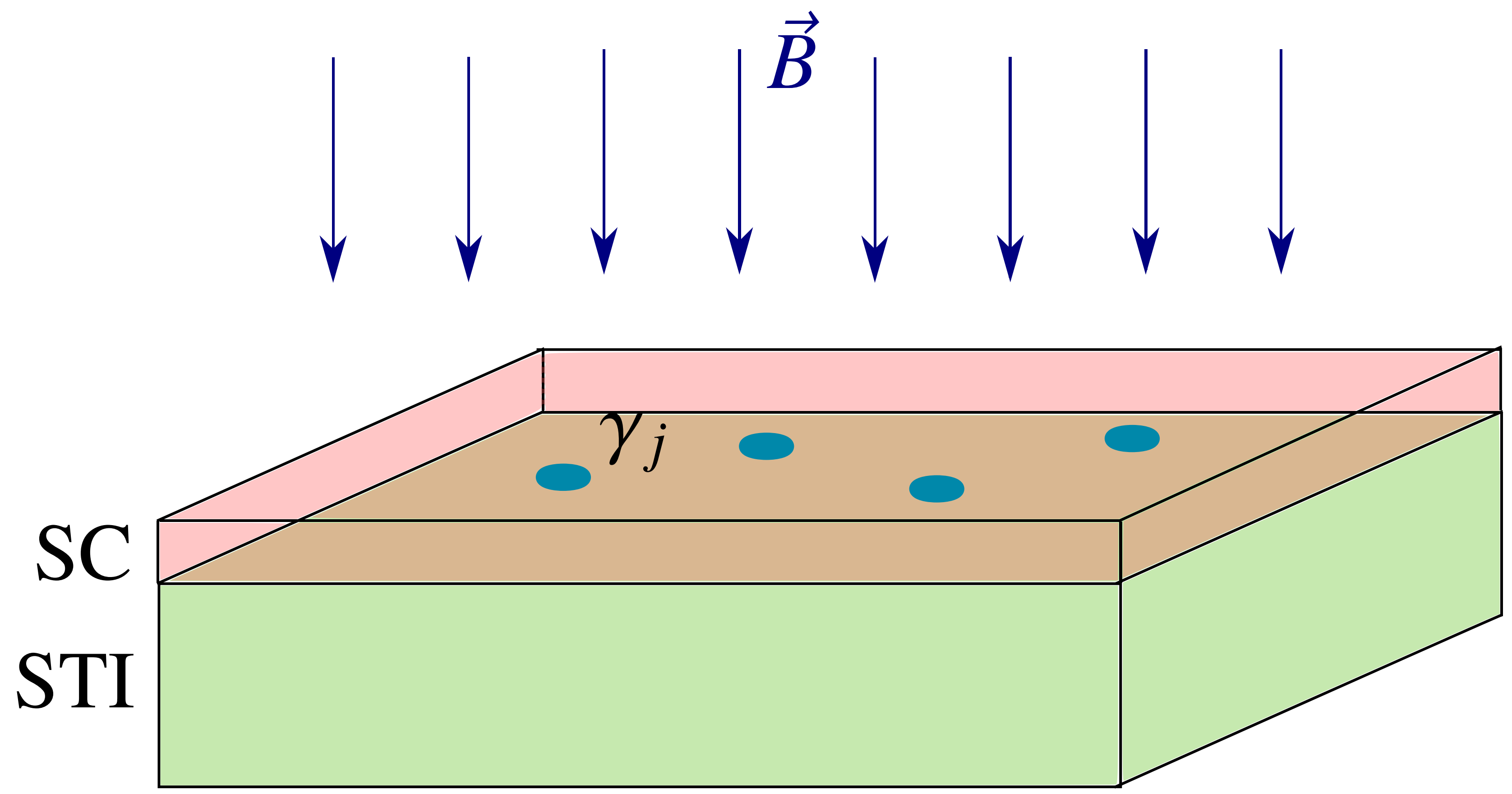


[Fu & Kane, PRL '08]
[Rahmani & Franz, Rep. Progr. Phys. '19]

Effective model:

tunable by gate voltage

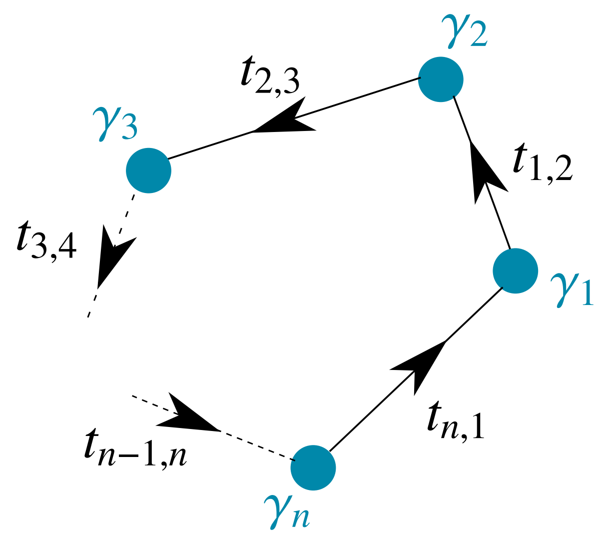
$$H = \sum_{ij} \mathrm{i} t_{ij} \gamma_i \gamma_j + \sum_{ijkl} g_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l + \dots$$

"Majorana-Hubbard models"

Grosfeld-Stern rule:

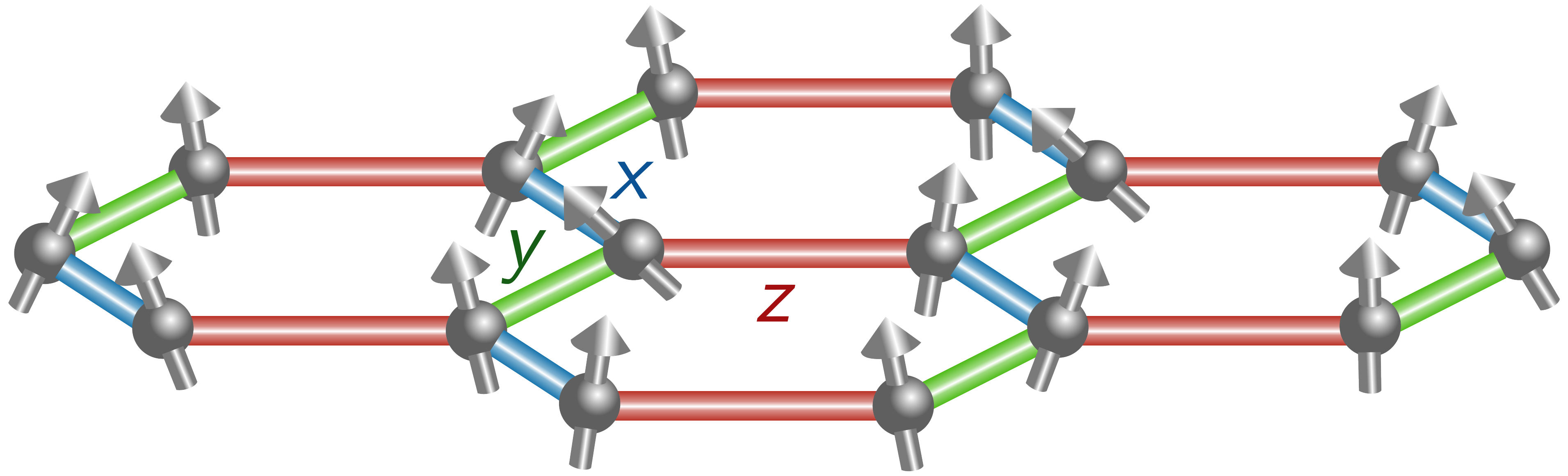
$$\arg\left(i^n t_{1,2} \dots t_{n-1,n} t_{n,1}\right) = \frac{\pi}{2}(n-2)$$

Square lattice: π flux
Honeycomb lattice: 0 flux



[Grosfeld & Stern, PRB '06]
[Liu & Franz, PRB '15]
[Wamer & Affleck, PRB '18]
[Li & Franz, PRB '18]
[Tummuru, Nocera, Affleck, PRB '21]
...

Motivation #2: Kitaev honeycomb model



Hamiltonian:

$$\mathcal{H} = K\left(\sum_{\langle ij\rangle_x}\sigma_i^x\sigma_j^x + \sum_{\langle ij\rangle_y}\sigma_i^y\sigma_j^y + \sum_{\langle ij\rangle_z}\sigma_i^z\sigma_j^z\right)$$

[Kitaev, Ann. Phys. '06]

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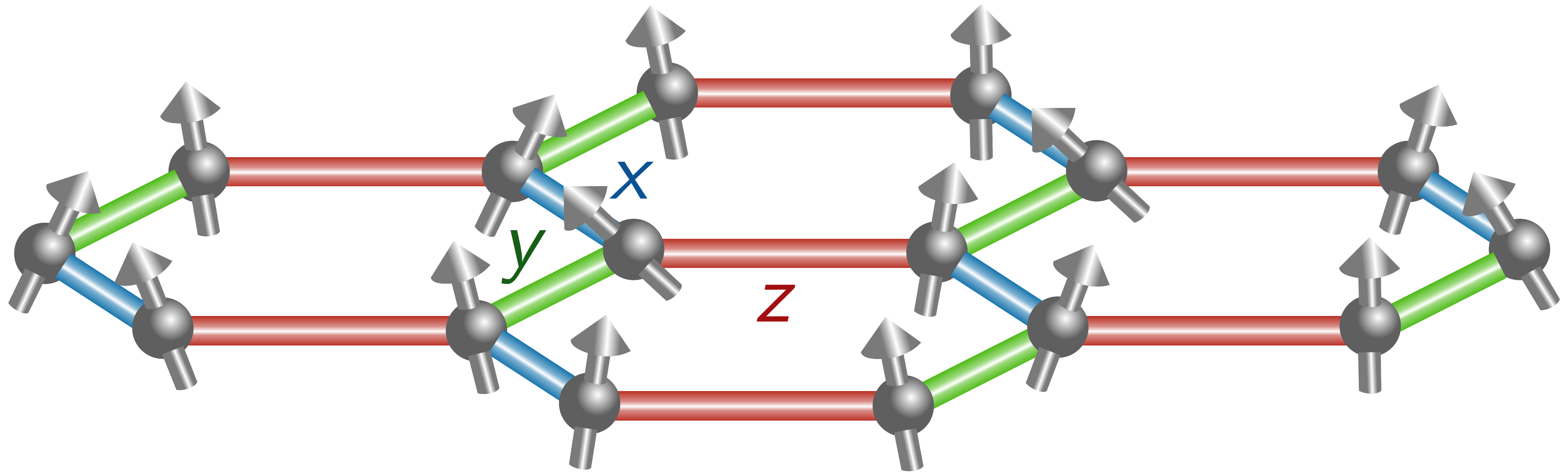

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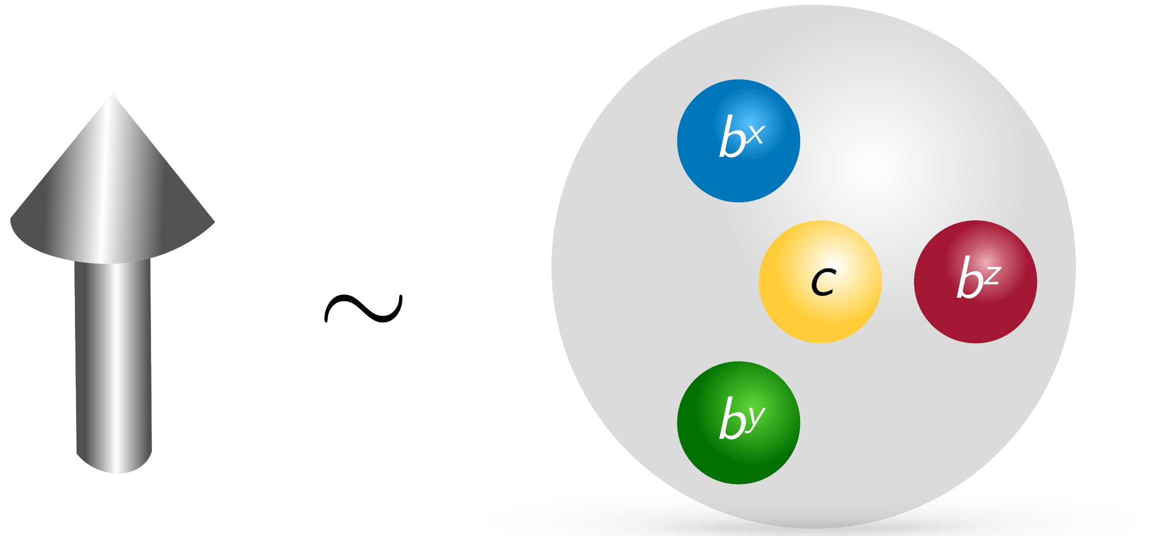
Majorana representation:

$$\sigma^x \sim i b^x c$$
$$\sigma^y \sim i b^y c$$
$$\sigma^z \sim i b^z c$$



[Kitaev, Ann. Phys. '06]

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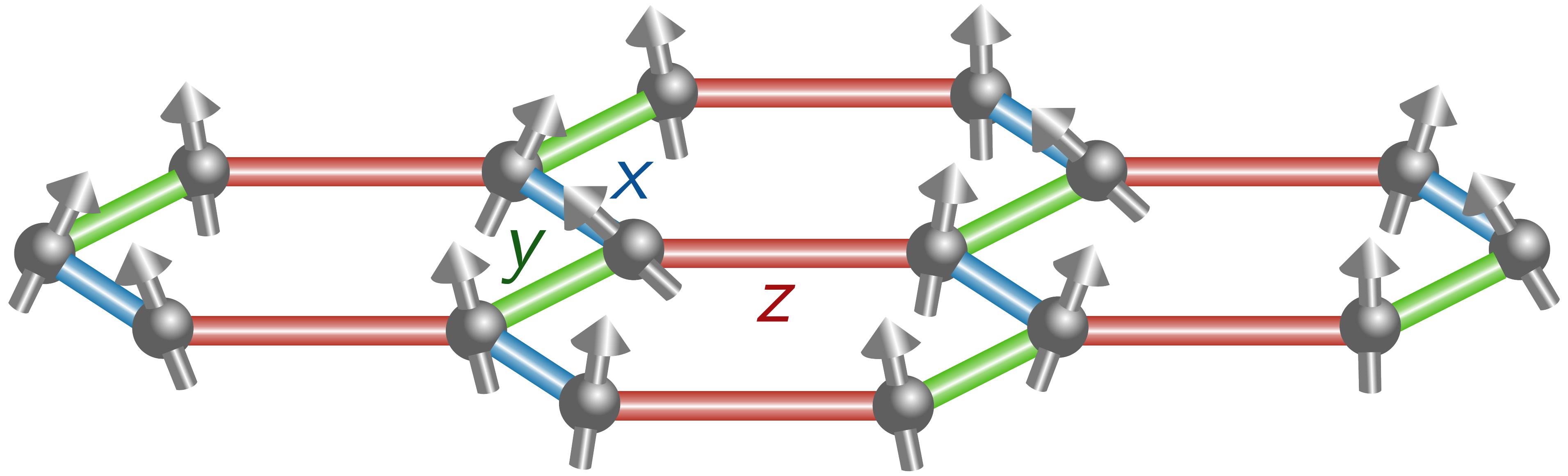

