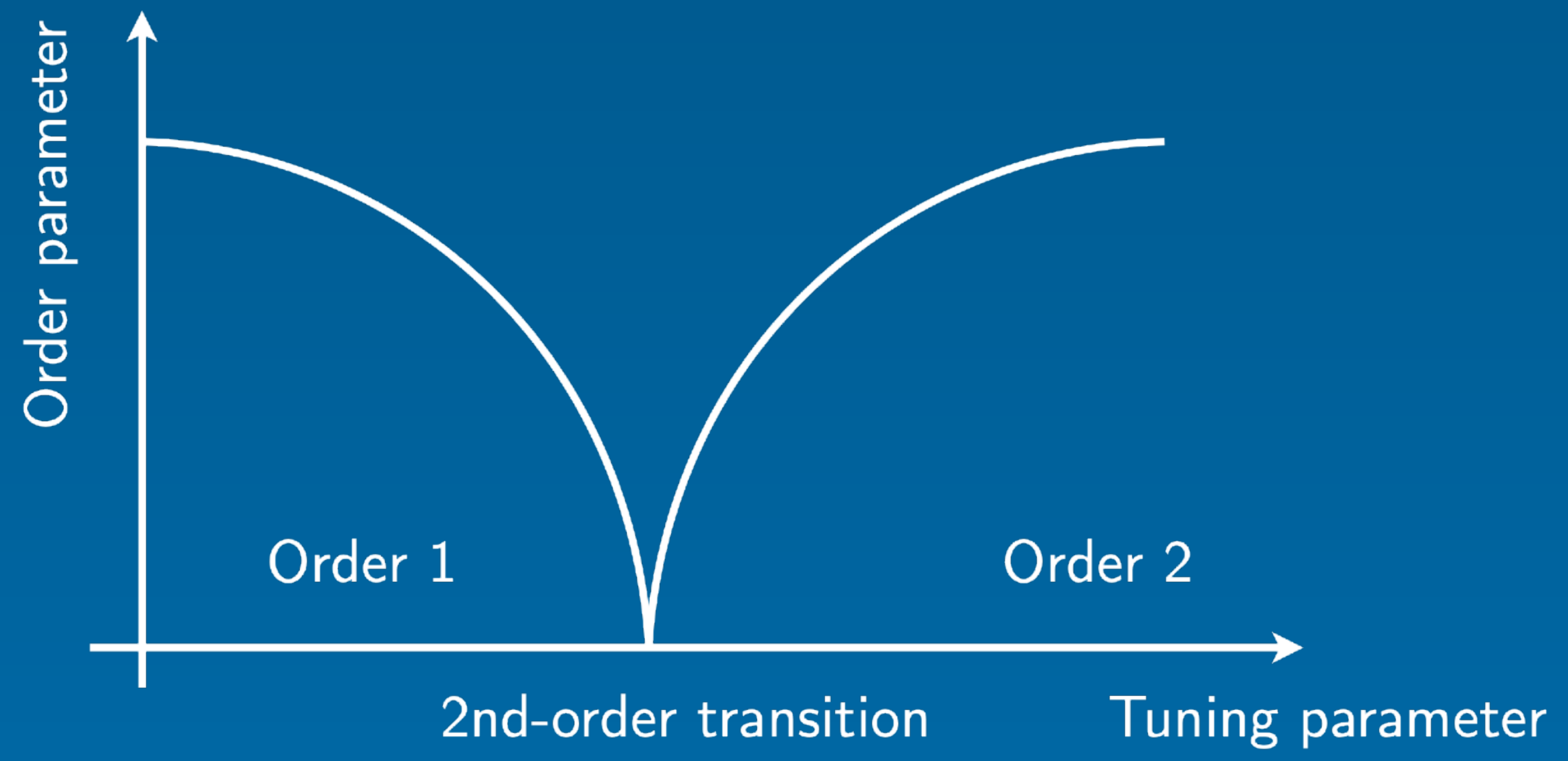
TECHNISCHE UNIVERSITÄT DRESDEN

DRESDEN concept

Continuous order-to-order transition from fixed-point annihilation

Lukas Janssen

David Moser



Emmy Noether-Programm

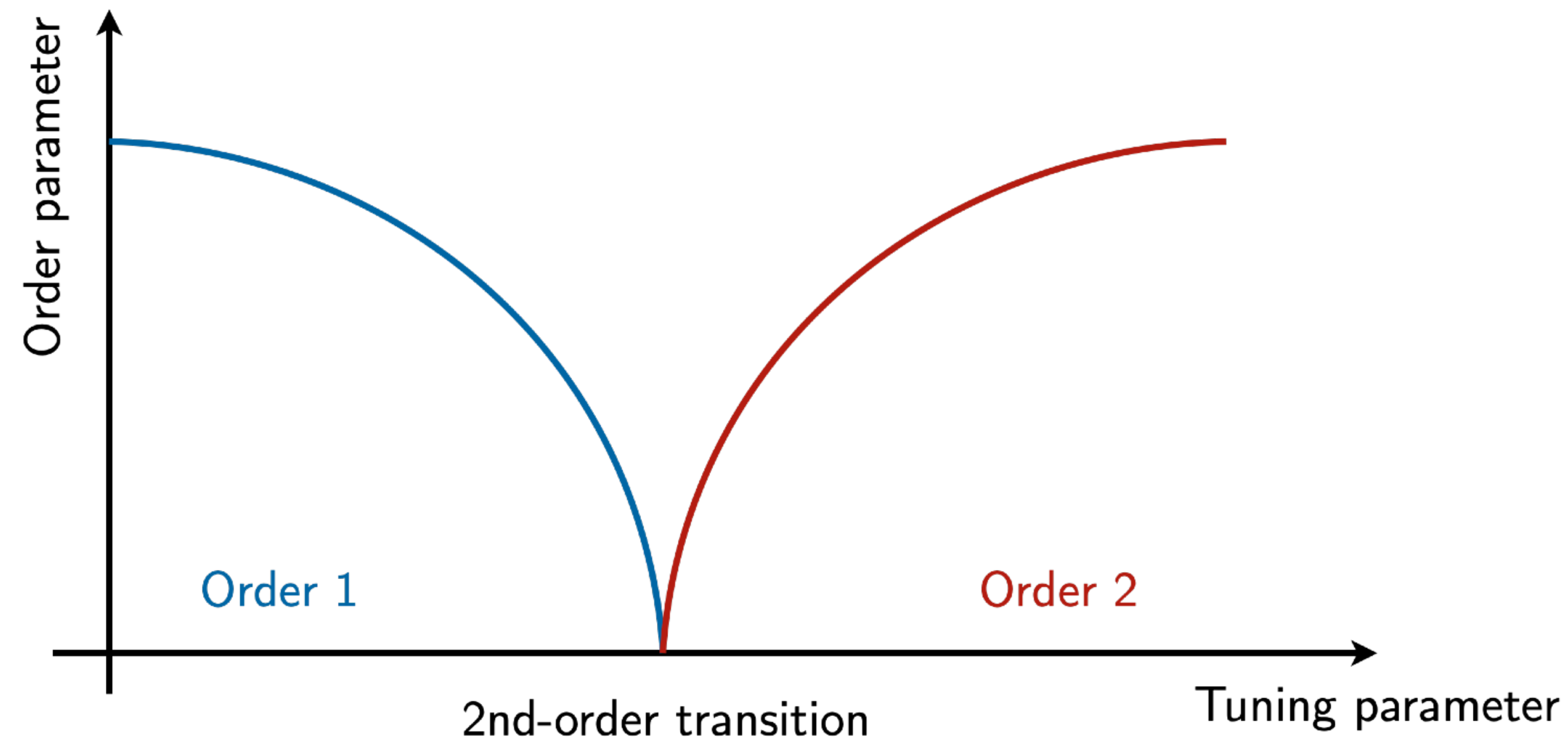
DFG Deutsche Forschungsgemeinschaft

ct.qmat Complexity and Topology in Quantum Matter

Outline

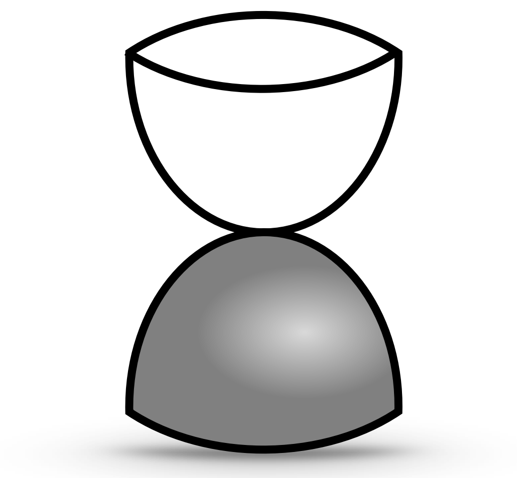
(1) Introduction

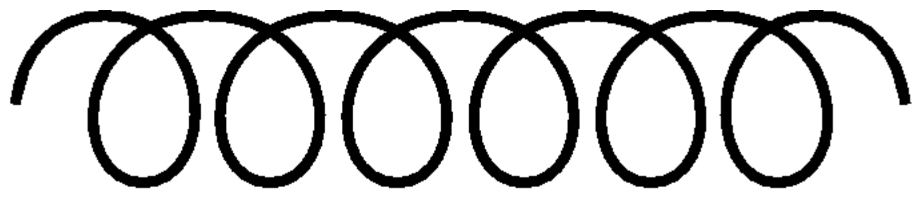
(2) Continuous order-to-order transition from fixed-point annihilation

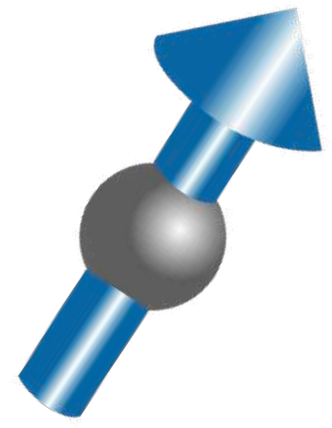


(3) Examples

- Luttinger semimetals
- QCD_4 plus 4-fermion interactions
- Spin-boson models



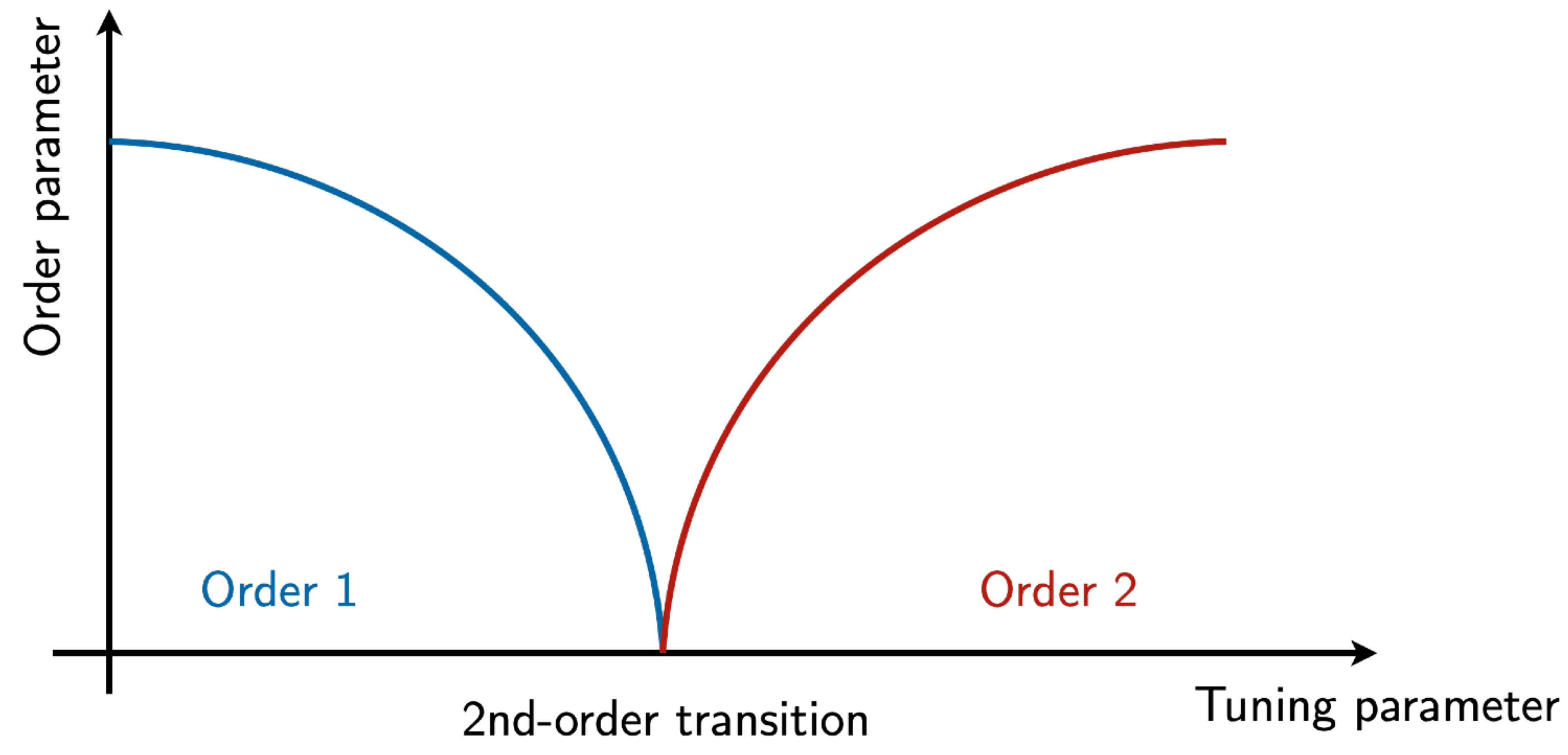




(4) Conclusions

Outline

(1) Introduction



(2) Continuous order-to-order transition from fixed-point annihilation

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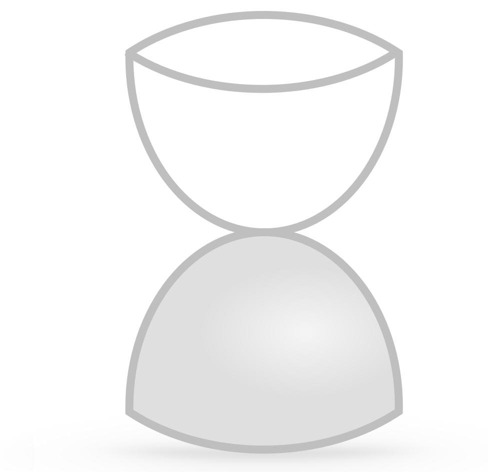
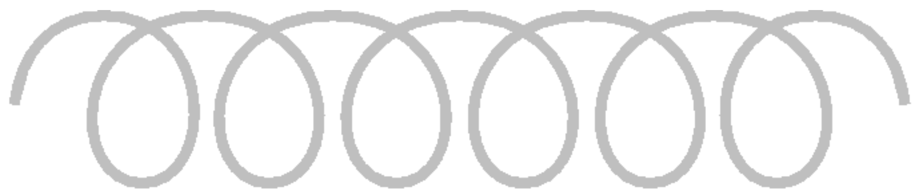
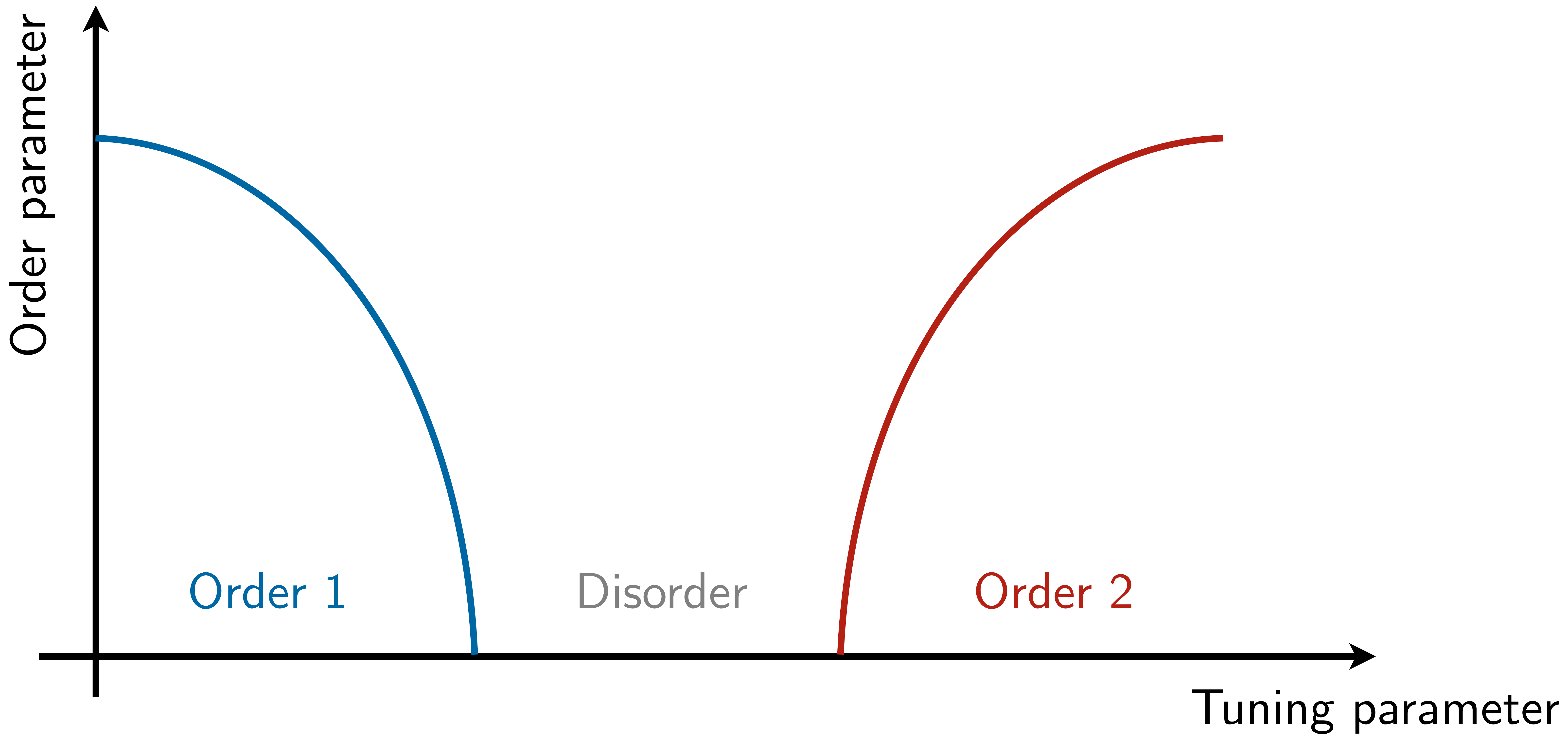