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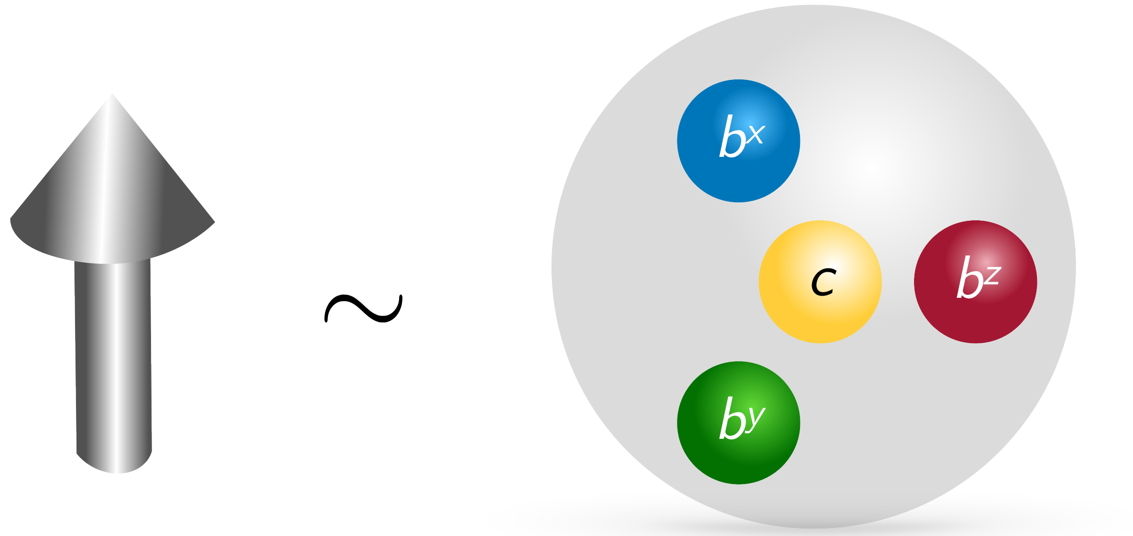


Fractionalization:

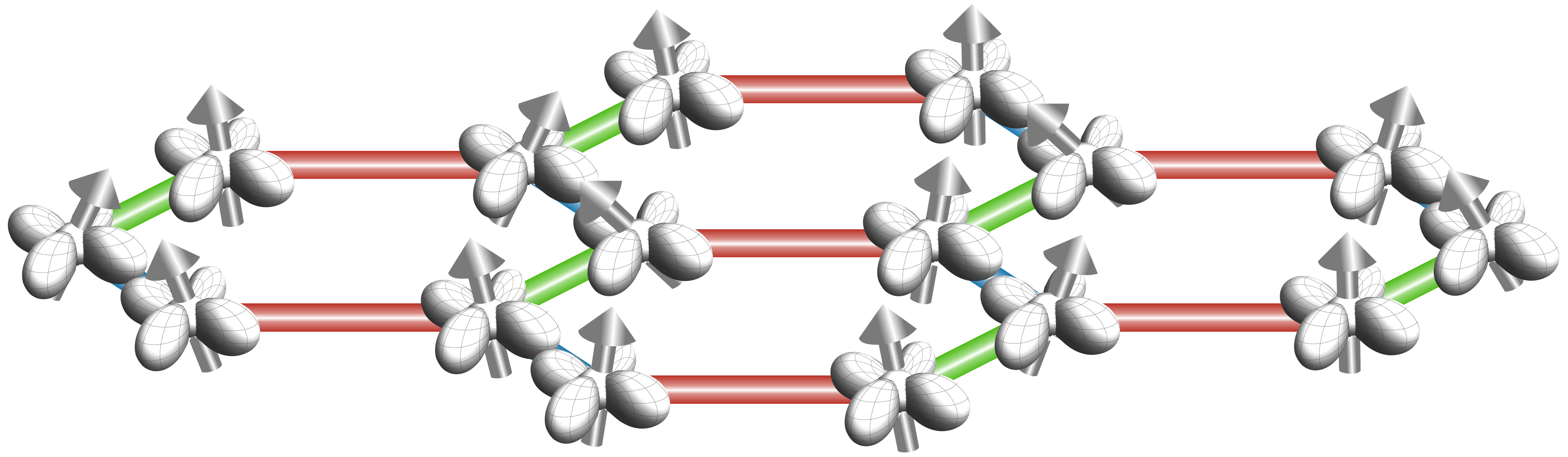
$$\mathcal{H} \sim iK \sum_{\langle ij\rangle_\alpha} \underbrace{(i b_i^\alpha b_j^\alpha)}_{\equiv \hat{u}_{ij} = \hat{u}_{ij}^\dagger} c_i c_j$$

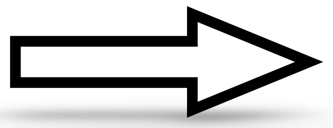
with $\left[\hat{u}_{ij}, \tilde{\mathcal{H}}\right] = 0 \quad \Rightarrow \quad$ static $\mathbb{Z}_2$ gauge field!

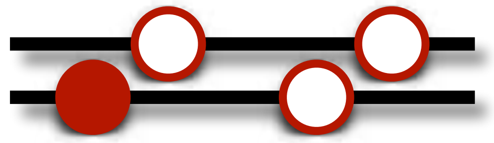
[Kitaev, Ann. Phys. '06]

Beyond Kitaev spin-1/2

Spin-orbital generalization:

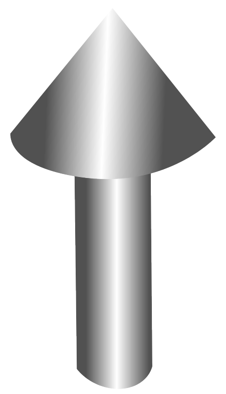




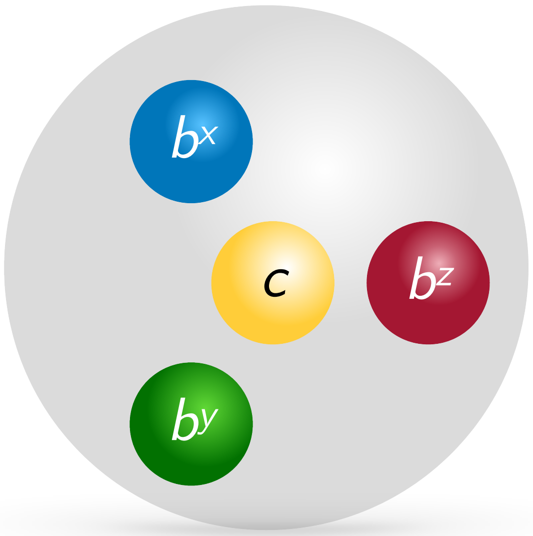


$\sigma^{\alpha} \quad 2 \times 2$

$\gamma^{i} = \sigma^{\alpha} \otimes \tau^{\beta} \quad 4 \times 4$

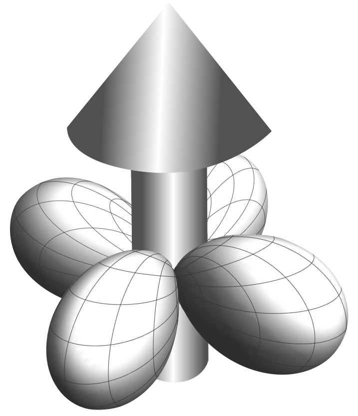


$\sim$



spin

4 Majoranas
with gauge constraint



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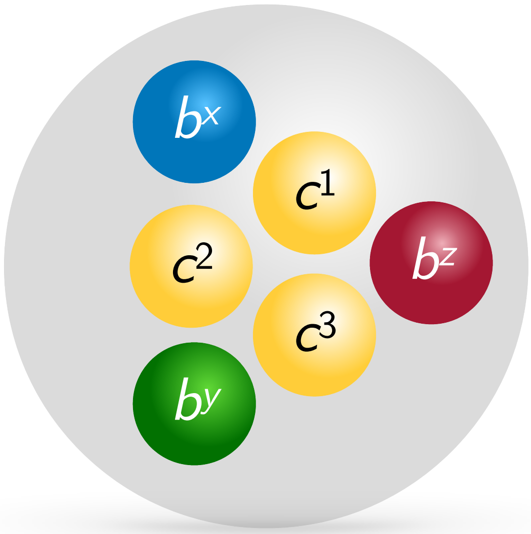

spin + orbital

6 Majoranas
with gauge constraint

[Chulliparambil, *et int.*, LJ, Tu, PRB '20]

SO(3) Kitaev-Heisenberg spin-orbital model

Hamiltonian:

$$\mathcal{H} = K \sum_{\langle ij \rangle_\alpha} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \tau_i^\alpha \tau_j^\alpha}_{\mapsto \ \hat{u}_{ij} c_i^\top c_j} + J \sum_{\langle ij \rangle} \underbrace{\vec{\sigma}_i \cdot \vec{\sigma}_j \otimes \mathbb{1}_i \mathbb{1}_j}_{\mapsto \ \frac{1}{4}(c_i^\top \vec{L} c_i) \cdot (c_j^\top \vec{L} c_j)}$$

spin-1 matrices

with $[\hat{u}_{ij}, \mathcal{H}] = 0$ still static!

[Seifert, Dong, *et int.*, LJ, PRL '20]

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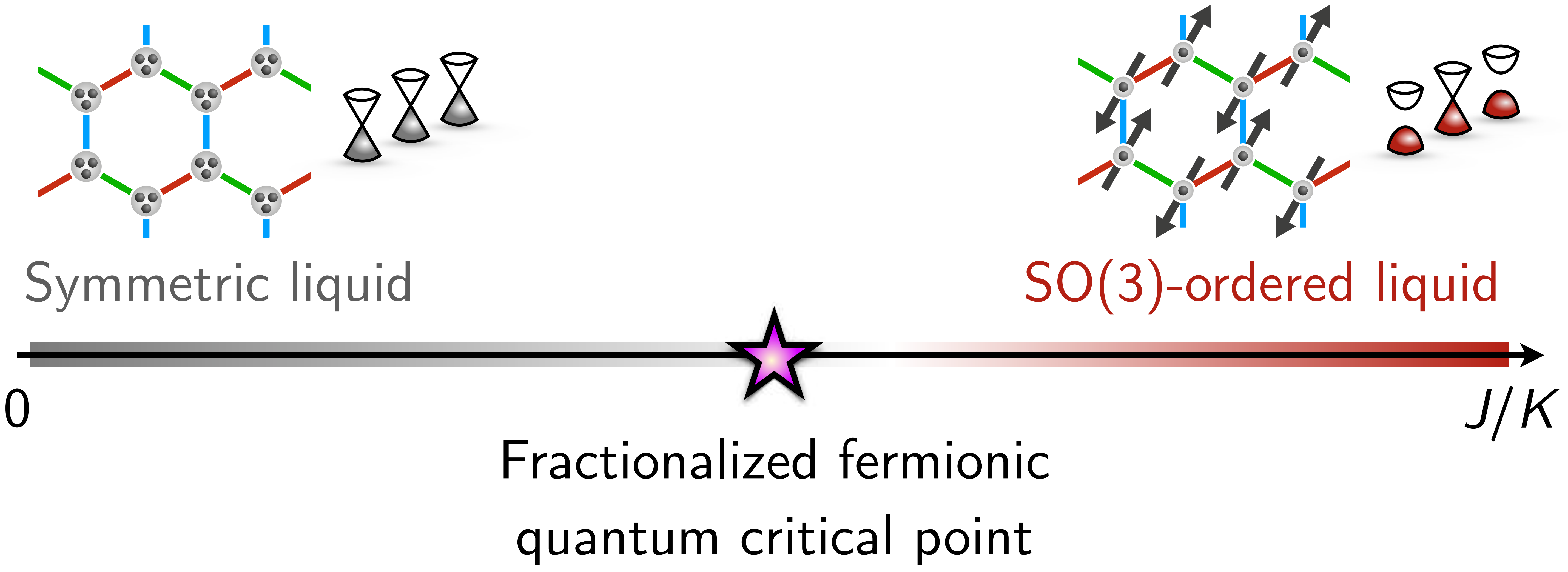
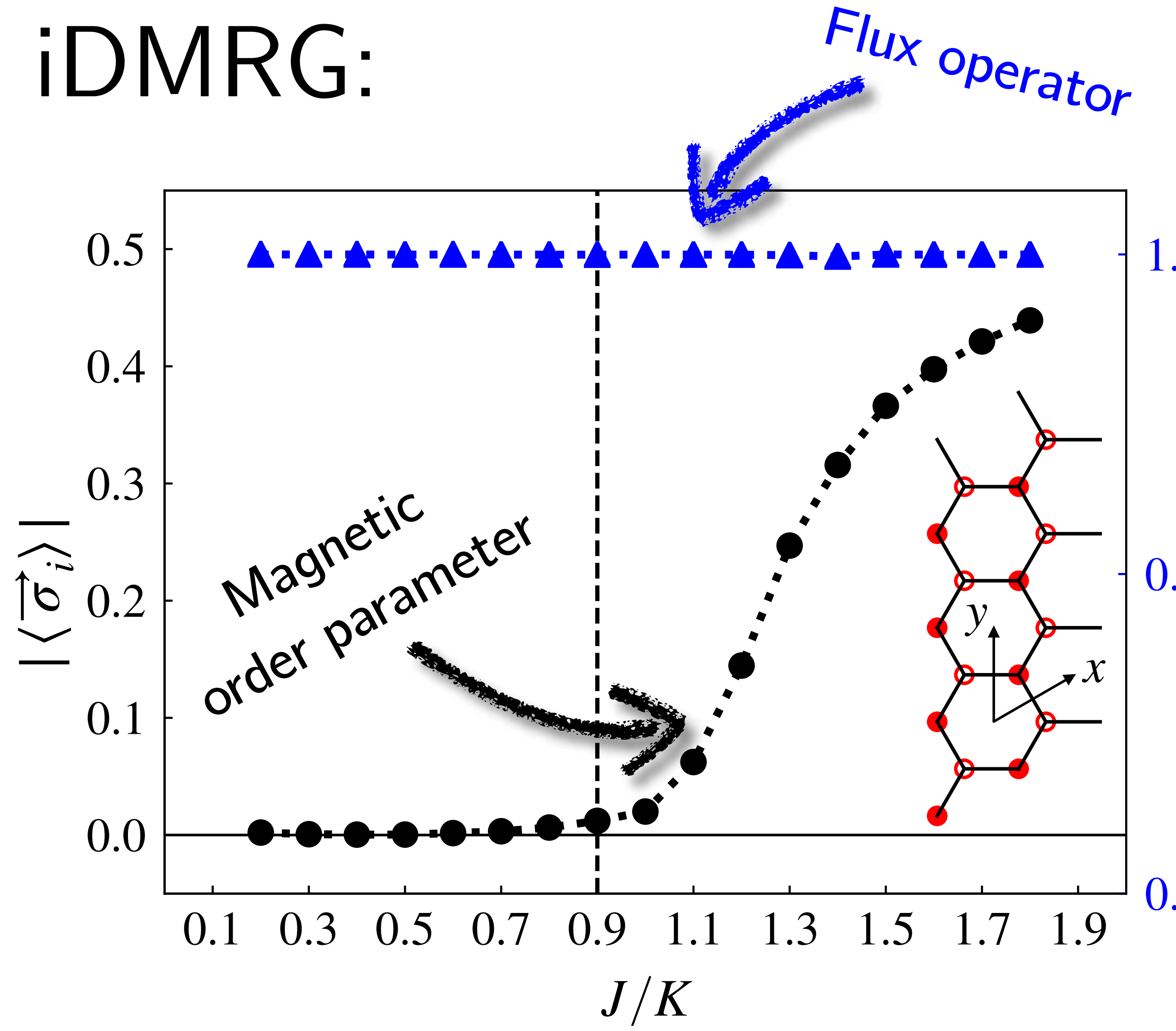
Phase diagram:



iDMRG:



[Seifert, Dong, *et int.*, LJ, PRL '20]

SO(N) Majorana-Hubbard models

Generators of SO(N)

Hamiltonian:

$$\mathcal{H} = \sum_{\langle ij \rangle} i t_{ij} c_i^\top c_j + J \sum_{\langle ij \rangle} \underbrace{\sum_{a<b} \left(\tfrac{1}{2} c_i^\top L^{ab} c_i\right) \left(\tfrac{1}{2} c_j^\top L^{ab} c_j\right)}_{=\frac{1}{2}(c_i^\top c_j)(c_i^\top c_j)}$$

with $c_i \equiv (c_i^1, \ldots, c_i^N)^\top$

"Majorana analog of SU(N) Hubbard-Heisenberg model"

SO(N) Majorana-Hubbard models

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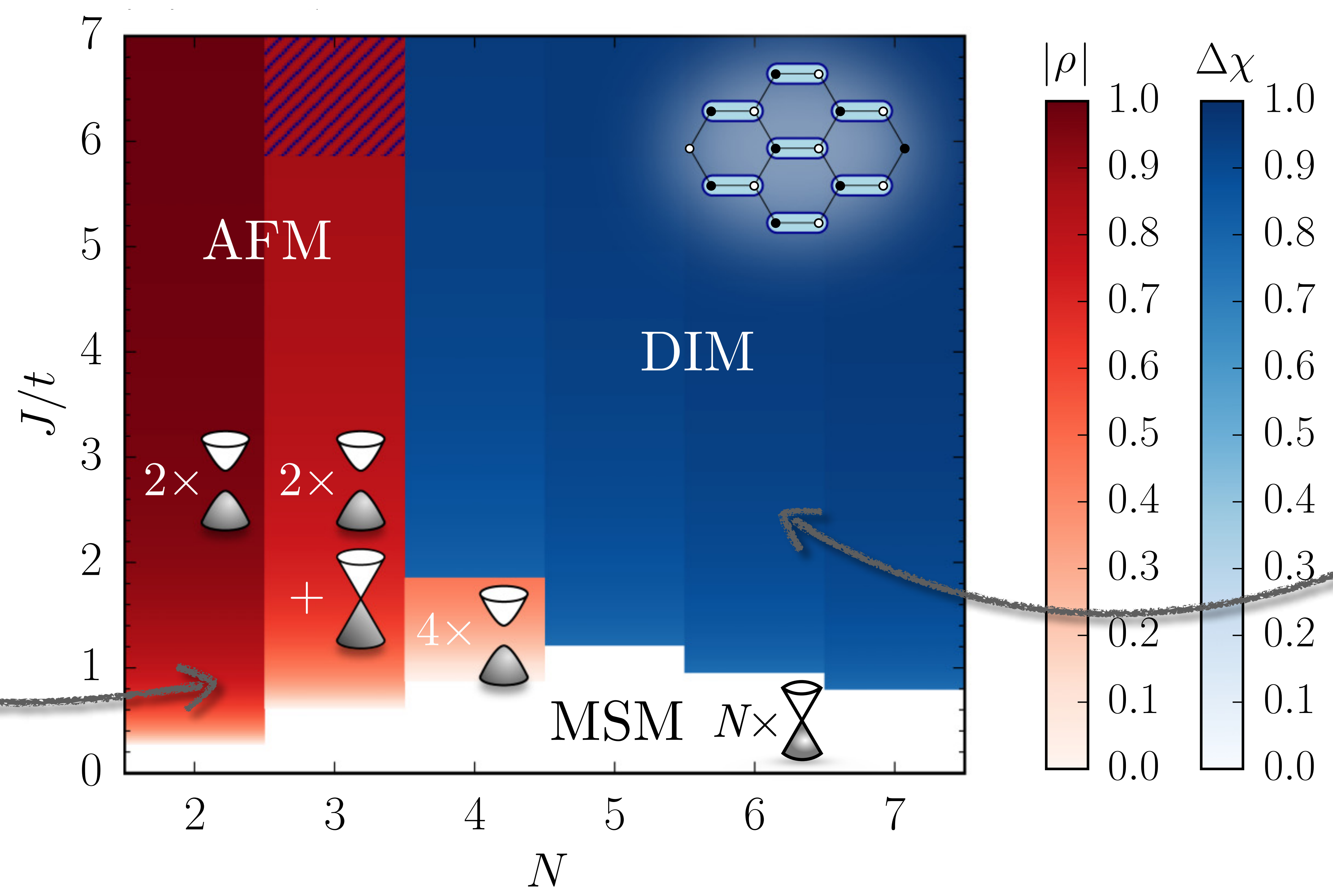
"Majorana analog of SU(N) Hubbard-Heisenberg model"

Phase diagram:

$\phi_i^{ab} = \langle \tfrac{1}{2} c_i^\top L^{ab} c_i \rangle$

SO(N) symmetry breaking



$\chi_{ij} = \langle i c_i^\top c_j \rangle / N$

Rotational symmetry breaking

... on honeycomb lattice

... and similar on π-flux square lattice

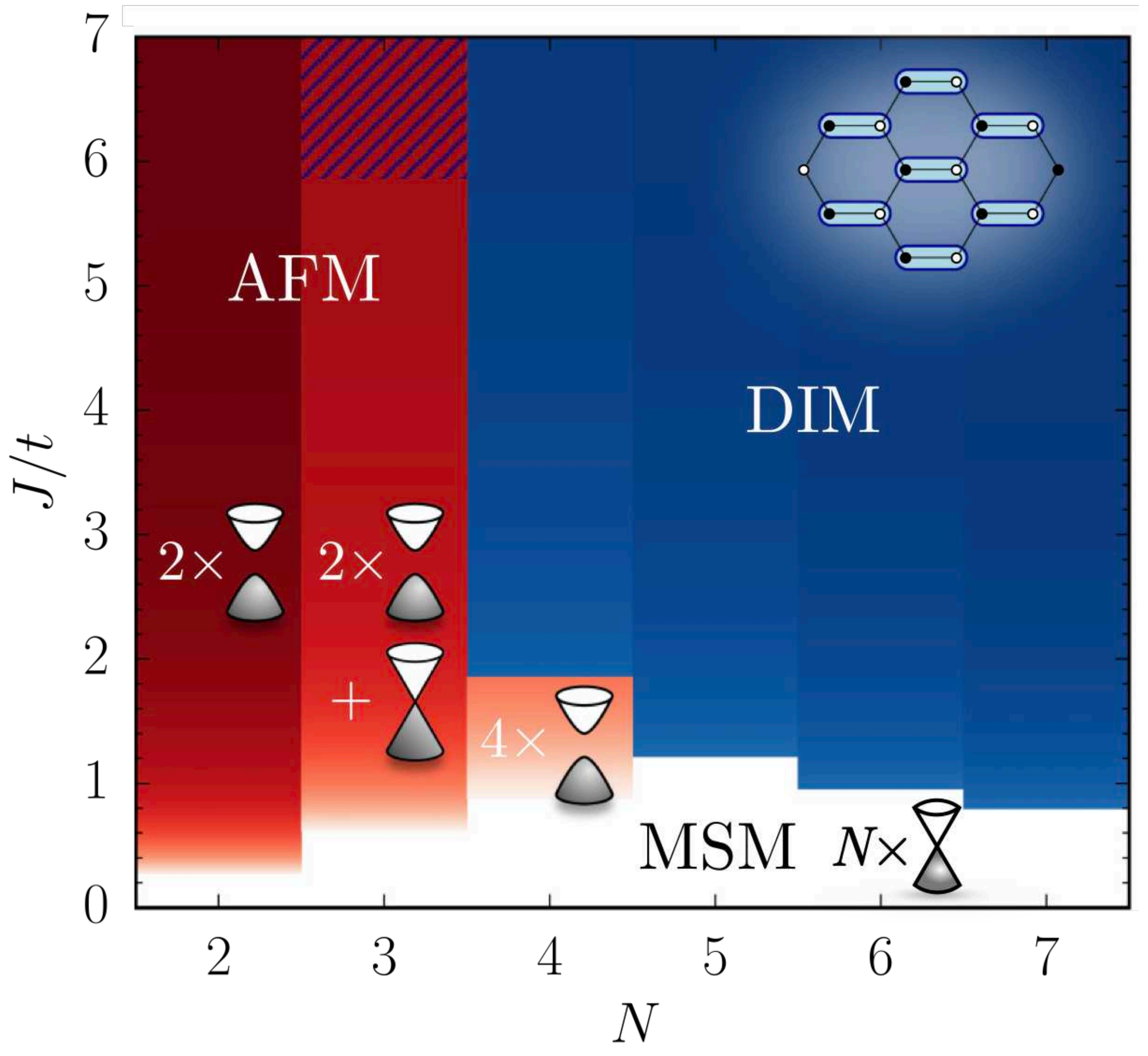
Quantum phase transitions

Effective model (MSM-to-AFM transition):

tensor order parameter

$$\mathcal{L} = \bar{\psi}_\alpha \gamma_\mu \partial_\mu \psi_\alpha + \frac{1}{4}\phi^{ab}\left(r - \partial_\mu^2\right)\phi^{ab} + \frac{g}{2}\phi^{ab}\bar{\psi}_\alpha (L^{ab})_{\alpha\beta}\psi_\beta$$

$$+ \frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da}$$



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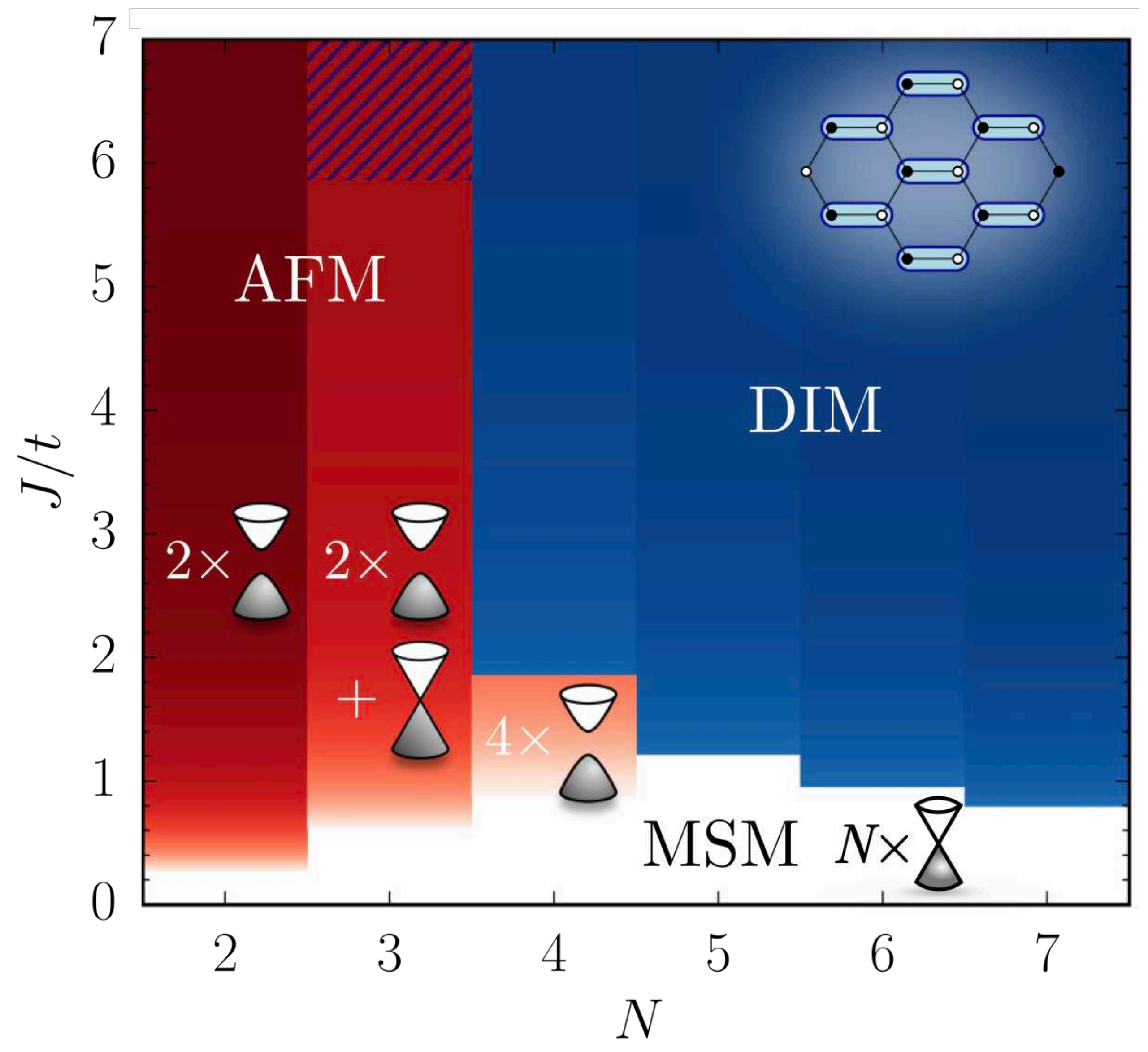