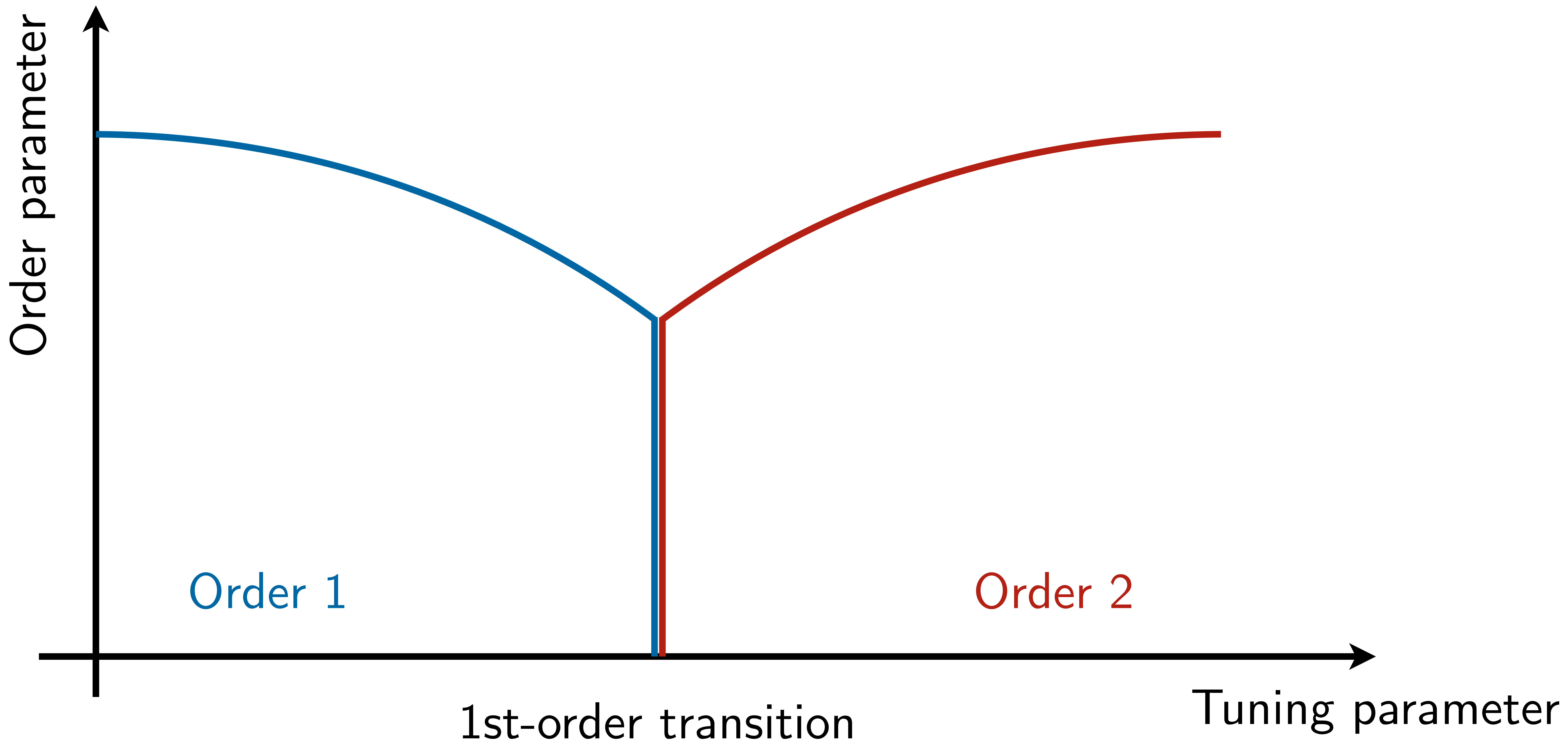


(4) Conclusions

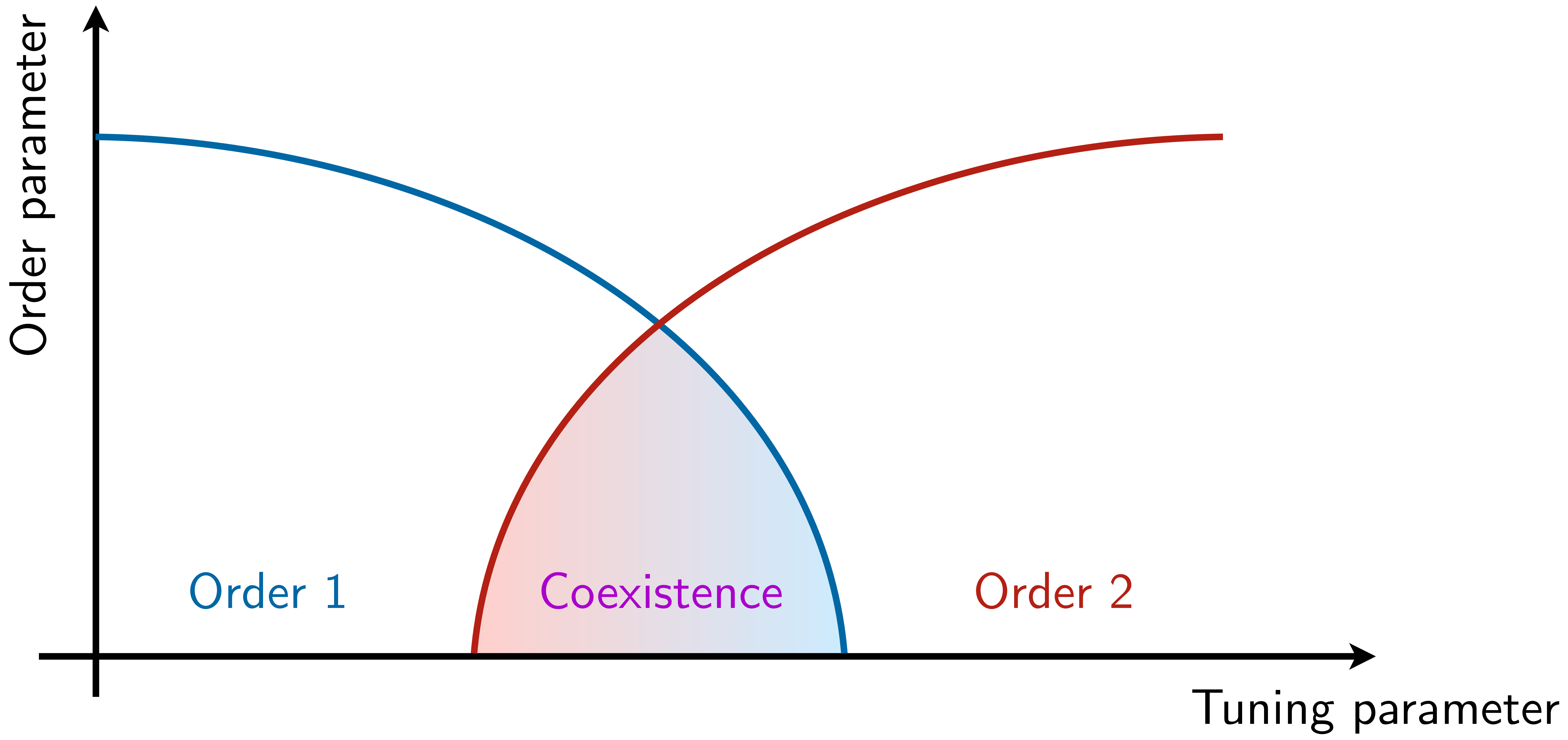
Competing orders



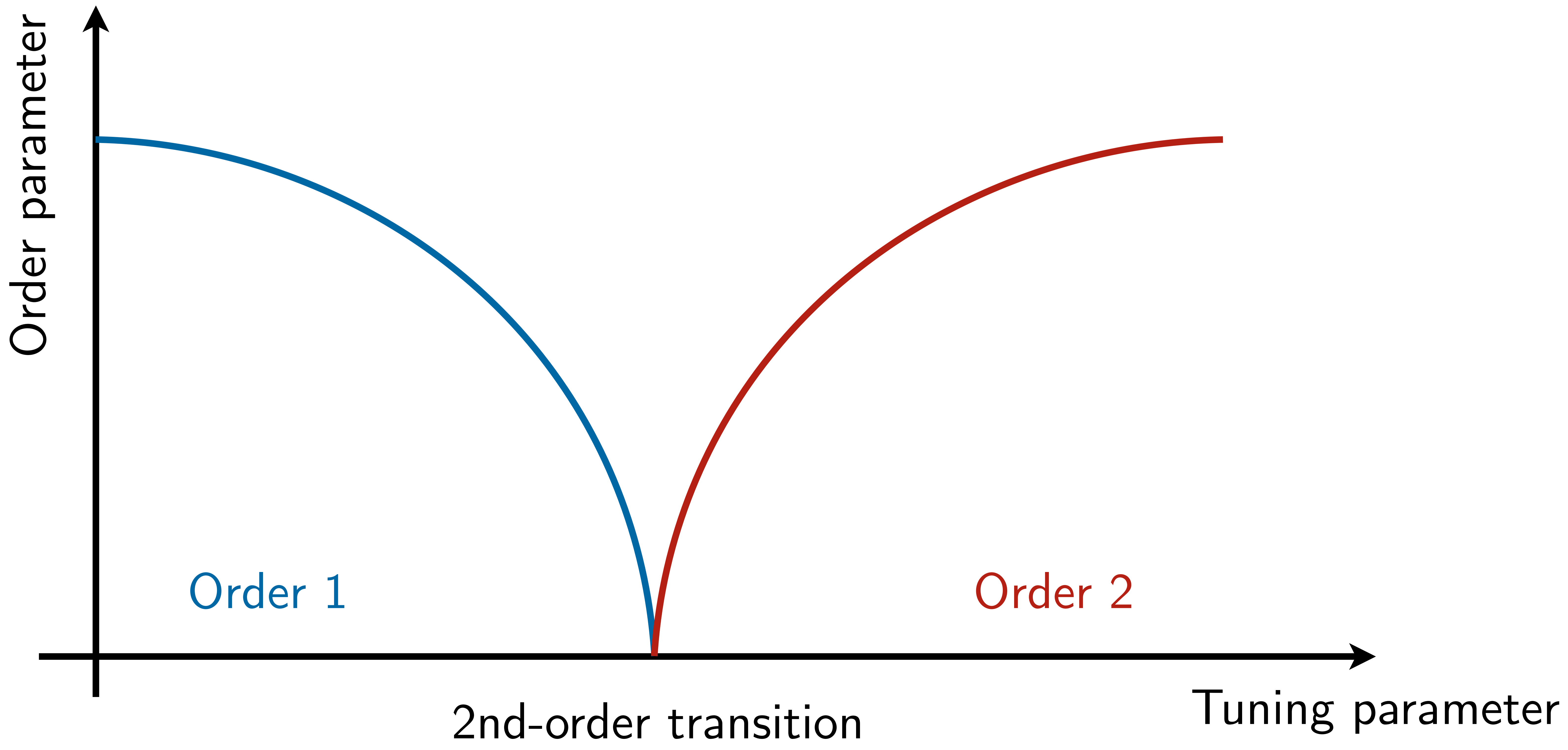
Competing orders



Competing orders



Competing orders

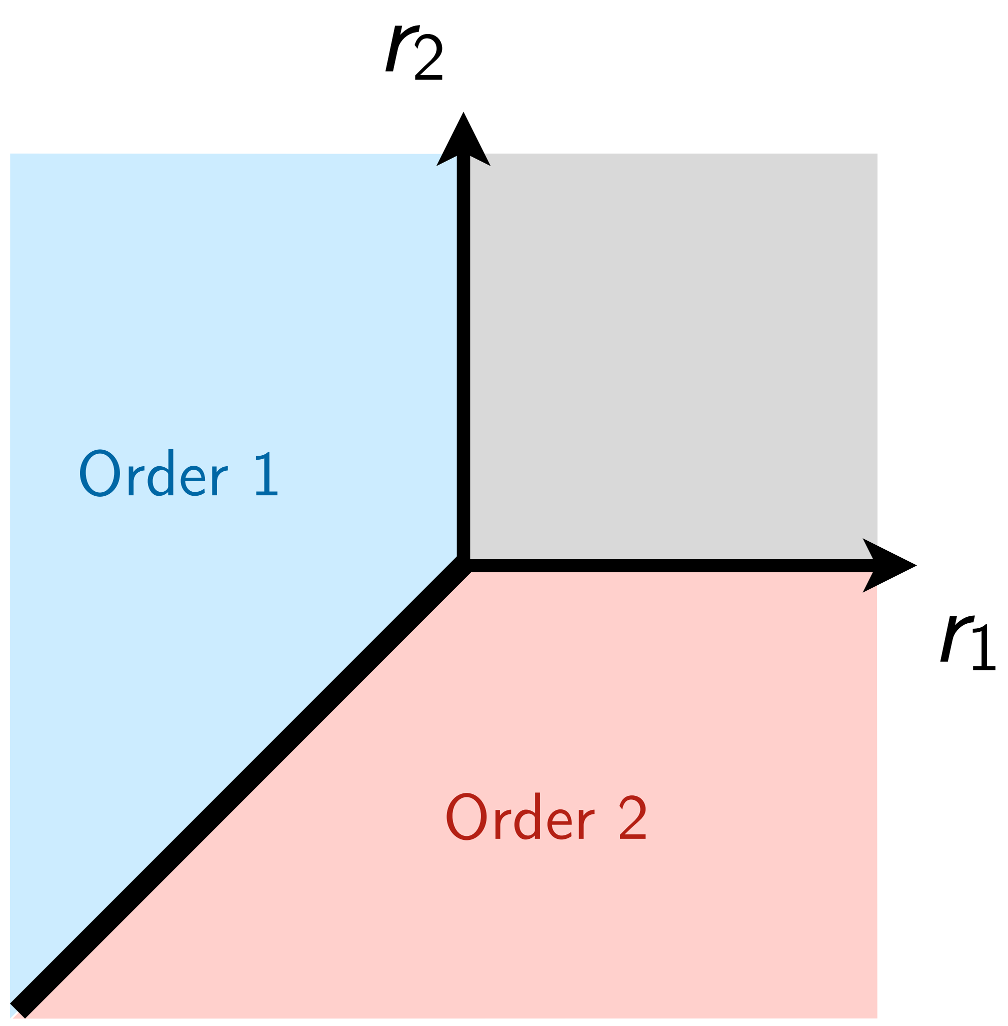


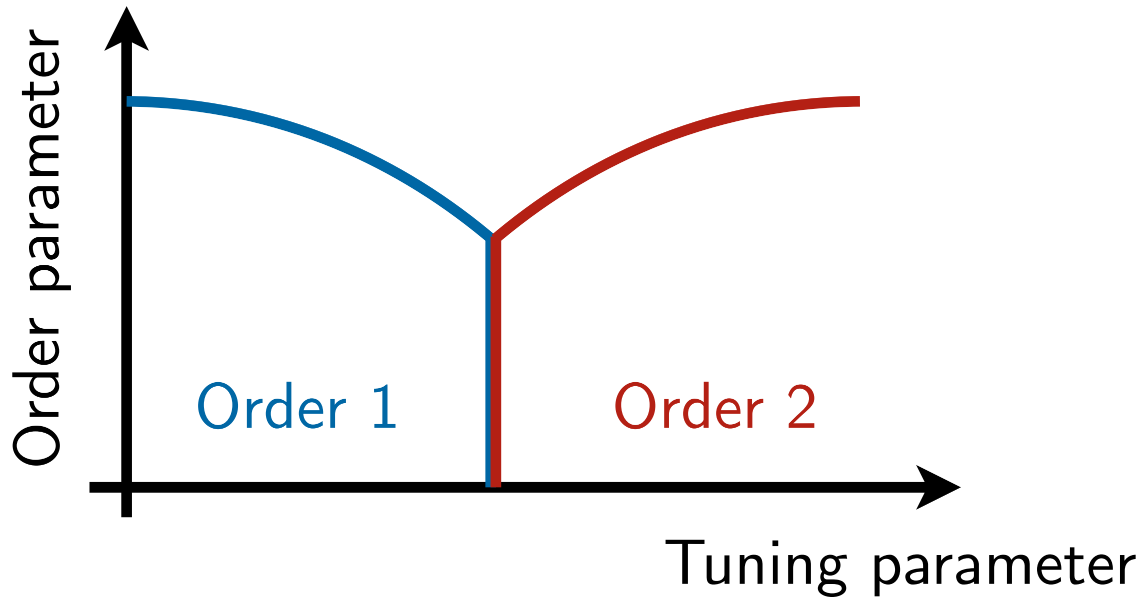
Landau

Landau theory

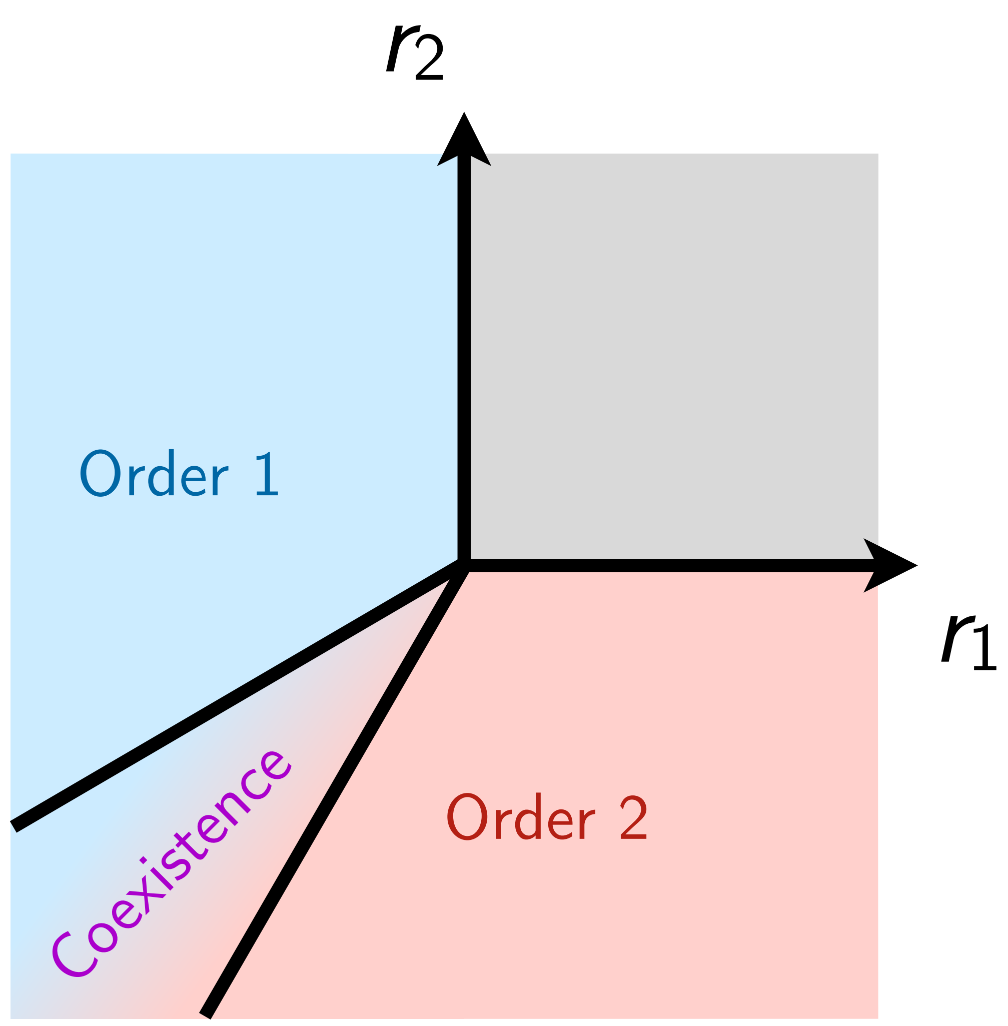
Landau functional:
$$F(\phi,\varphi) = \frac{r_1}{2}\phi^2 + \frac{r_2}{2}\varphi^2 + \lambda_1\phi^4 + \lambda_2\varphi^4 + 2\lambda_{12}\phi^2\varphi^2 + \mathcal{O}((\phi,\varphi)^6)$$

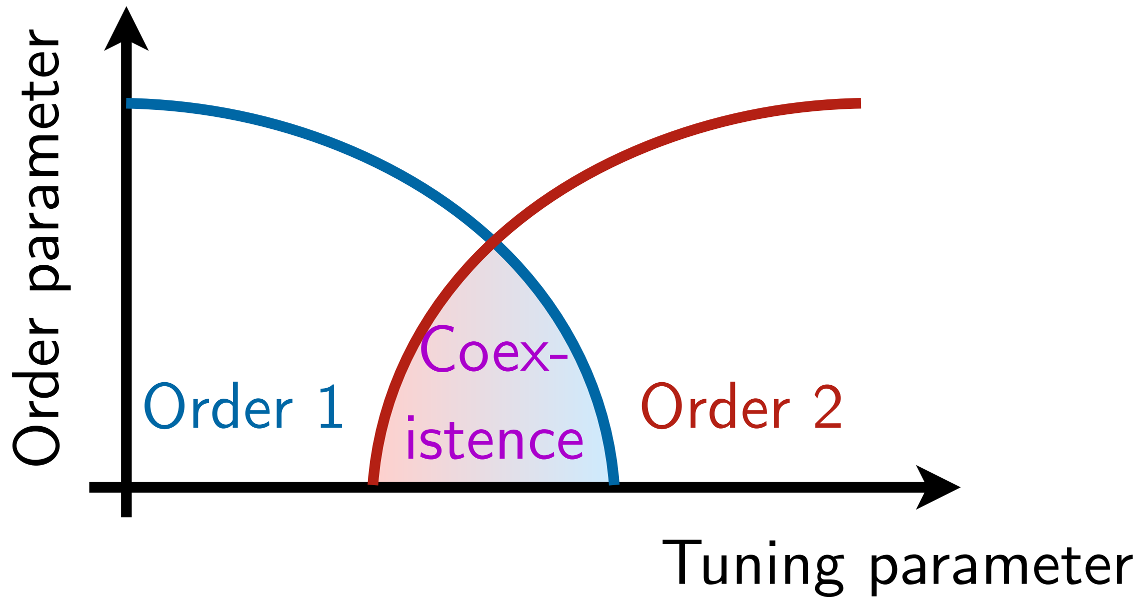
$$\lambda_1\lambda_2 - \lambda_{12}^2 \leq 0$$





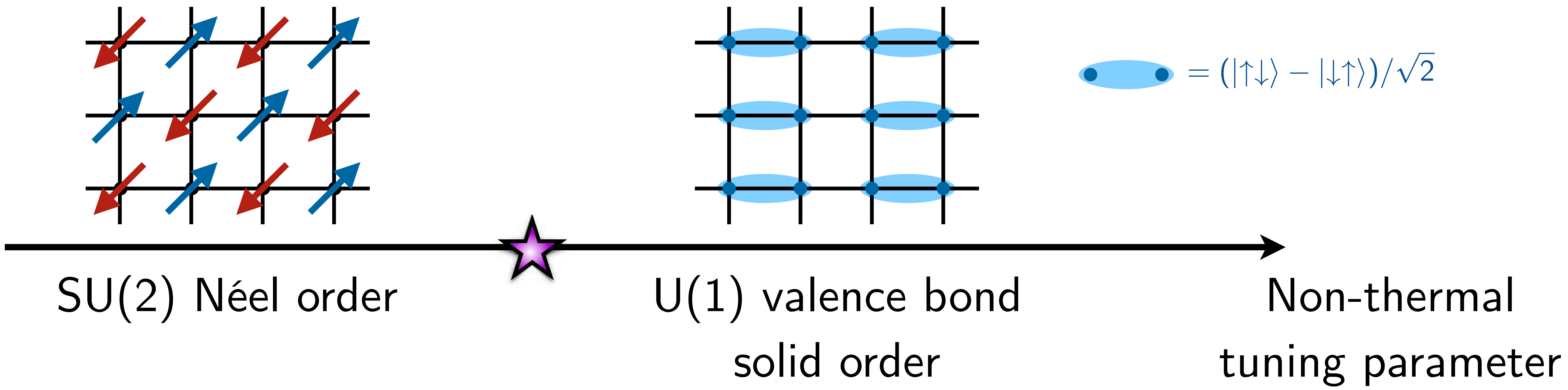
$$\lambda_1\lambda_2 - \lambda_{12}^2 > 0$$





Deconfined criticality

SU(2)-to-U(1) transition:



... driven by defects

[Senthil *et al.*, Science '04]

Effective field theory:

$$\mathcal{L} = (D_\mu z)^\dagger D_\mu z \quad \text{with} \quad z^\dagger z = 1 \qquad \text{“}\mathbb{C}P^1 \text{ model”}$$

where $D_\mu = \partial_\mu - i a_\mu$

CP¹ model

Effective field theory:

$$\mathcal{L} = (D_\mu z)^\dagger D_\mu z \quad \text{with} \quad z^\dagger z = 1$$

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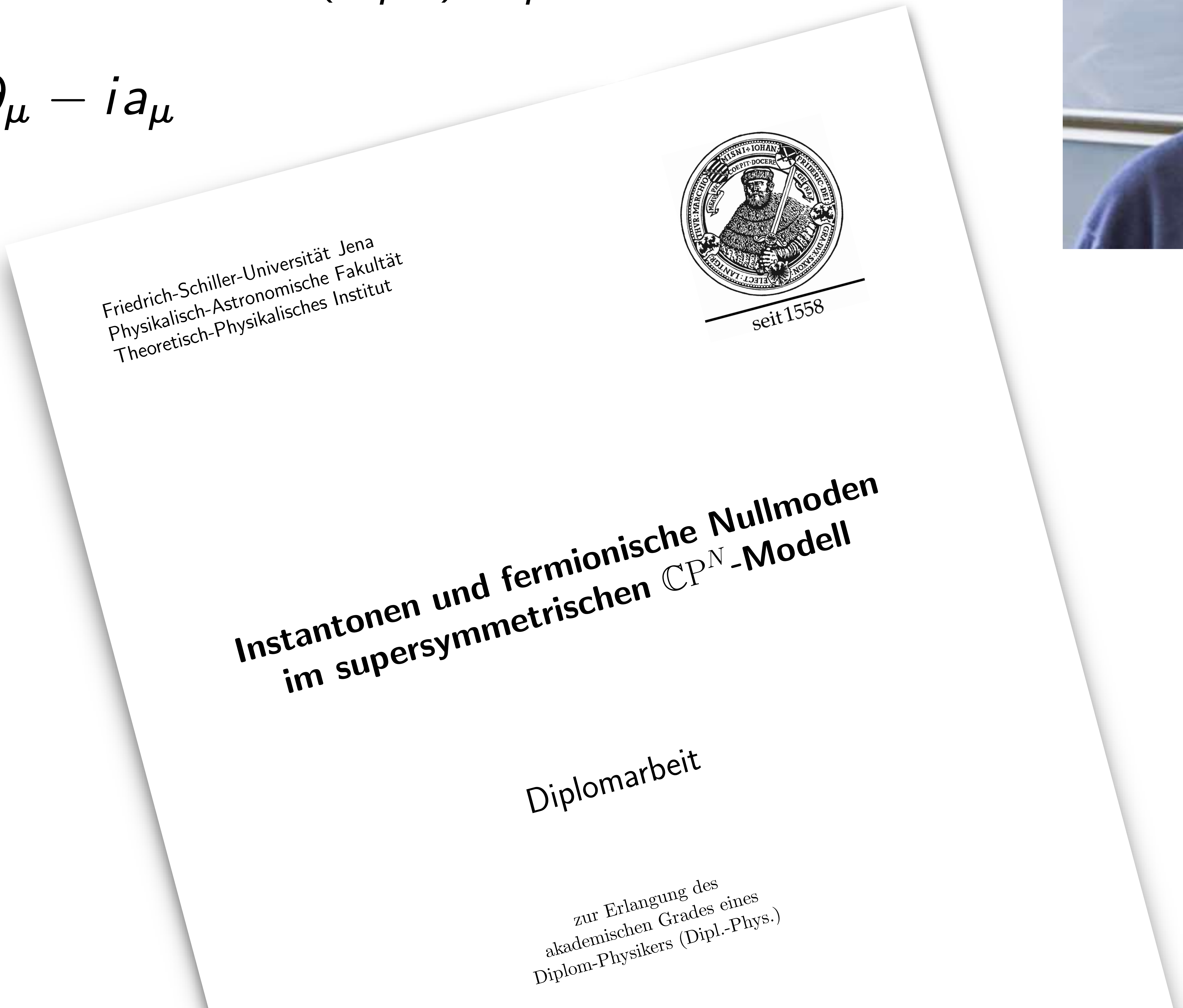
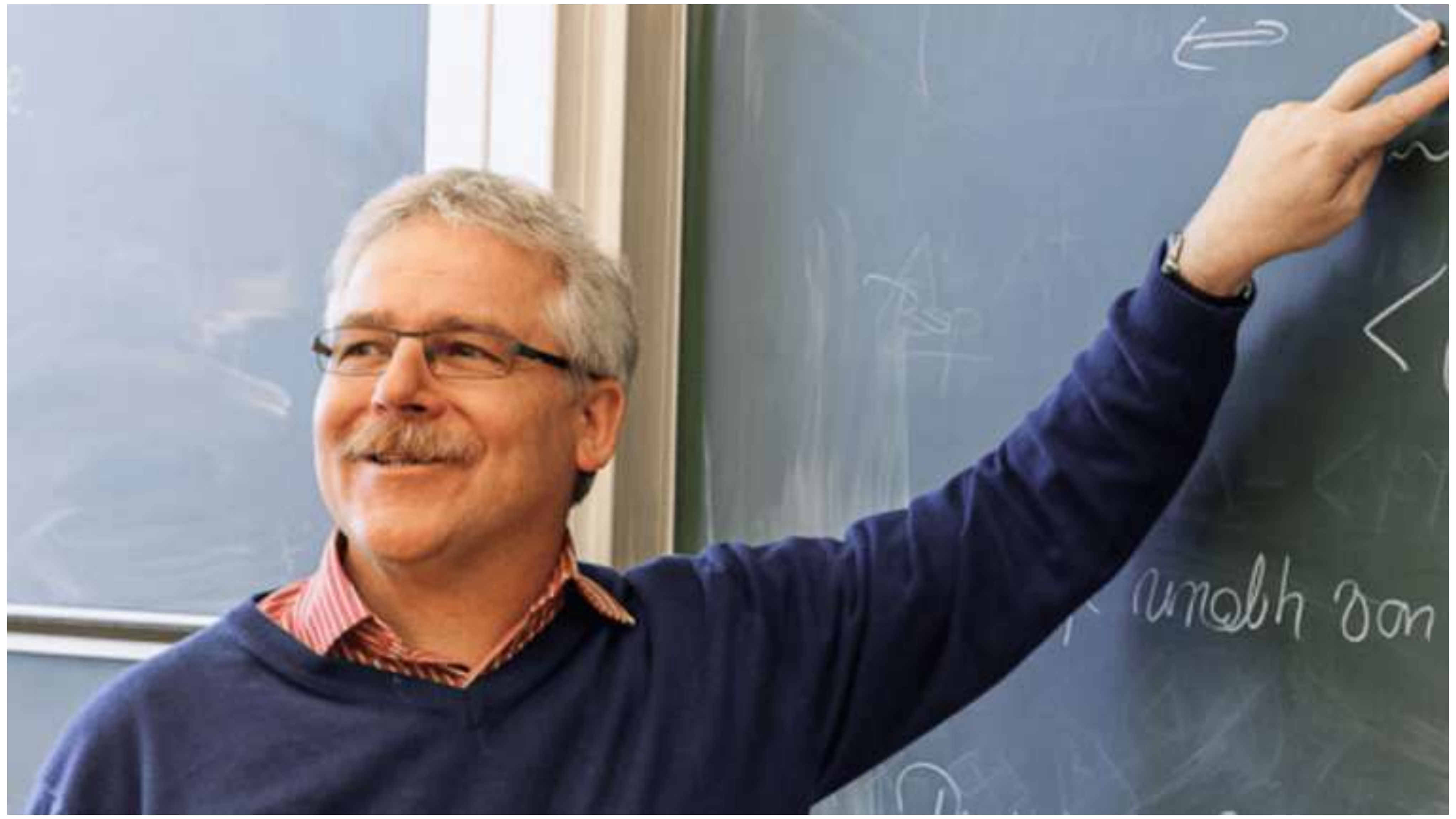

Friedrich-Schiller-Universität Jena
Physikalisch-Astronomische Fakultät
Theoretisch-Physikalisches Institut

seit 1558

Instantonen und fermionische Nullmoden im supersymmetrischen $\mathbb{C}P^N$-Modell

Diplomarbeit

zur Erlangung des
akademischen Grades eines
Diplom-Physikers (Dipl.-Phys.)



Deconfined criticality

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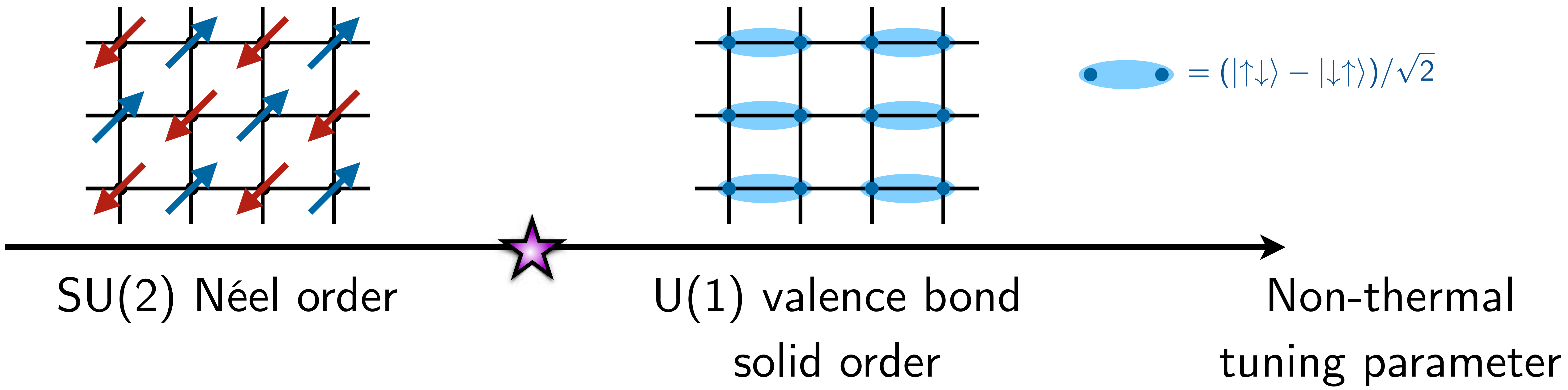

... driven by defects

[Senthil *et al.*, Science '04]

Effective field theory:

odel"

where $D_\mu = \partial_\mu - ia_\mu$

Scenario very beautiful, but ...

... quite complex

... very specific

... probably not realized

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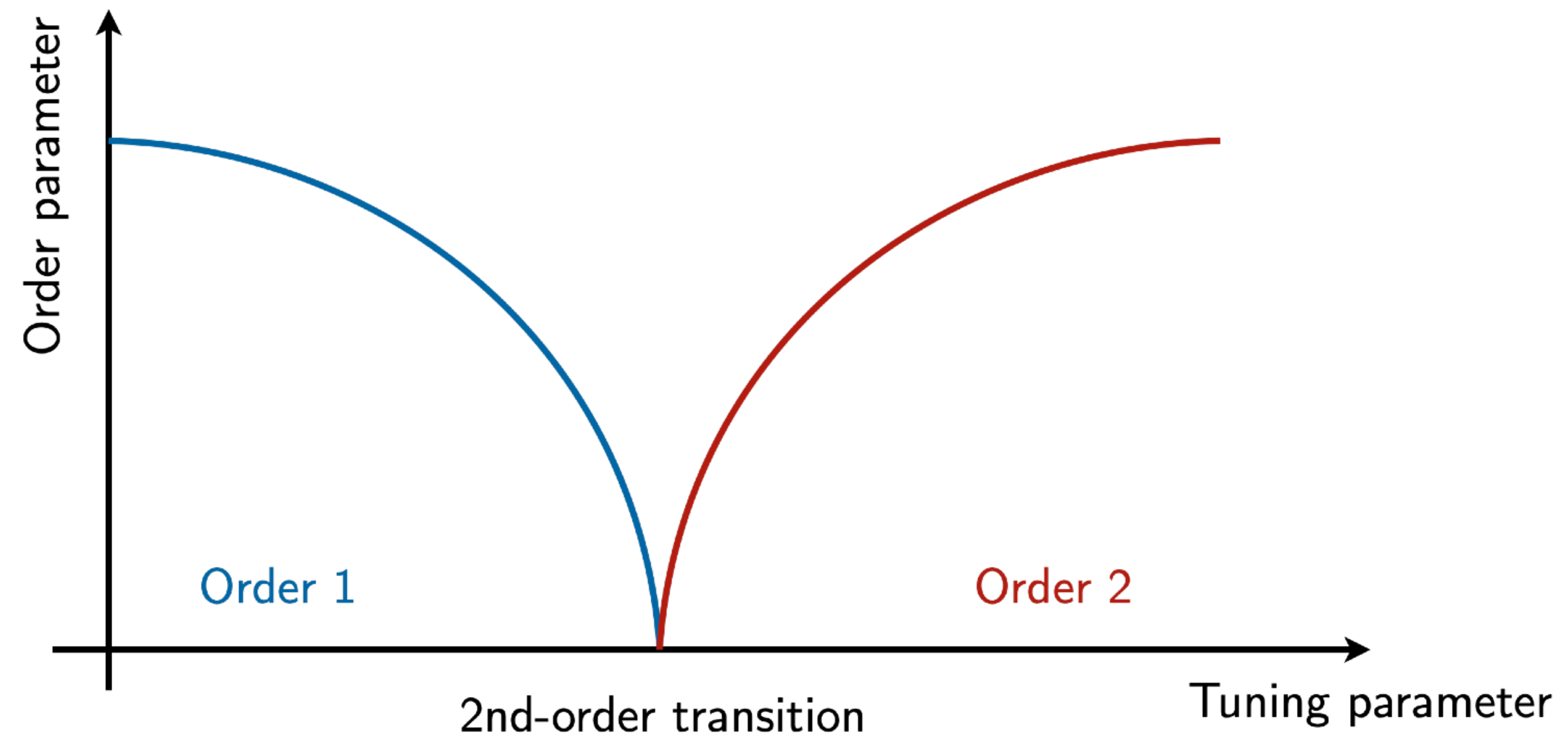
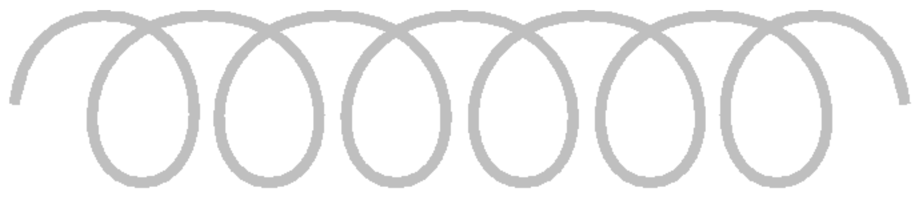
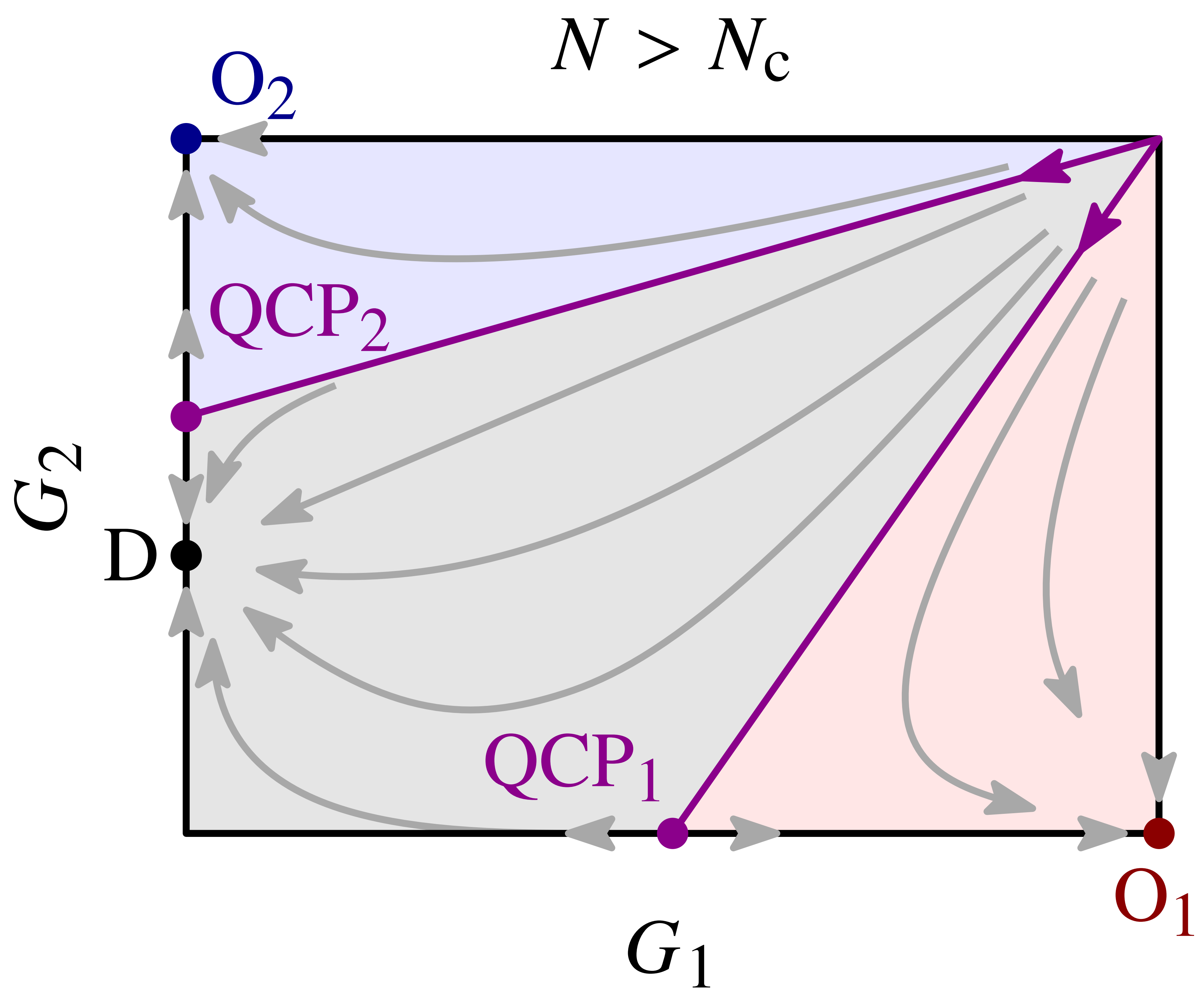
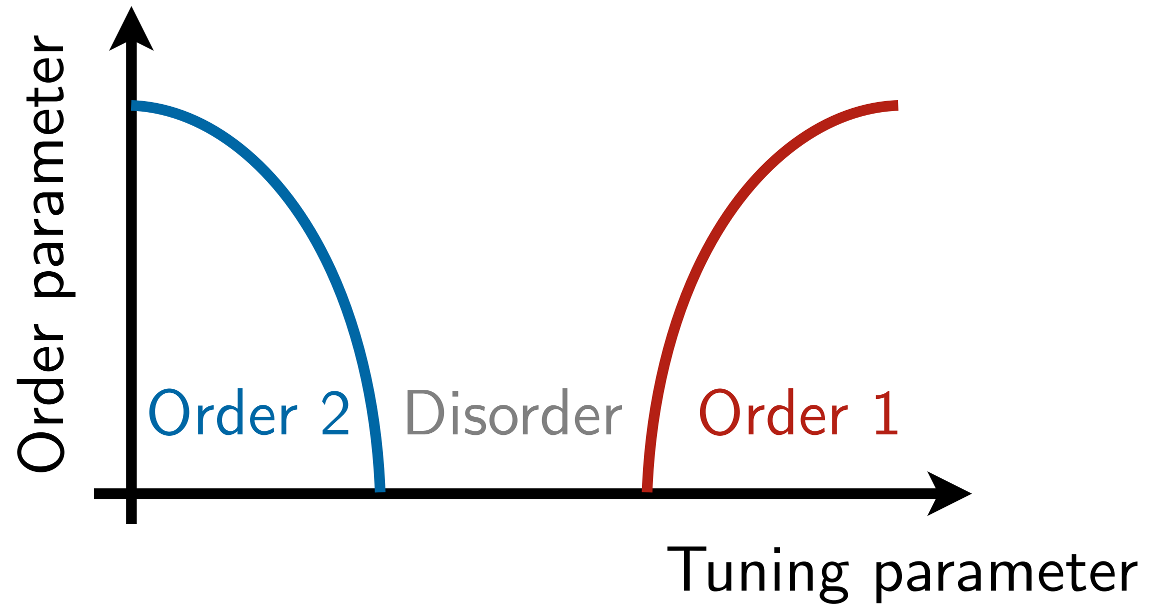
(4) Conclusions