$N \leq 3$:
$$\frac{\lambda_1}{4}\left(\phi^{ab}\phi^{ab}\right)^2 + \lambda_2 \phi^{ab}\phi^{bc}\phi^{cd}\phi^{da} = \frac{\lambda_1 + 2\lambda_2}{2}\left[\mathrm{Tr}\left(\phi^2\right)\right]^2$$

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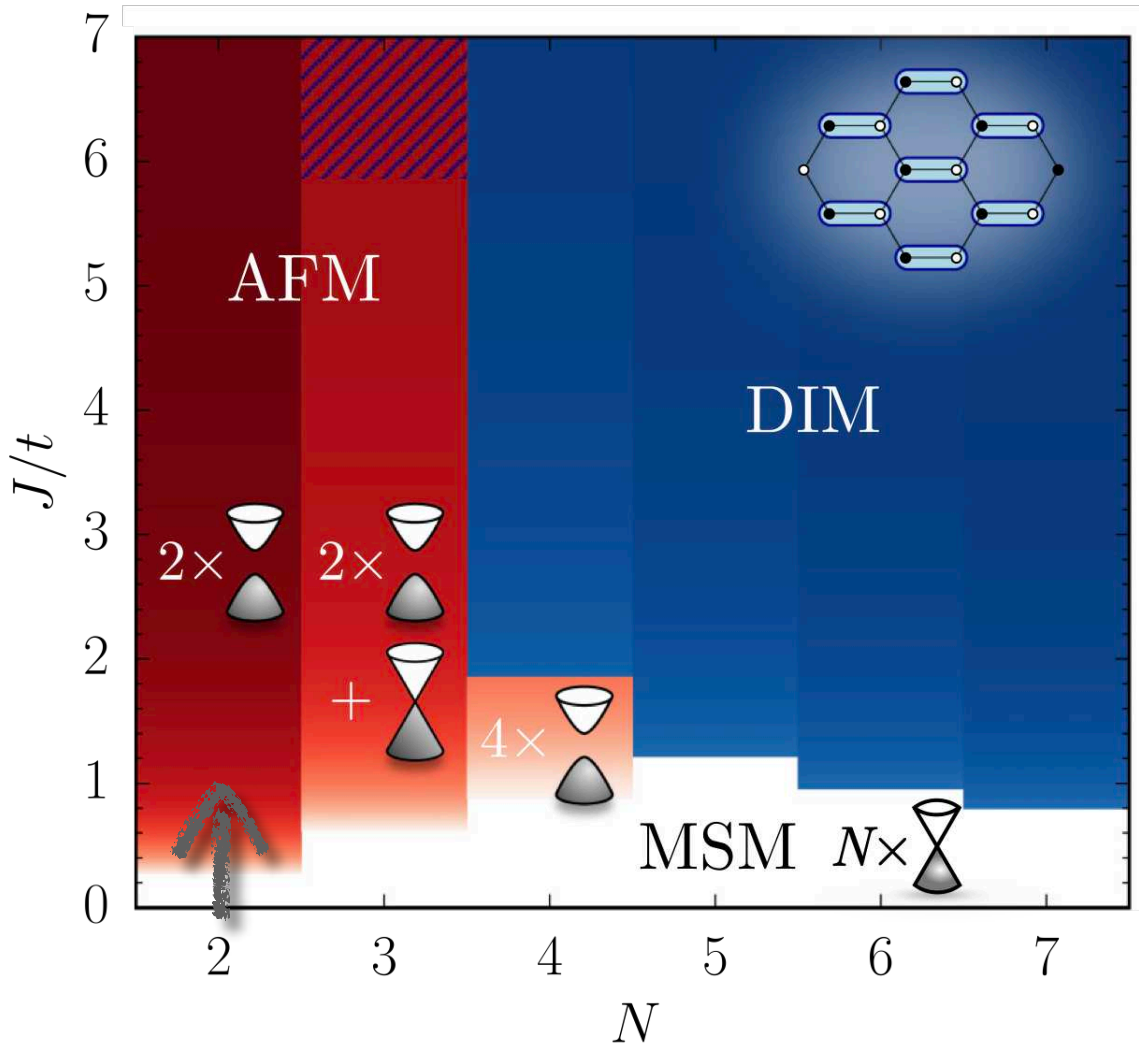

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$N = 2$:

$$(\phi^{ab}) = \begin{pmatrix} 0 & \phi \\ -\phi & 0 \end{pmatrix}$$

"Gross-Neveu-Ising"

[Wang, Corboz, Troyer, NJP '14]
[Li, Jiang, Yao, NJP '15]
[Huffman & Chandrasekharan, PRD '17; PRD '20]
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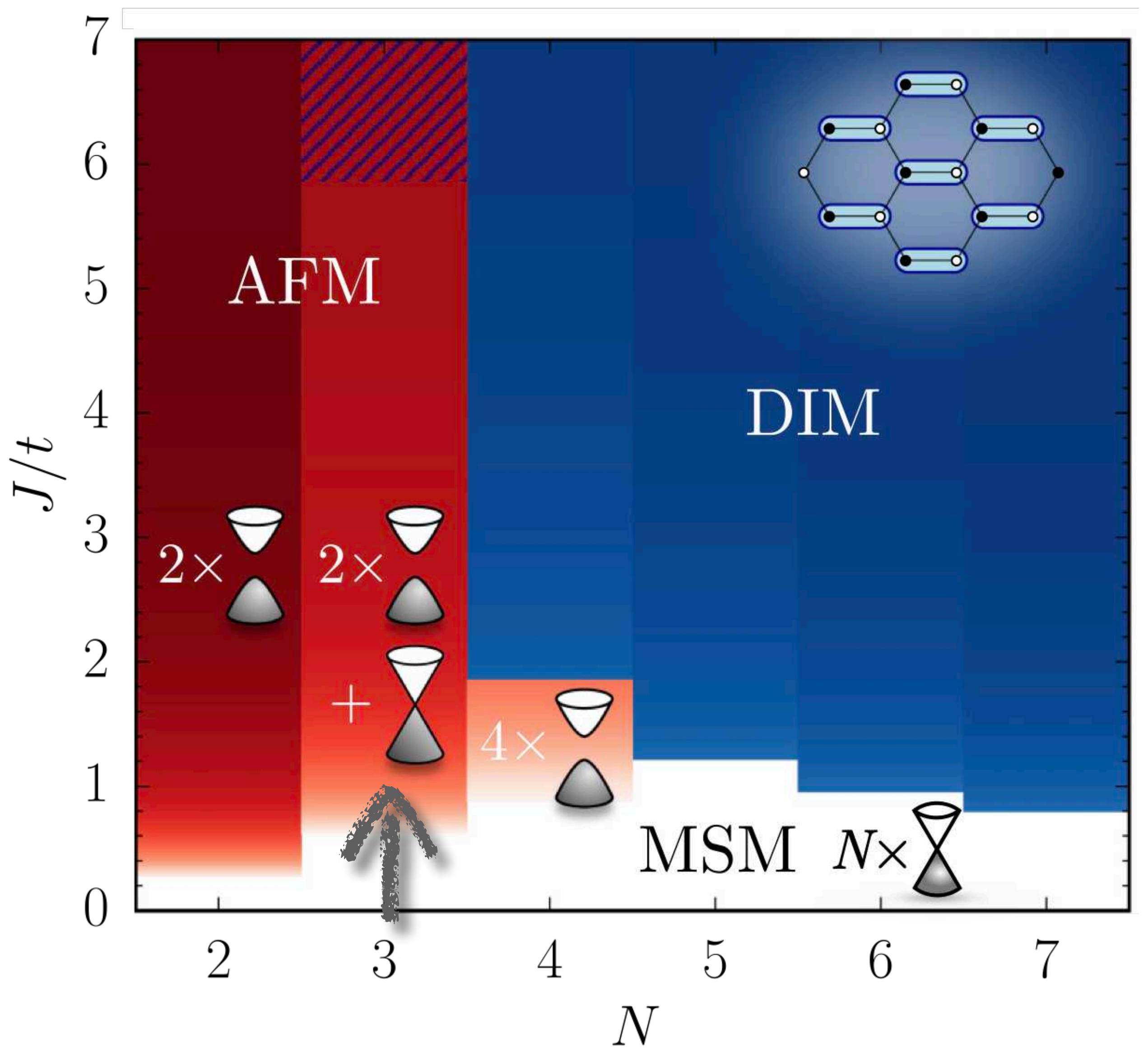

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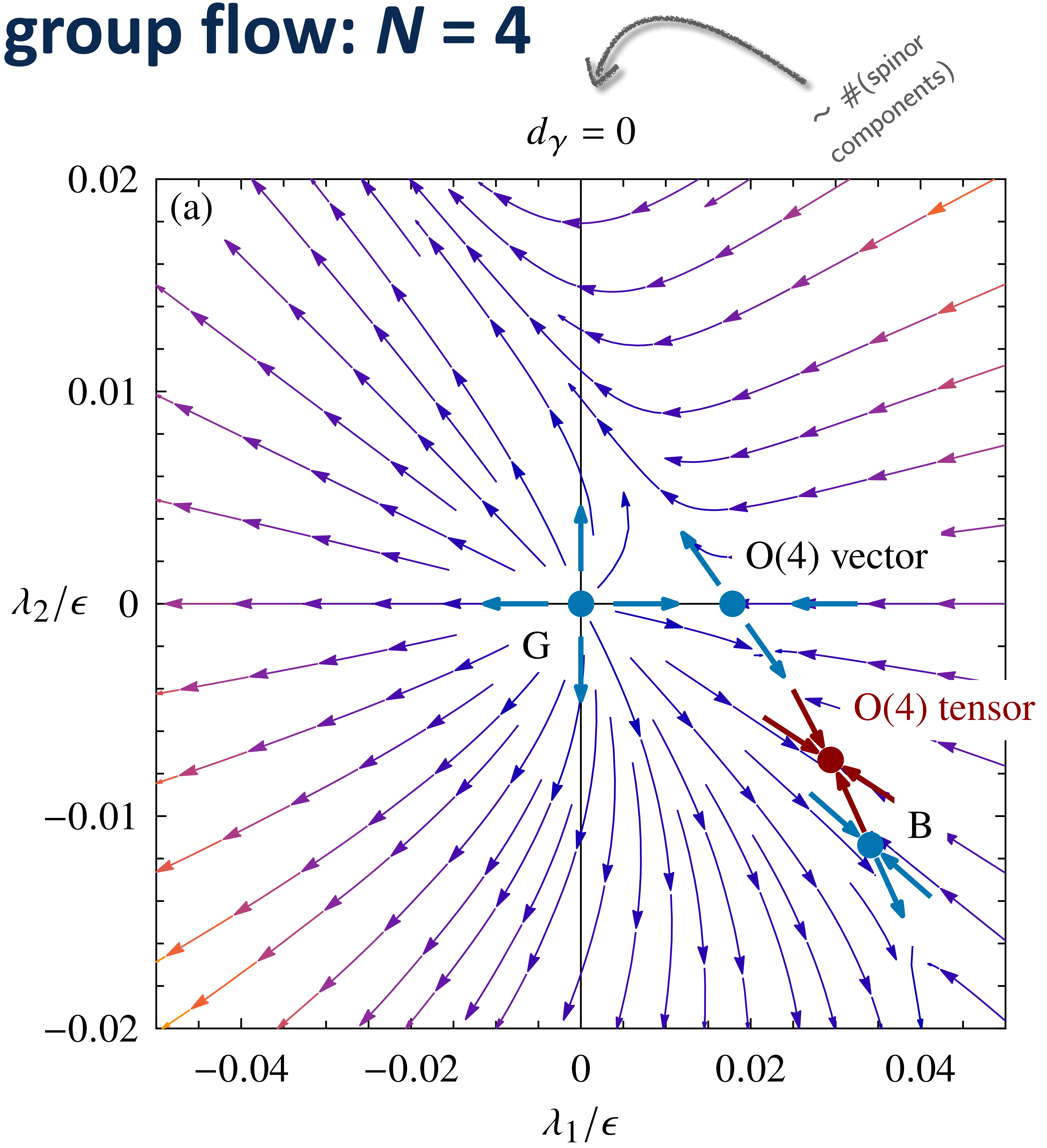
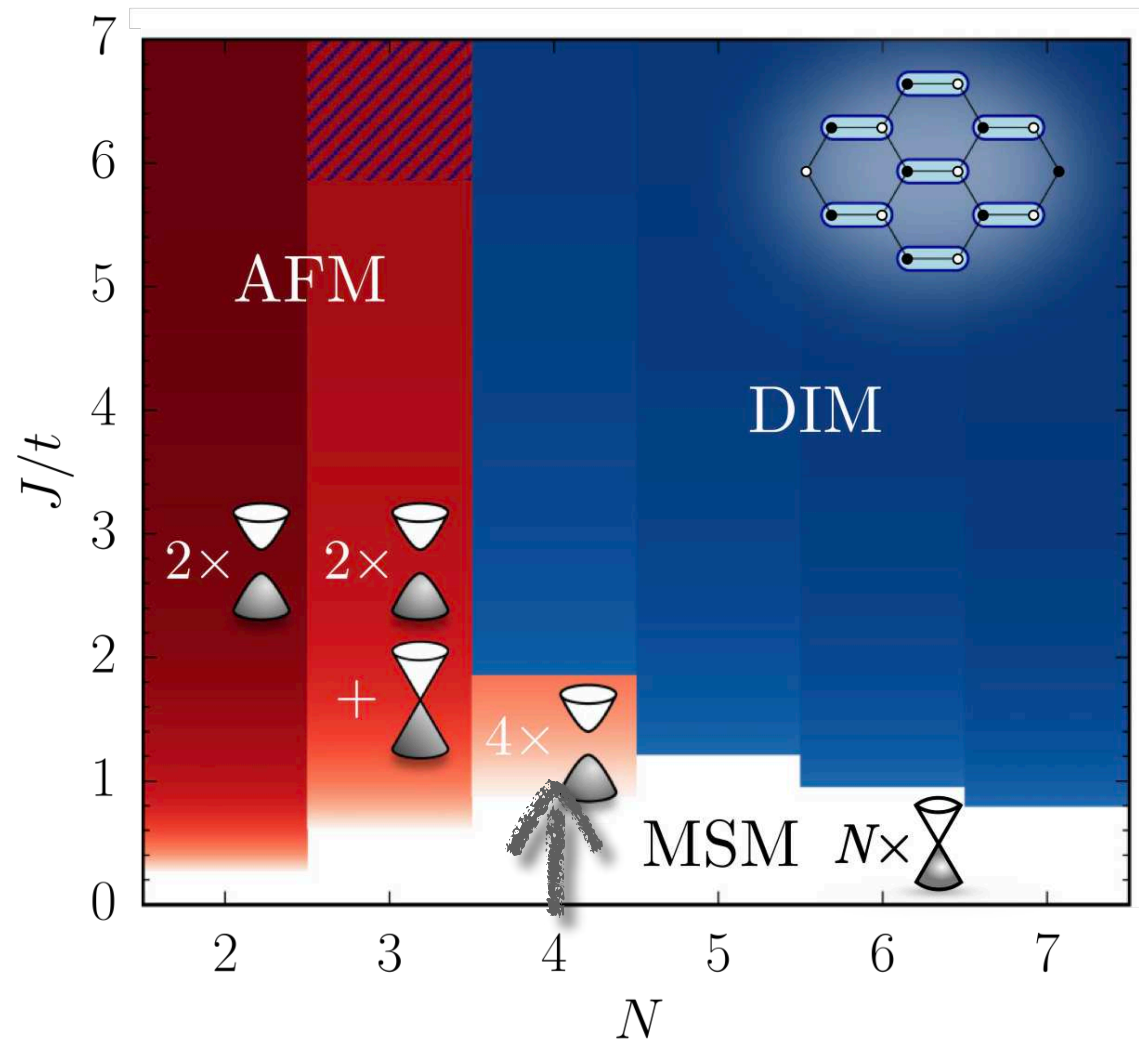
"Gross-Neveu-SO(3)"