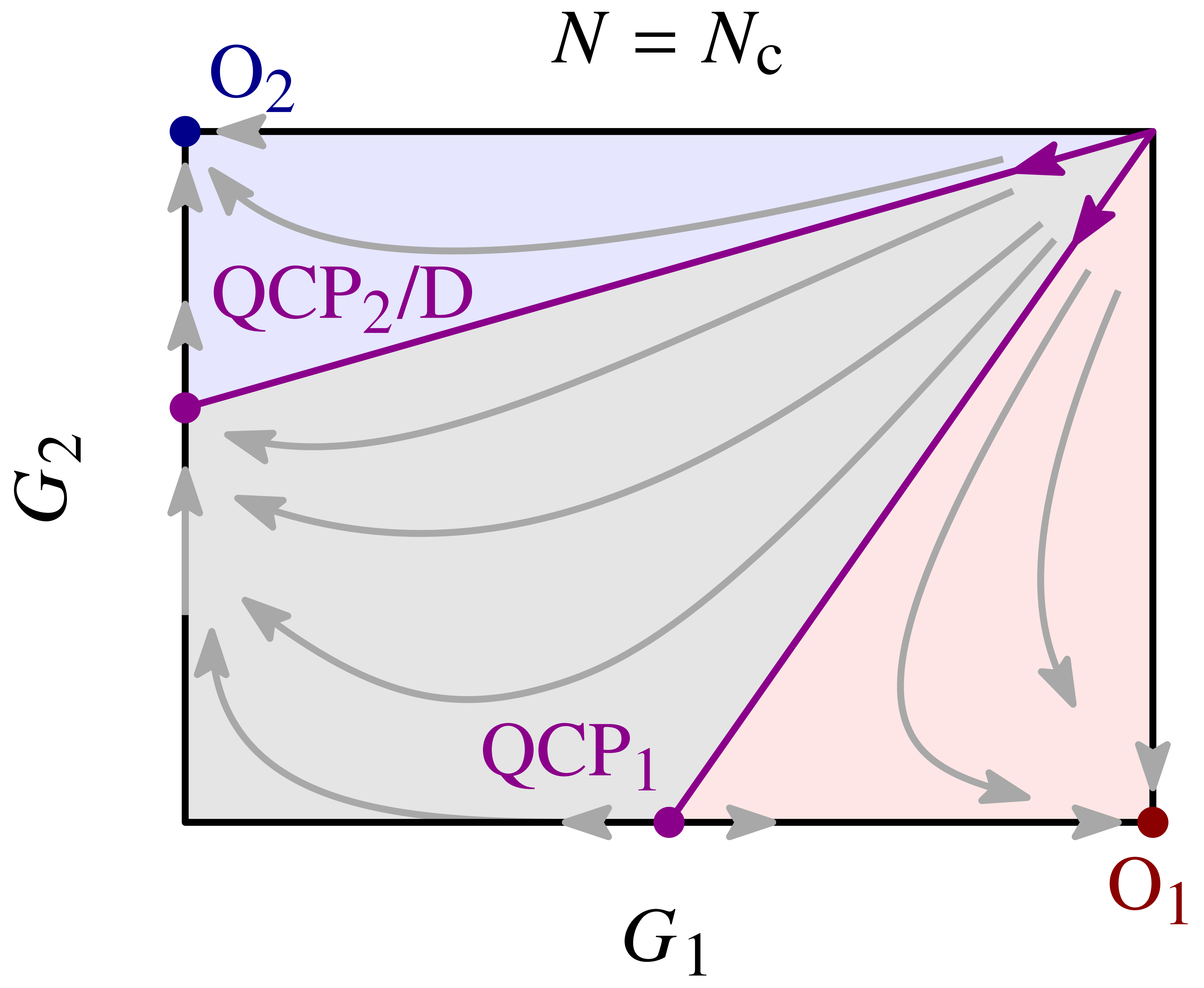


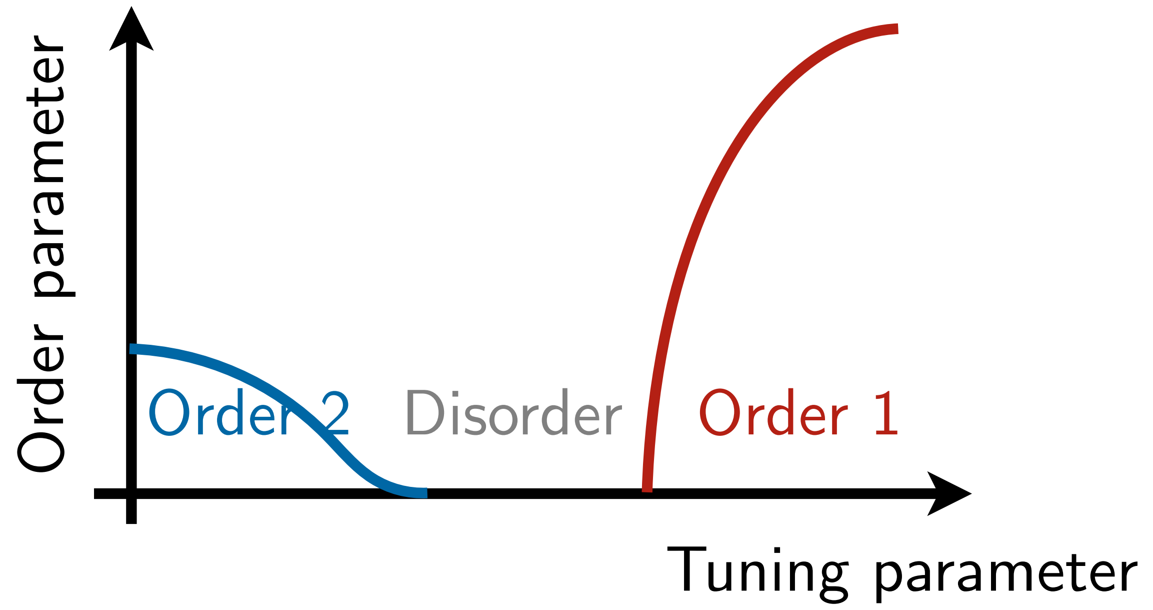
Mechanism


$N > N_c$
O_2
QCP_2
G_2
D
QCP_1
O_1
G_1


Order parameter
Order 2
Disorder
Order 1
Tuning parameter

Mechanism


$N = N_c$
O_2
QCP_2/D
G_2
QCP_1
O_1
G_1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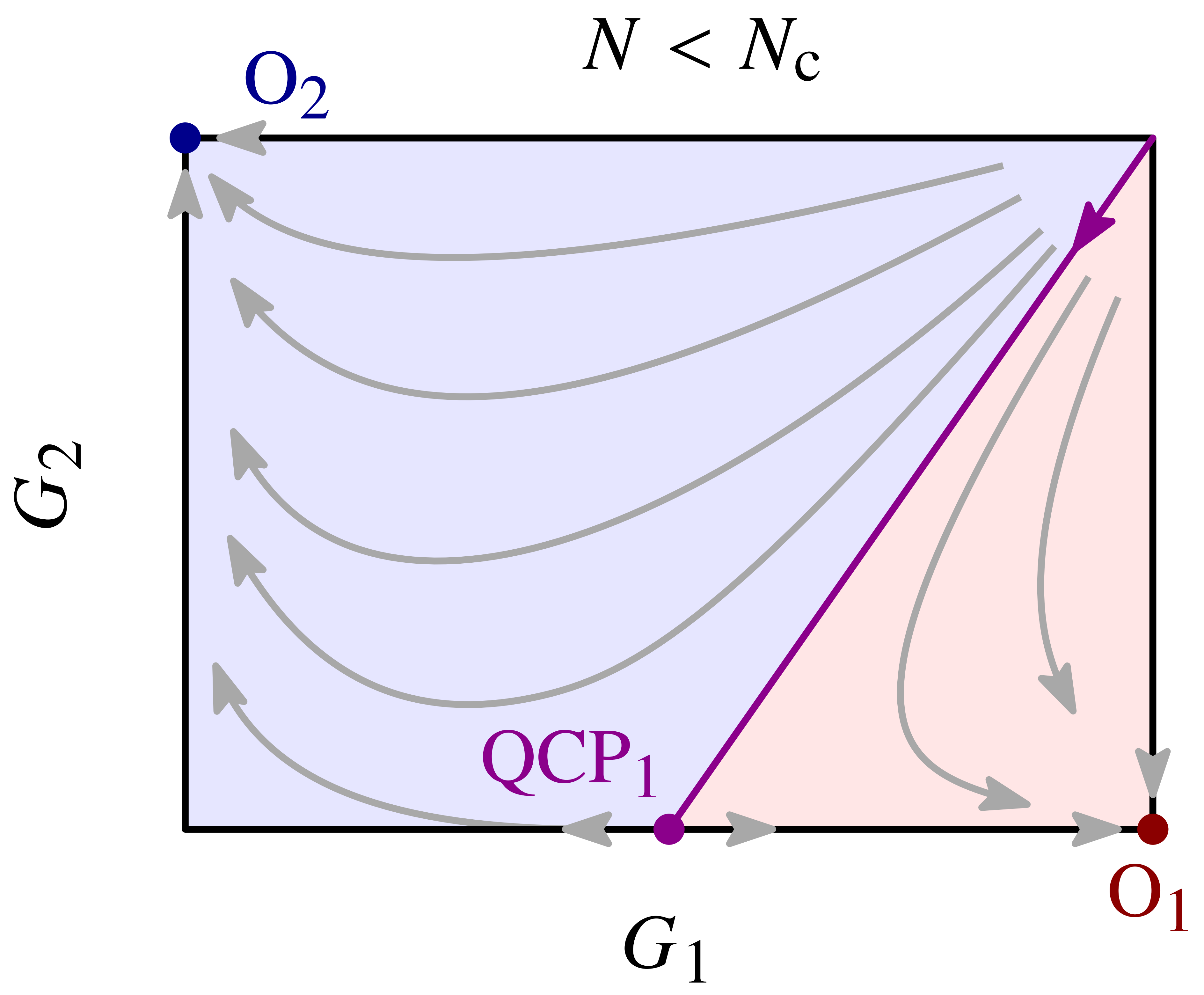
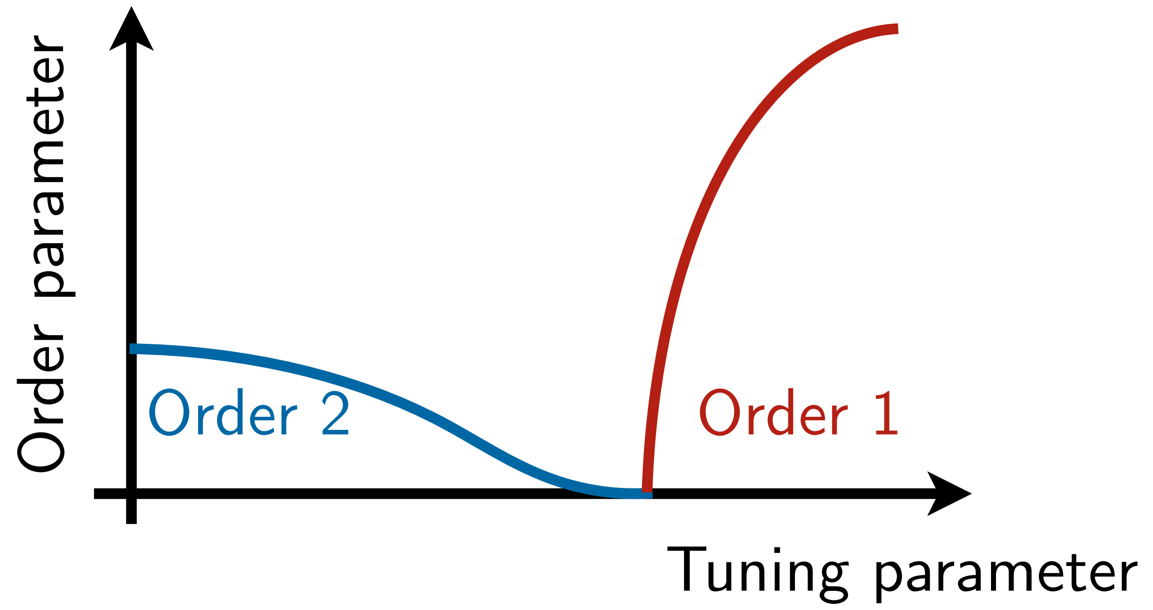

$N < N_c$
O_2
G_2
QCP_1
O_1
G_1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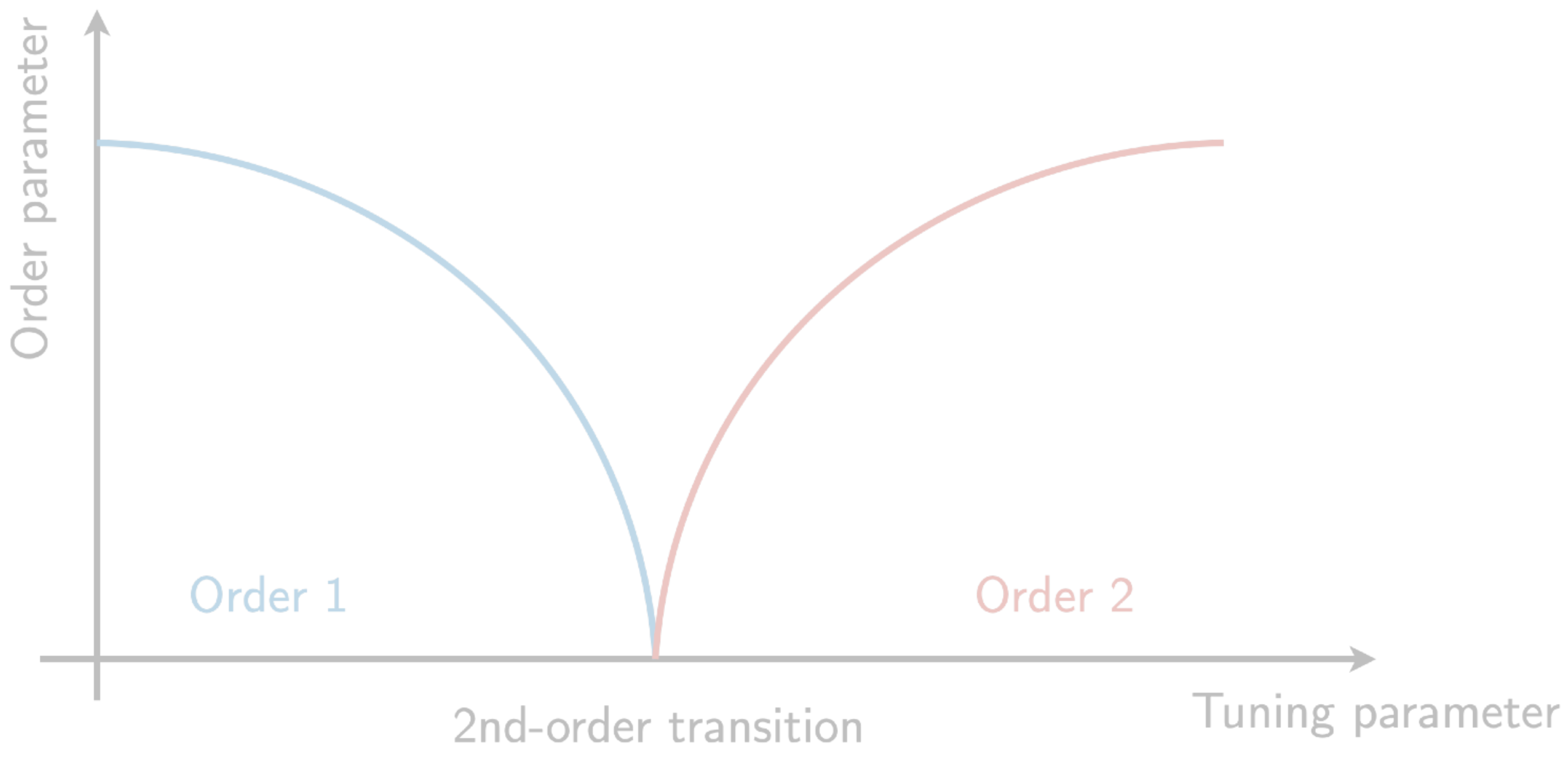
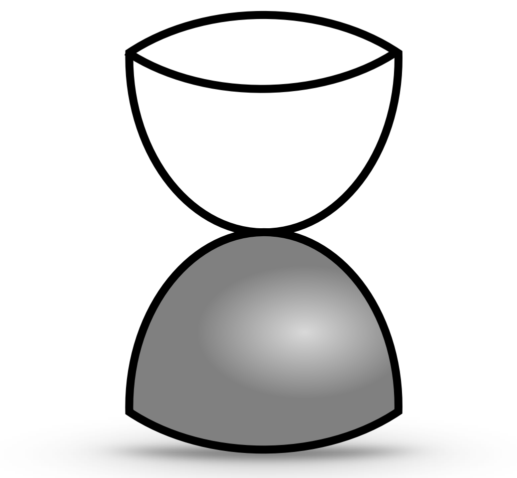
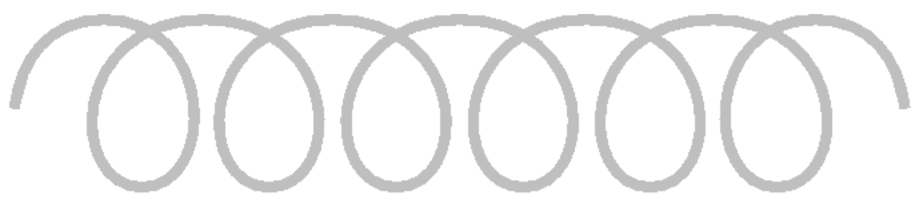