[Seifert, Dong, Chulliparambil, Vojta, Tu, LJ, PRL '20]
[Ray, Ihrig, Gracey, Scherer, LJ, PRB '21]
[Liu, Vojta, Assaad, LJ, PRL '22 (Editors' Suggestion)]

Renormalization group flow: $N = 4$

Bosons only:

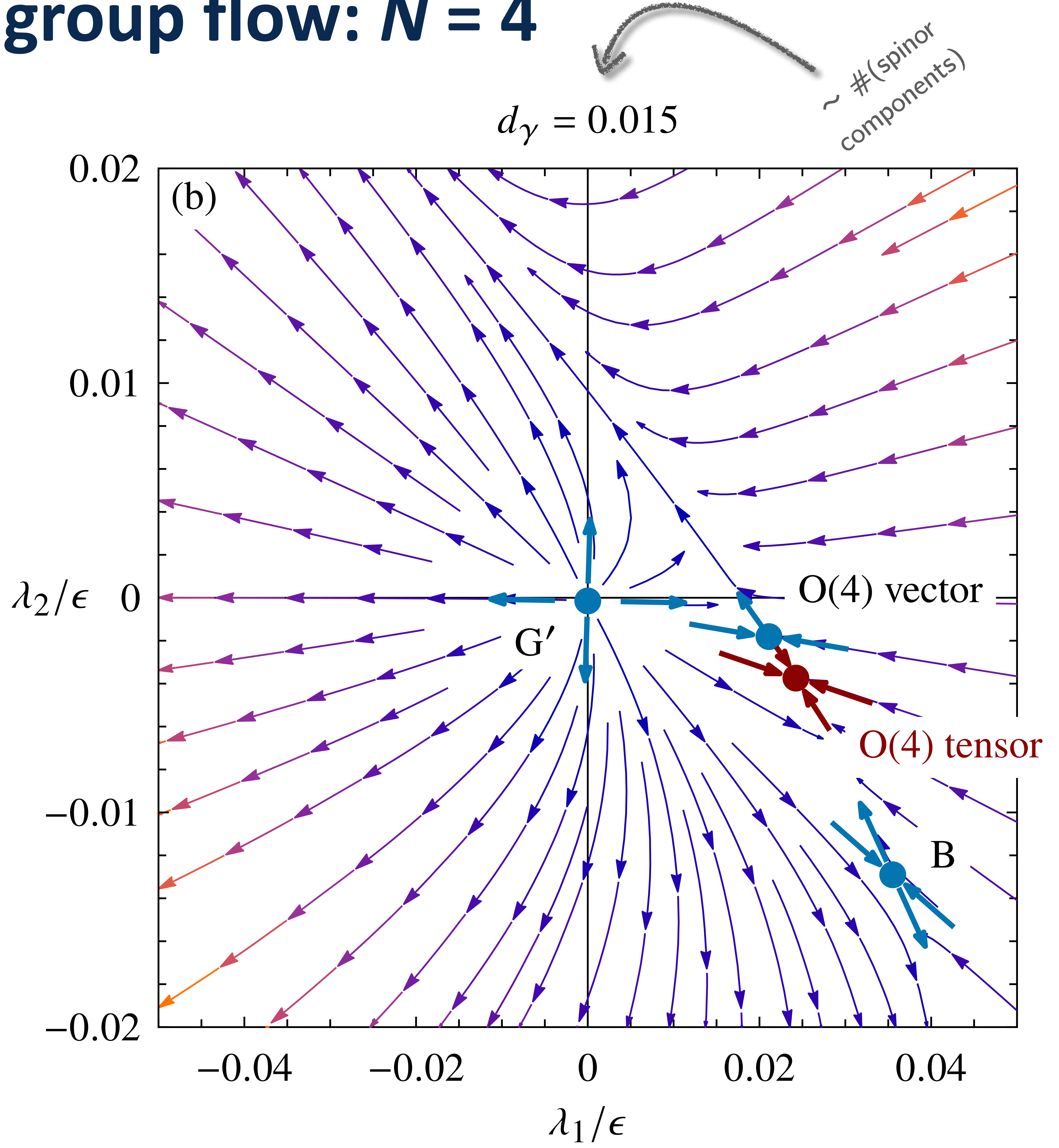


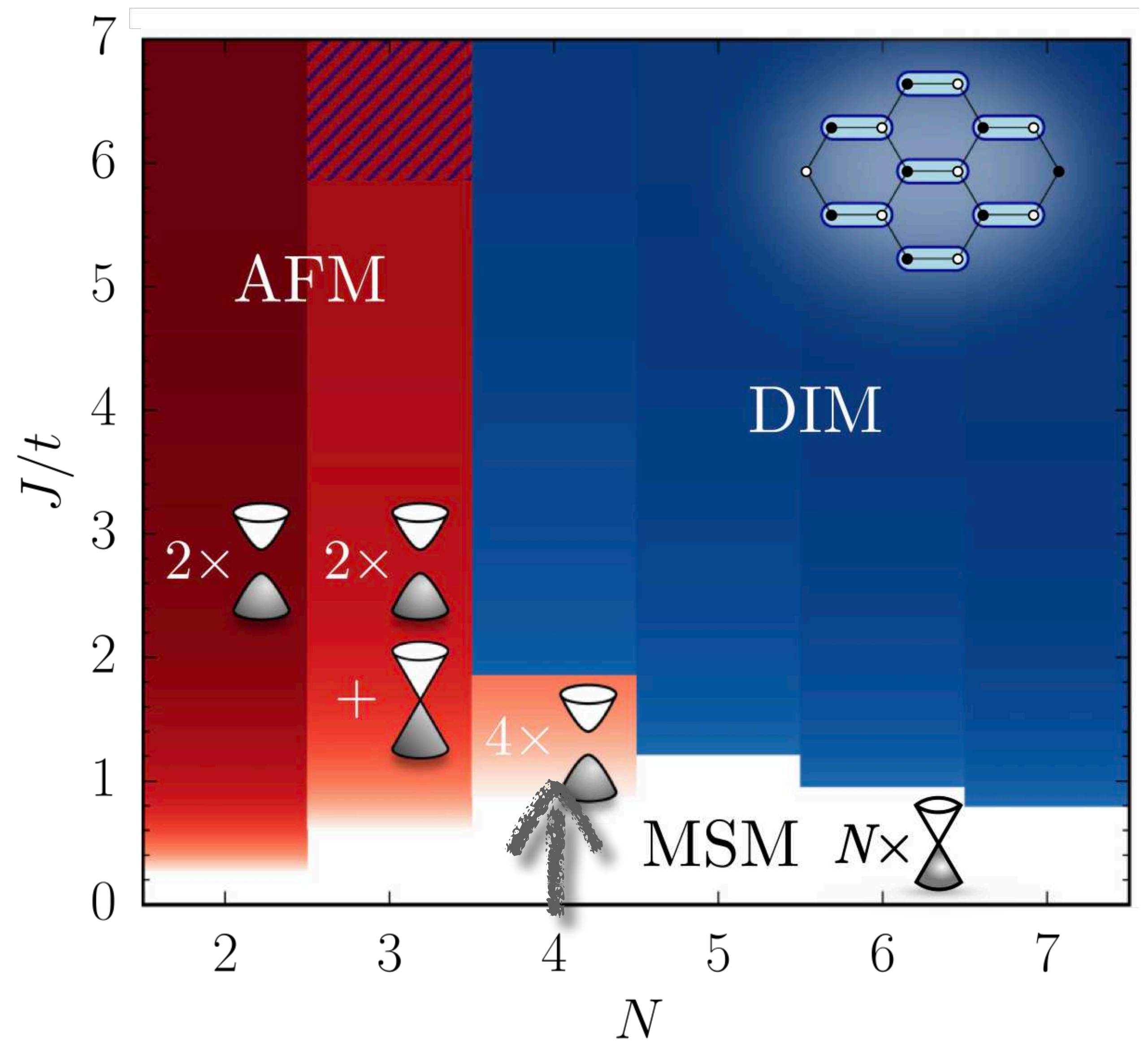


[LJ & Seifert, PRB '22]

Renormalization group flow: N = 4

Bosons
+ "few" fermions:



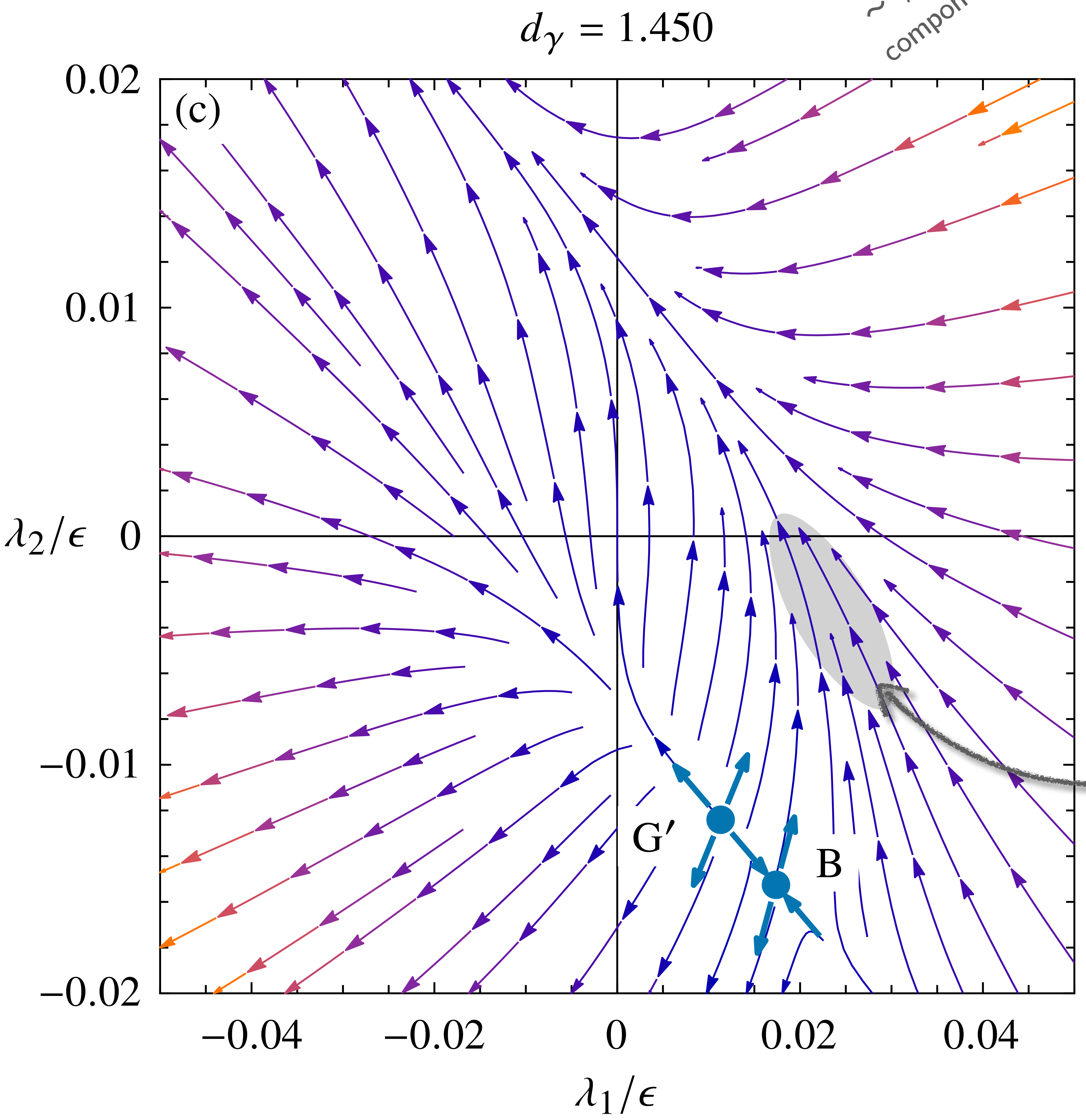


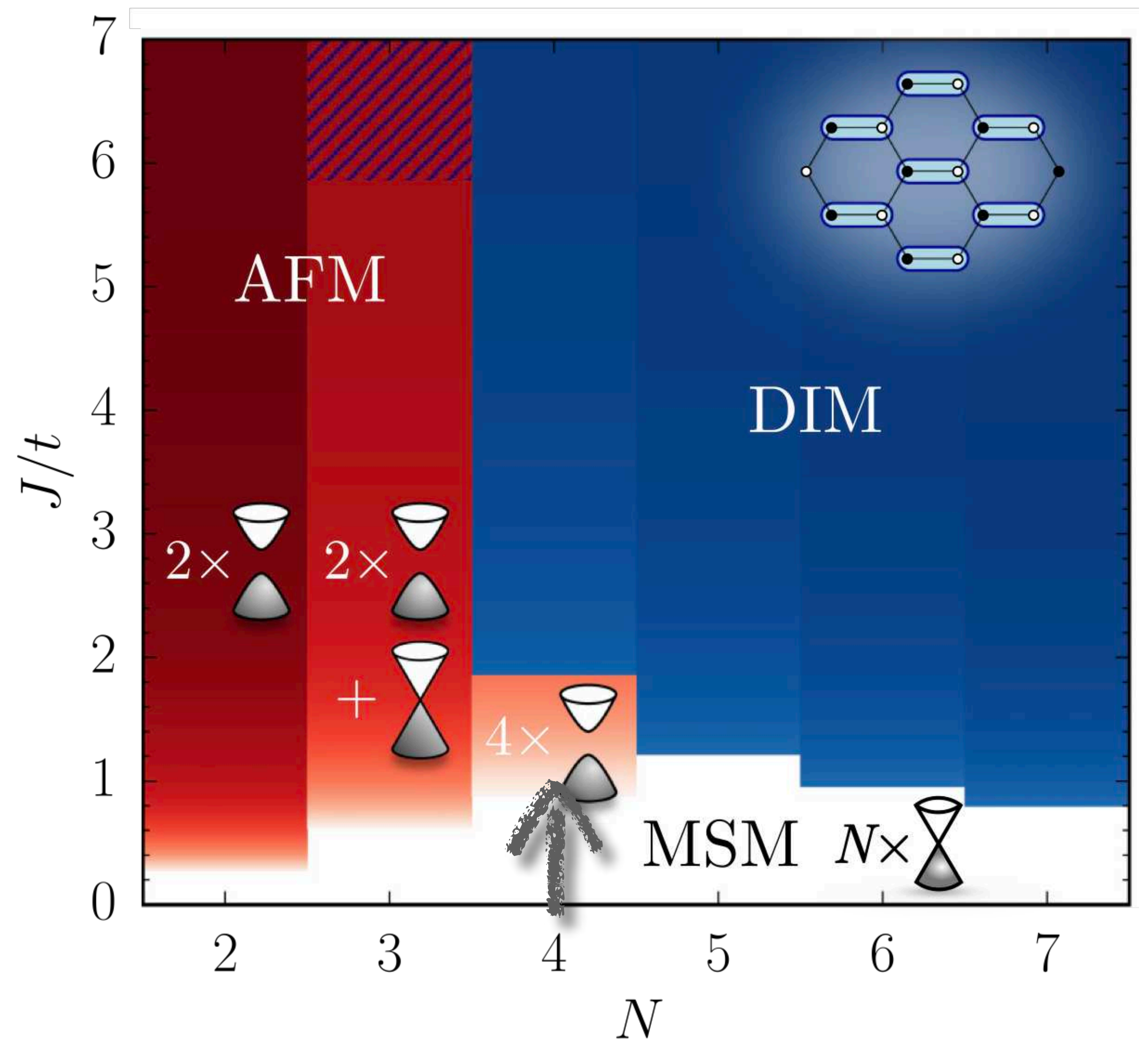
[LJ & Seifert, PRB '22]

Renormalization group flow: *N* = 4

Bosons

+ "few more" fermions:





Fixed-point annihilation!

[LJ & Seifert, PRB '22]

Renormalization group flow: *N* = 4

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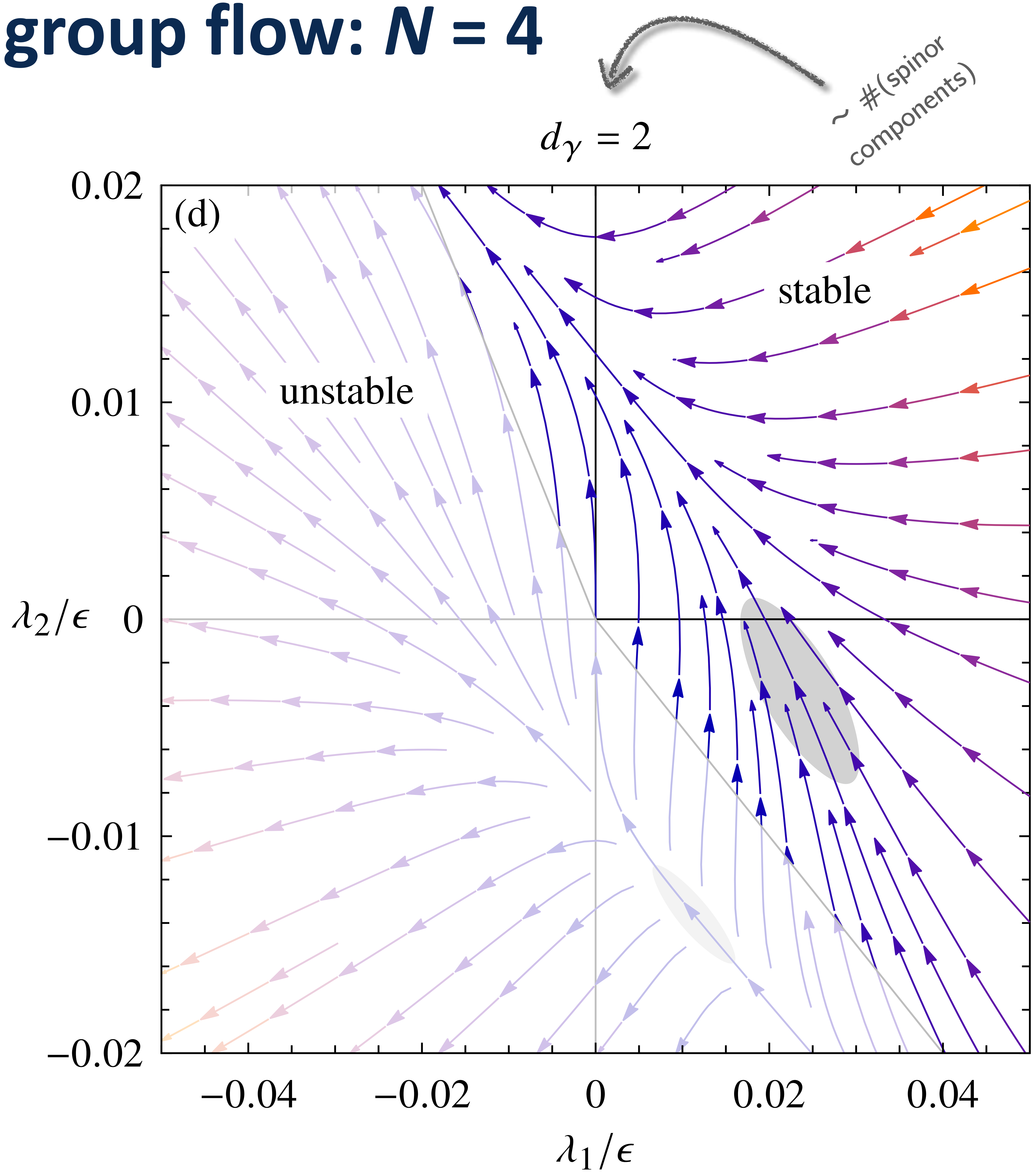
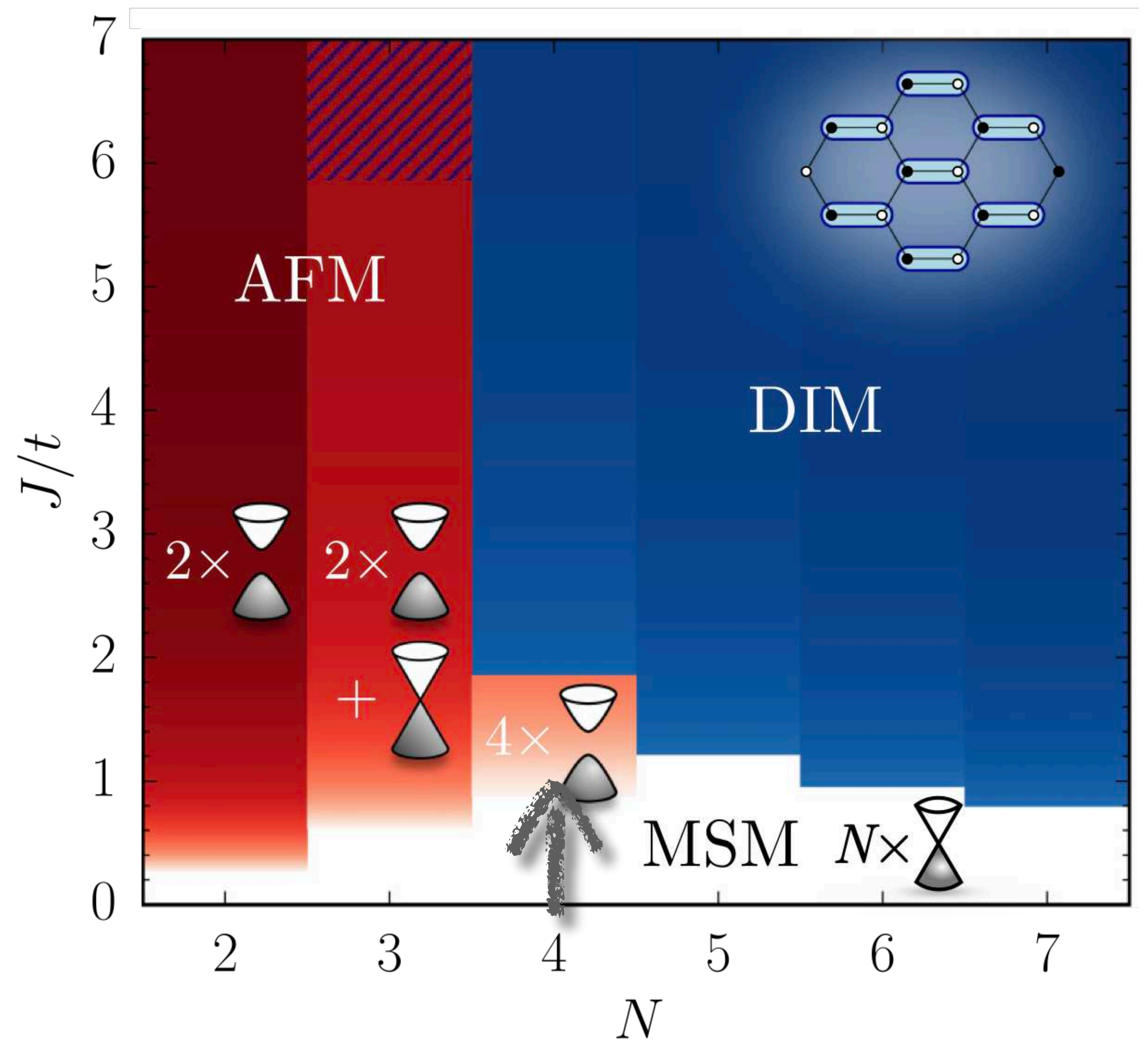
+ “all” fermions:





Only runaway flow!

[LJ & Seifert, PRB ’22]

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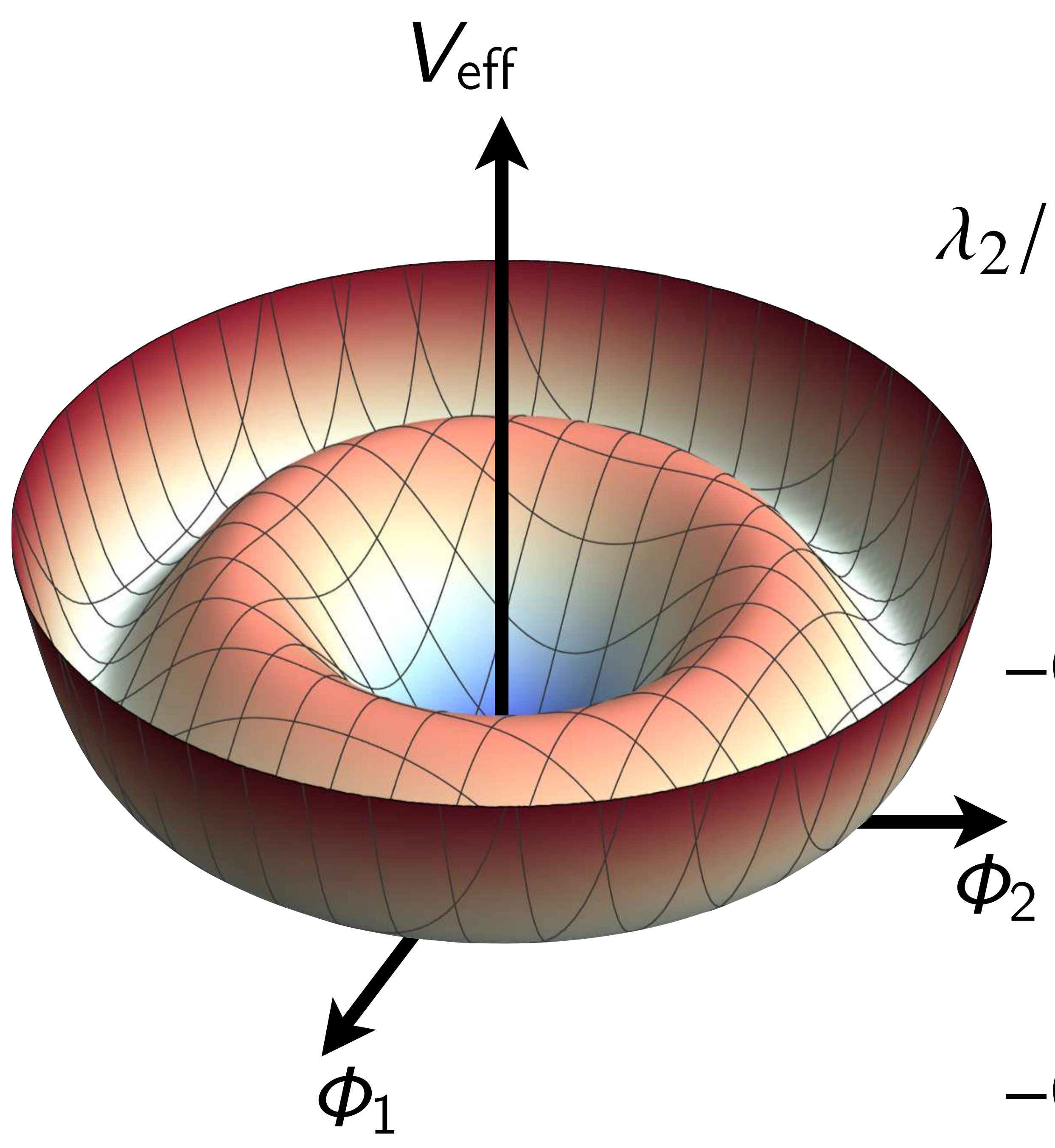
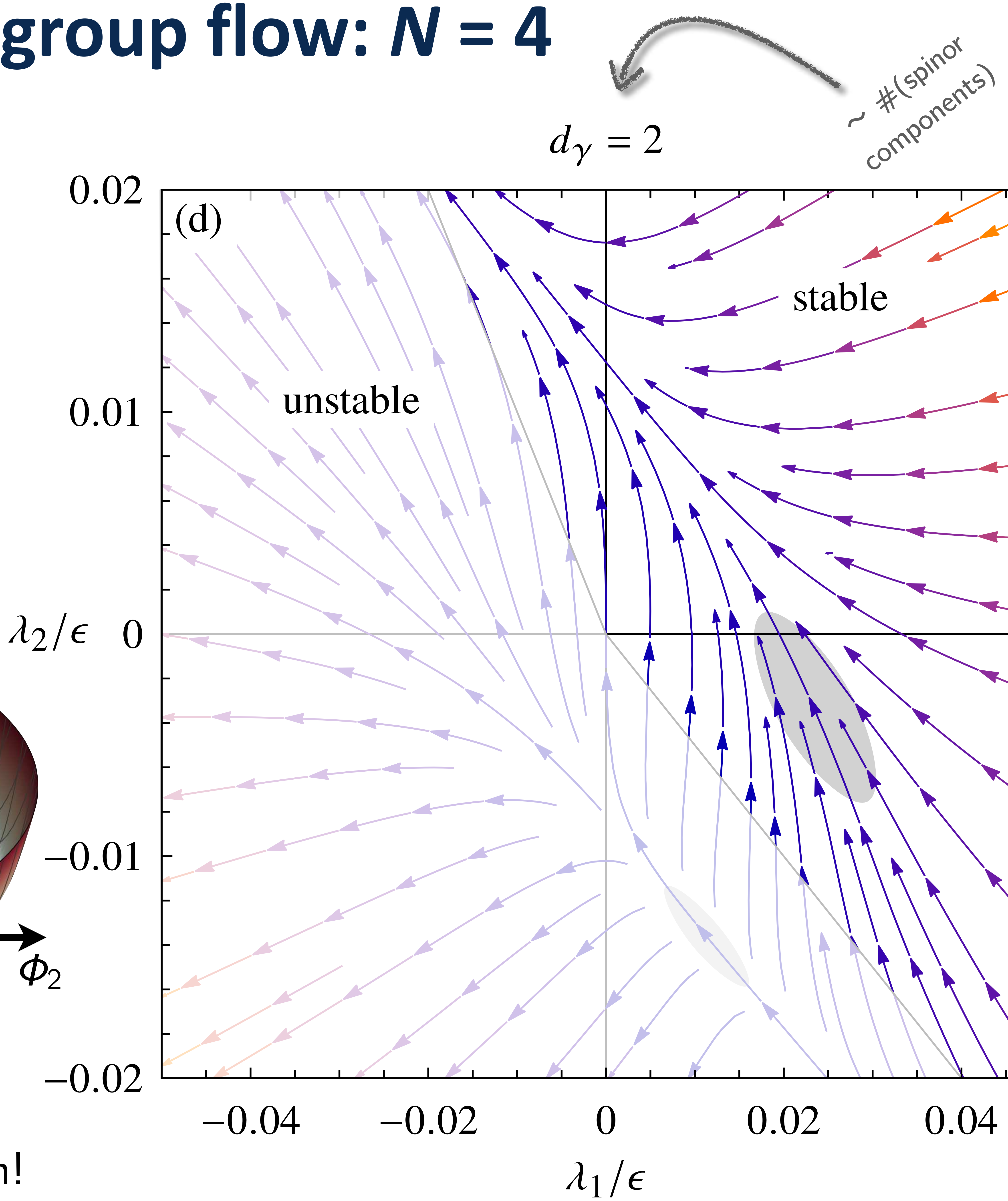
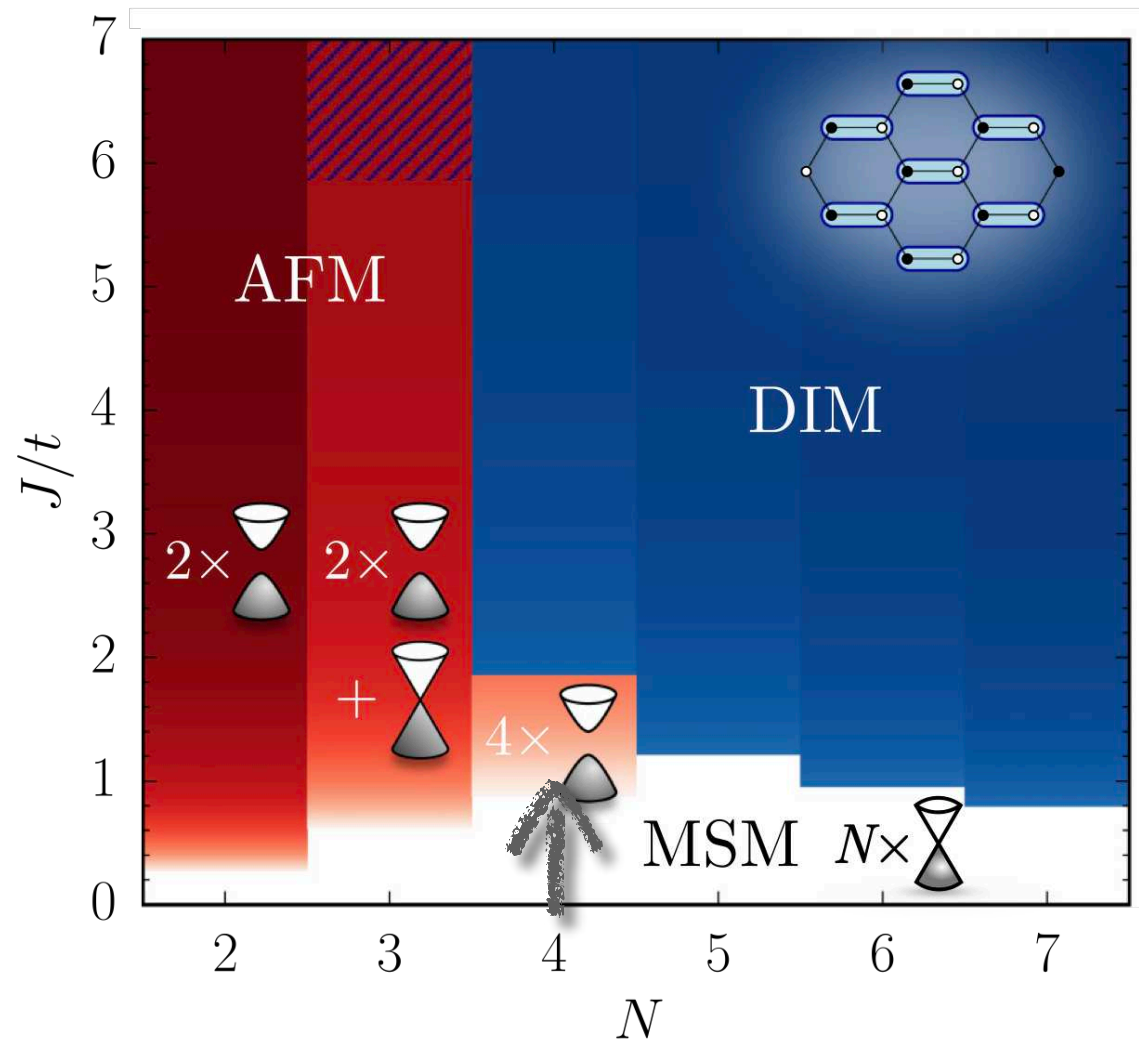


Weak first-order transition!





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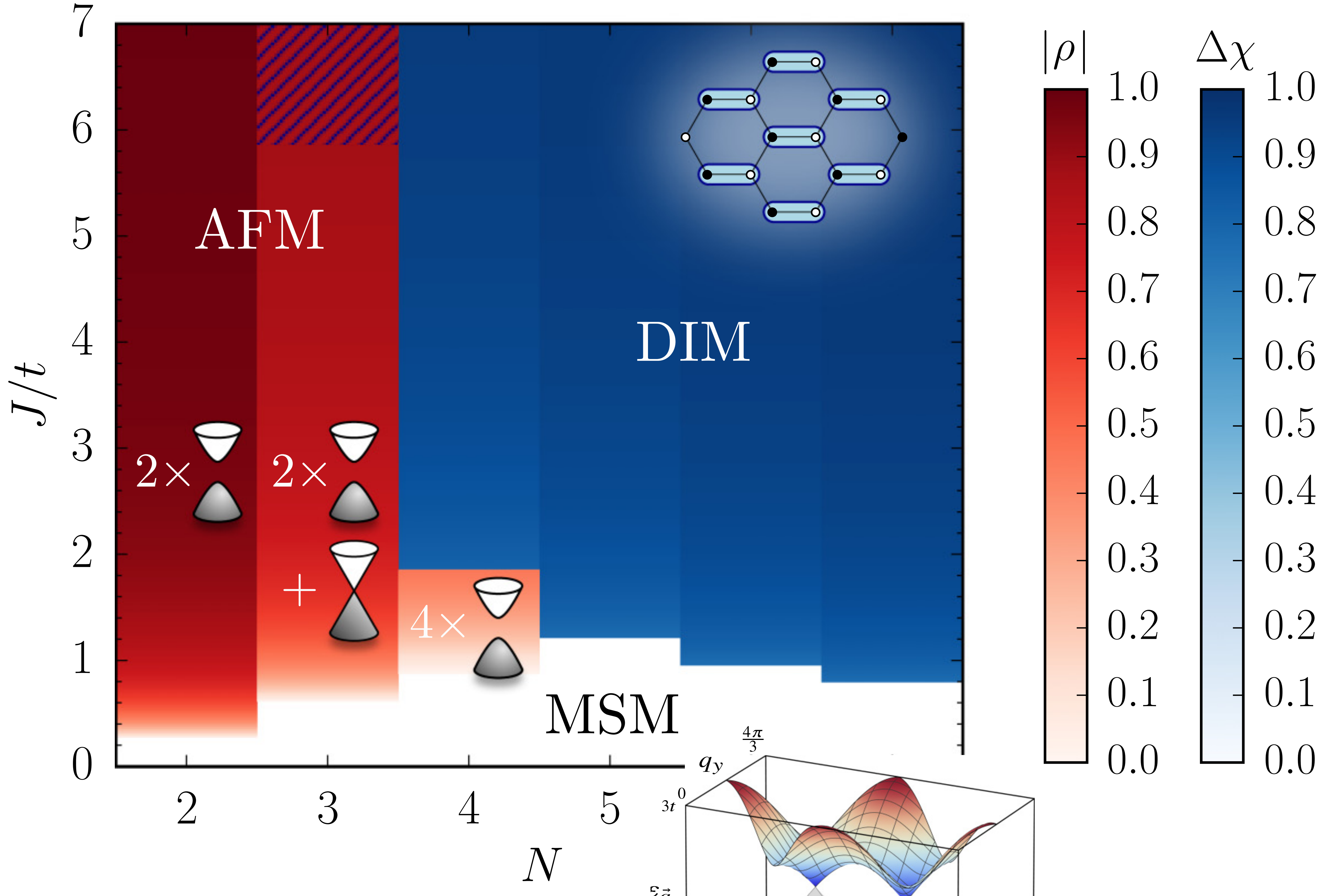
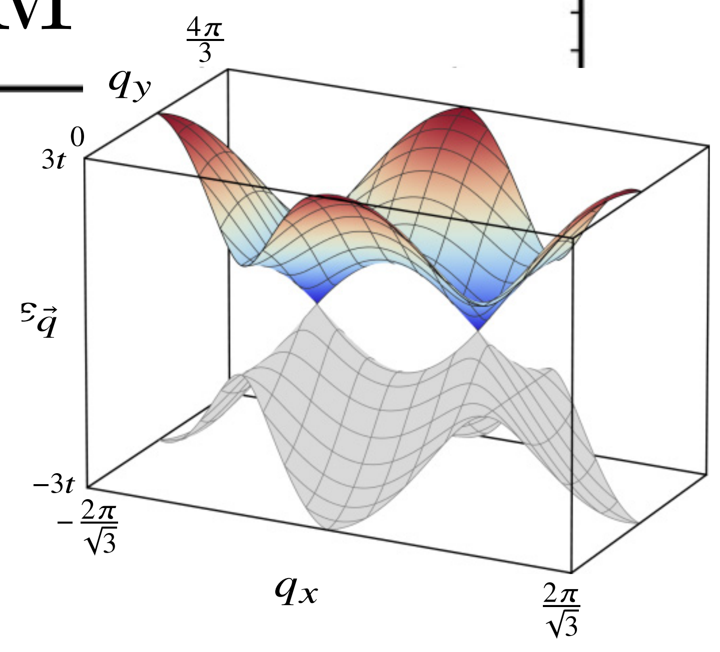
[LJ & Seifert, PRB '22]

Conclusions

SO(N) Majorana-Hubbard models

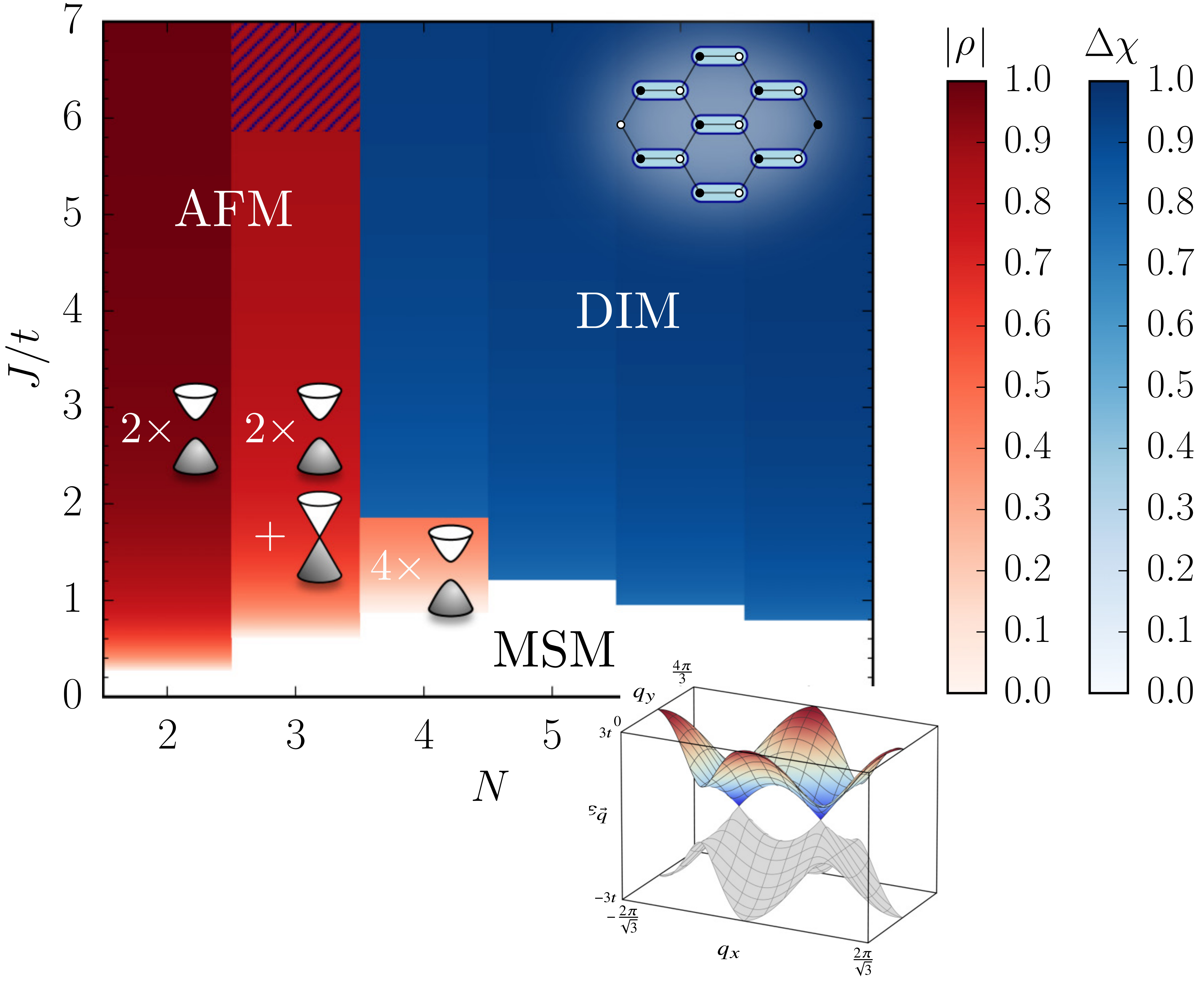




[LJ & Seifert, PRB '22]

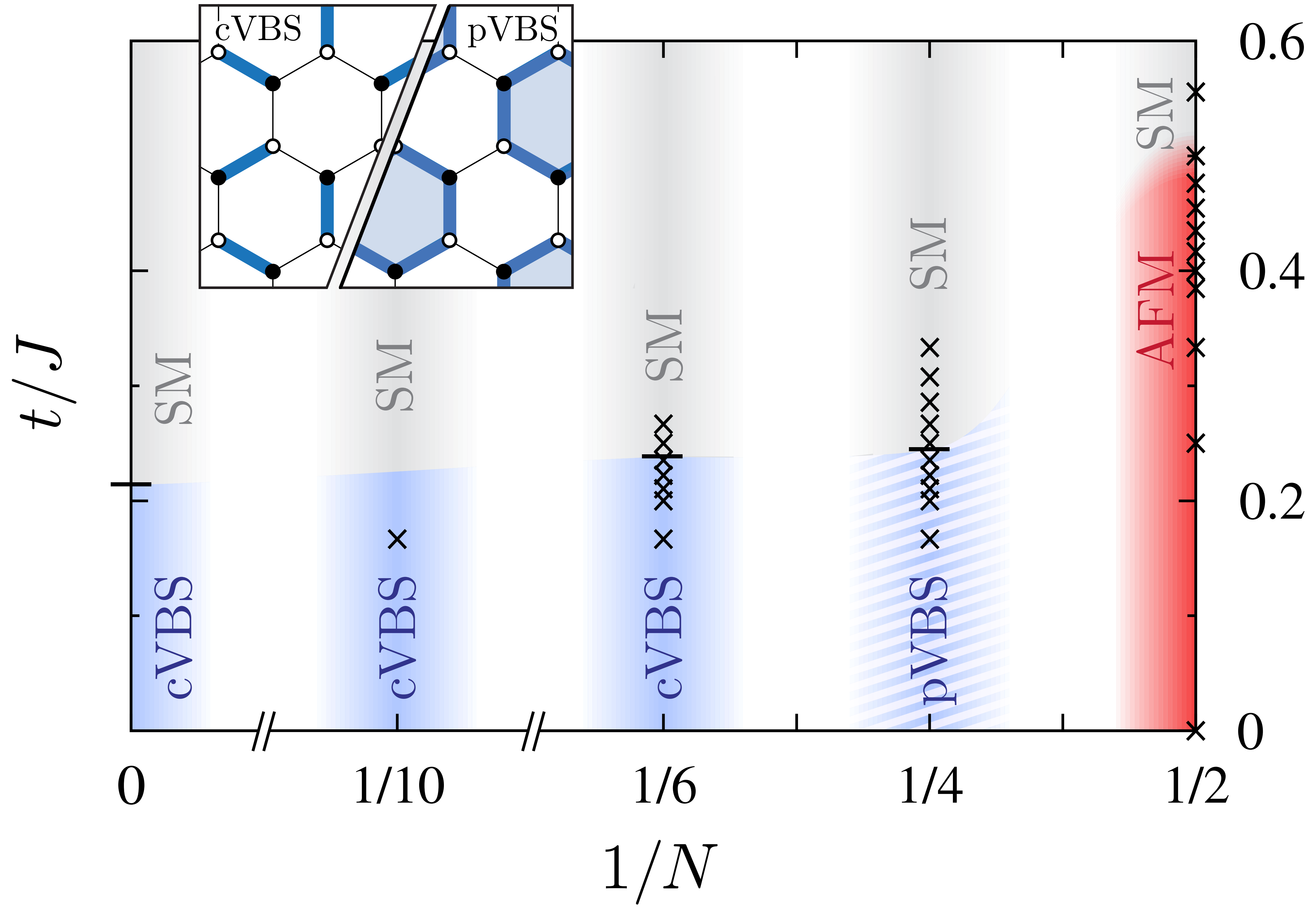
Conclusions

SO(*N*) Majorana-Hubbard models



[LJ & Seifert, PRB '22]

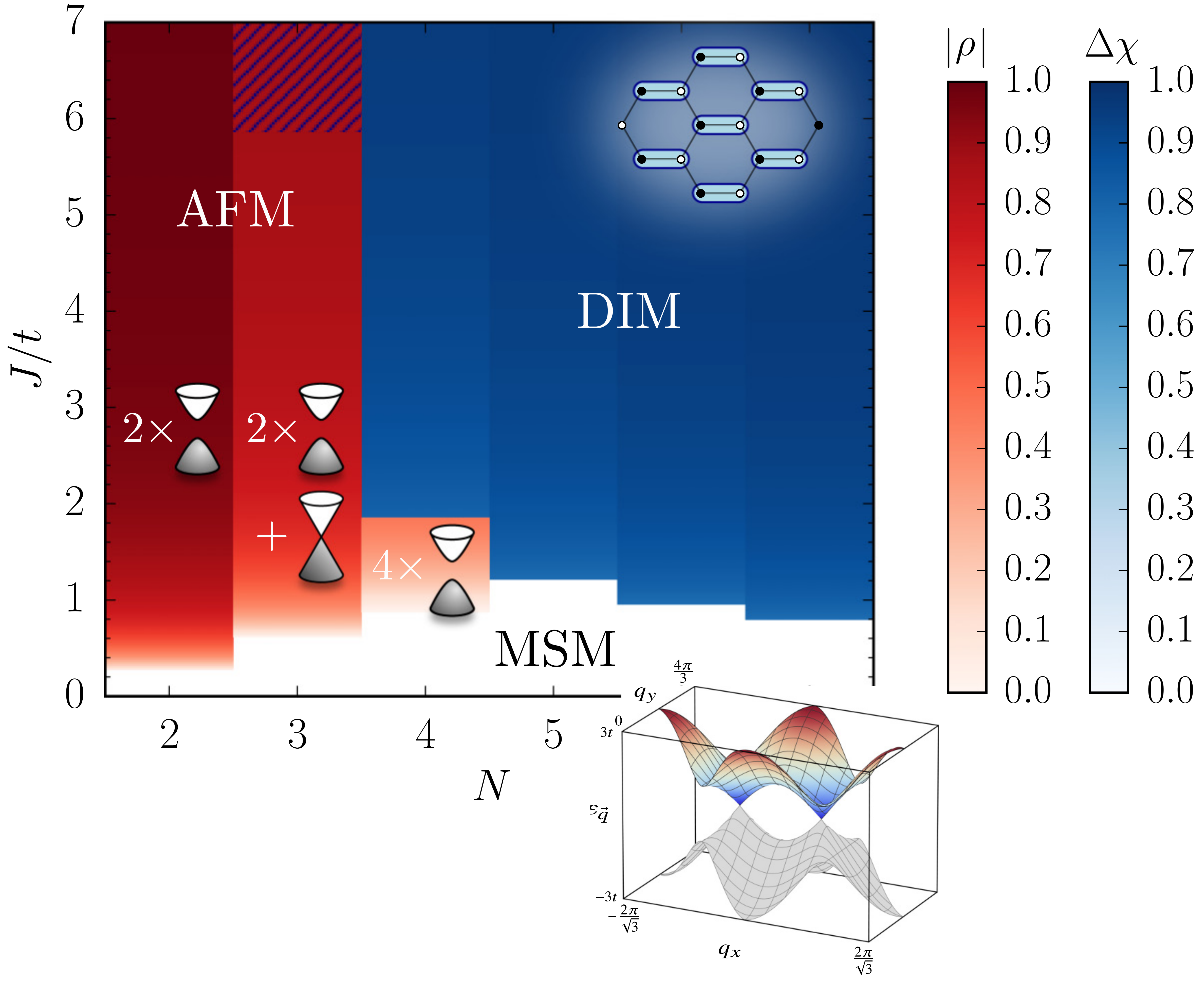
SU(*N*) Hubbard-Heisenberg models



[Affleck & Marston, PRB '88]
[Read & Sachdev, NPB '89]
[Lang, Meng, Muramatsu, Wessel, Assaad, PRL '13]

Conclusions

SO(*N*) Majorana-Hubbard models



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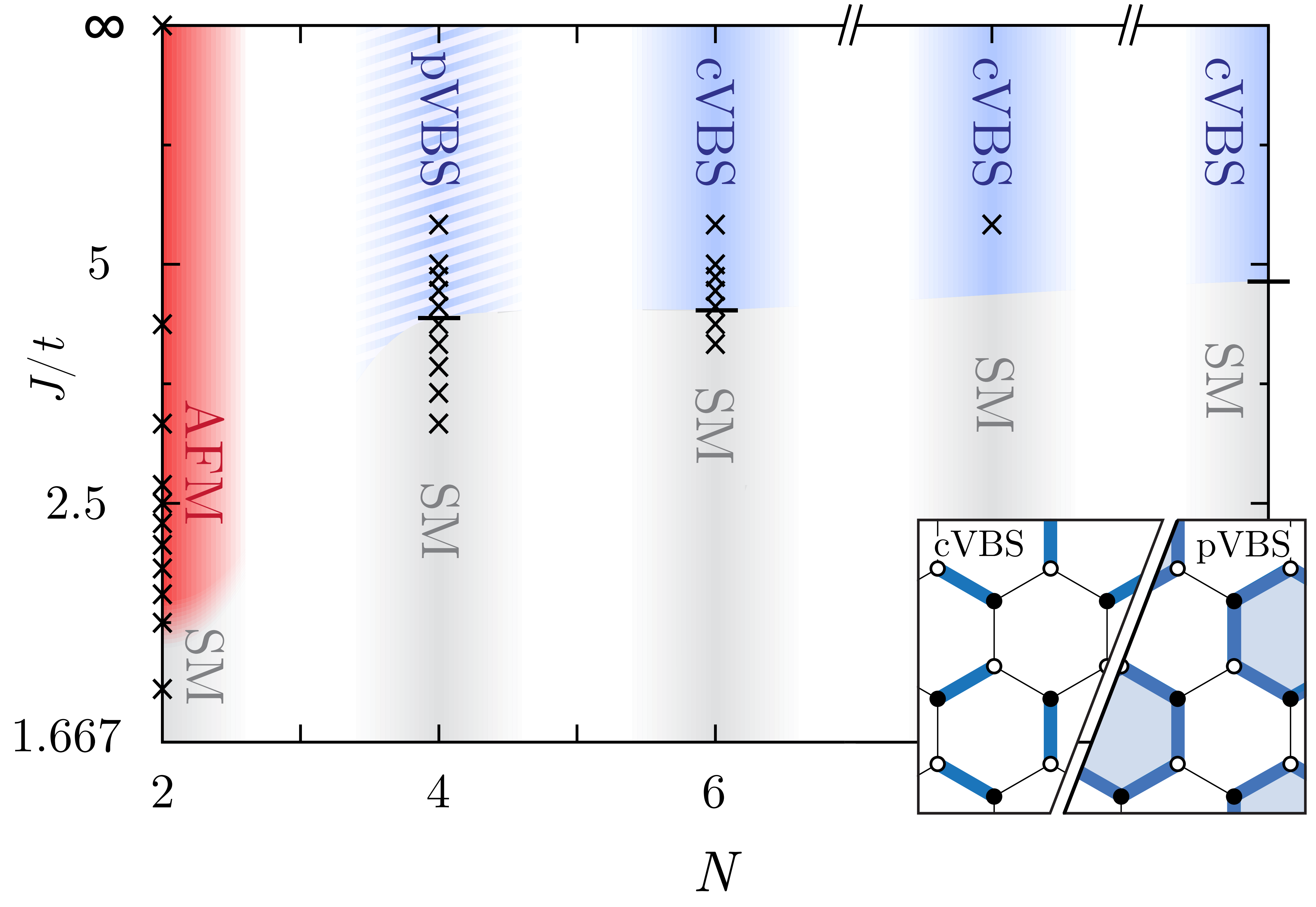

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