(3) Examples

- Luttinger semimetals



- QCD_4 plus 4-fermion interactions

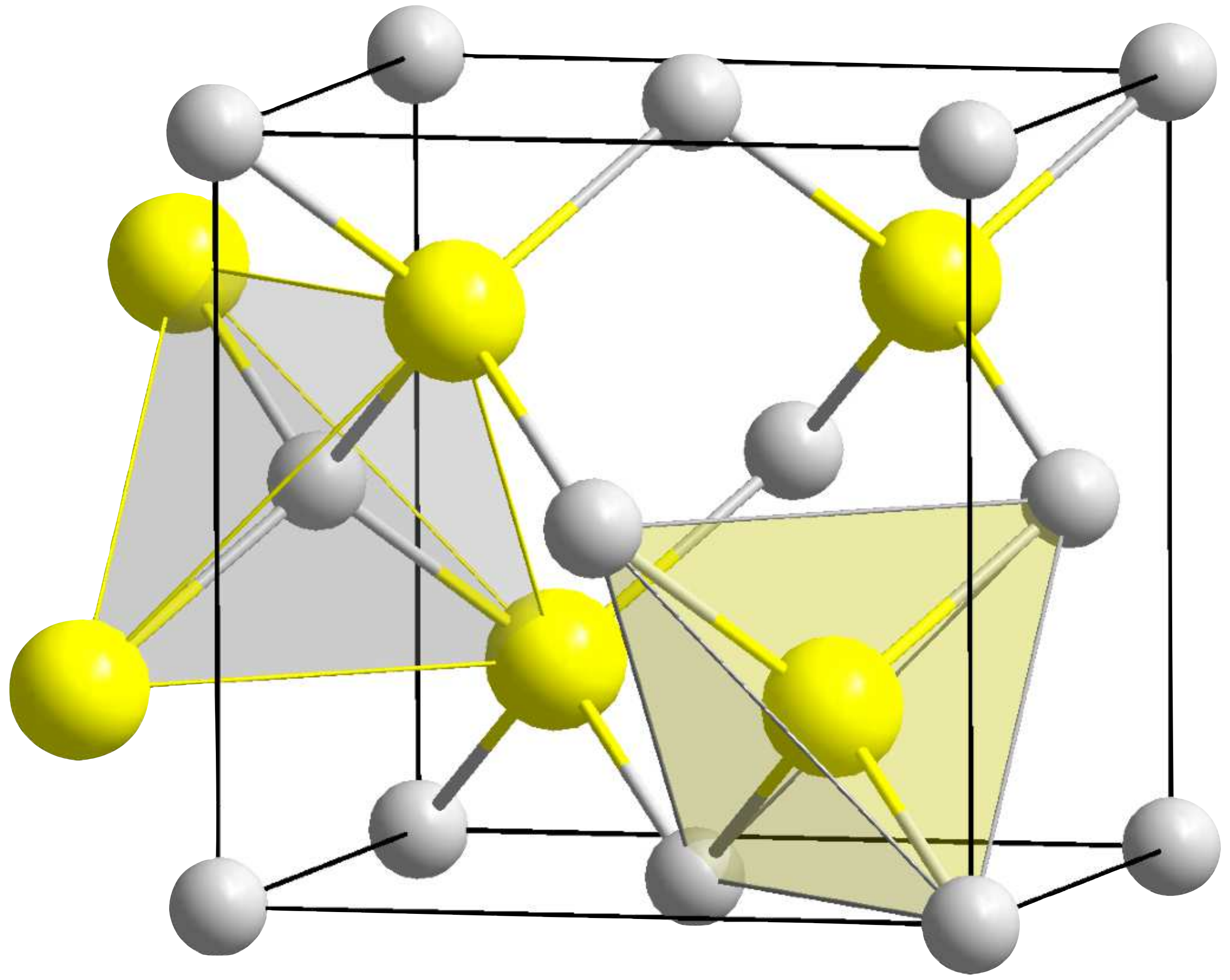


- Spin-boson models

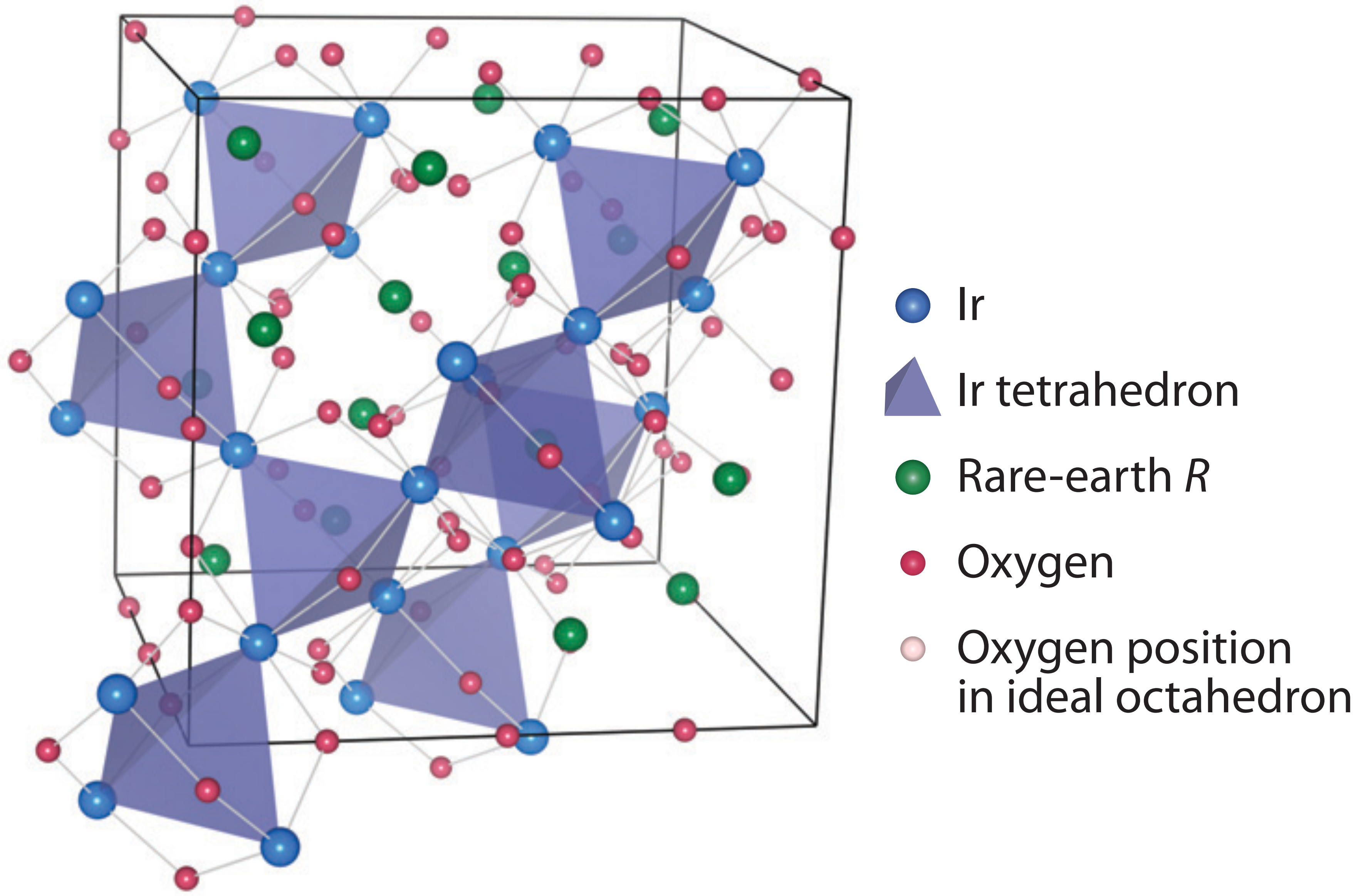
(4) Conclusions

Luttinger semimetals

Material realizations:

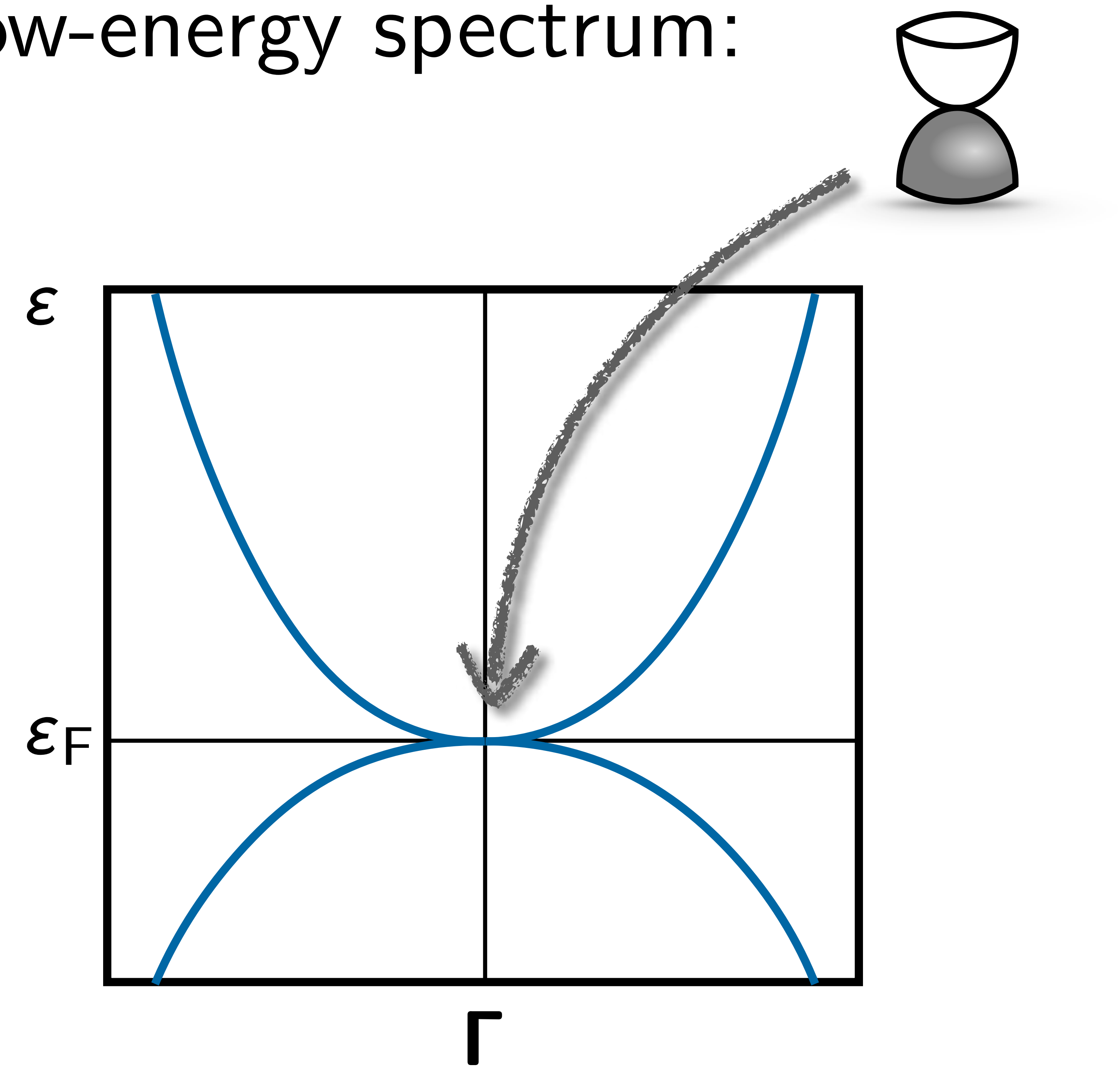


α-Sn, HgTe



$R_2Ir_2O_7$ (R = Pr, Nd)

Low-energy spectrum:



[Armitage, Mele, Vishwanath, RMP '18]

Effective model

spherical harmonics $(d_a) = (\sqrt{3}p_y p_z, \sqrt{3}p_x p_z, \sqrt{3}p_x p_y, \frac{\sqrt{3}}{2}(p_x^2 - p_y^2), \frac{1}{2}(2p_z^2 - p_x^2 - p_y^2))$

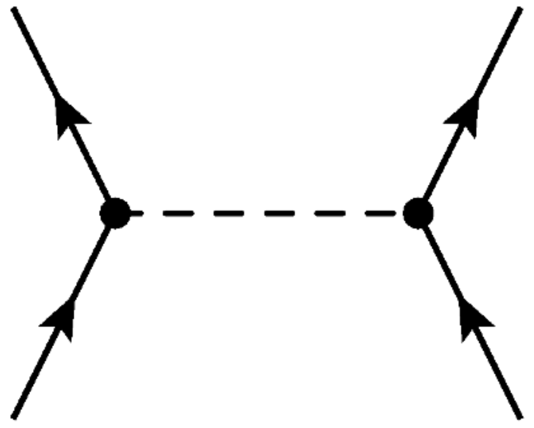
$(s_a) = (+,+,+,-,-)$

cubic anisotropy $\delta \in [-1, 1]$

Lagrangian:

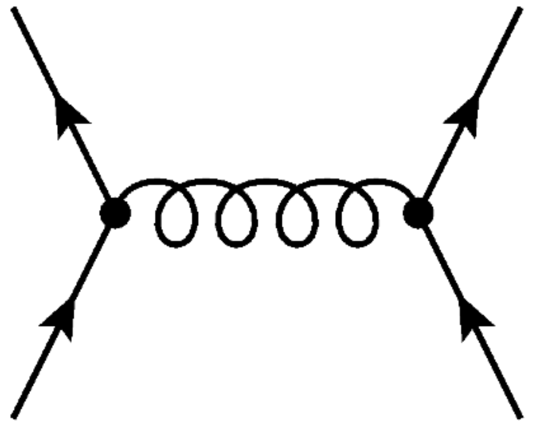
$$\mathcal{L} = \sum_{i=1}^{N} \psi_i^\dagger \left(\partial_\tau + \sum_{a=1}^{5} (1 + s_a\delta) d_a(-\mathrm{i}\nabla)\gamma_a \right) \psi_i$$

$$+ \frac{G_1}{2N} (\psi^\dagger \gamma_{45} \psi)^2$$



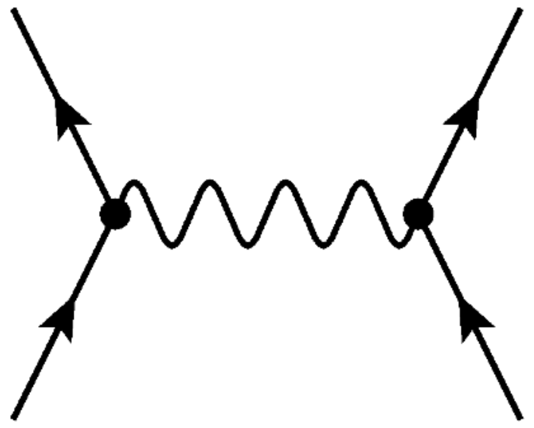
$\mathbb{Z}_2$ scalar

$$+ \frac{G_2}{2N} \sum_{a=1}^{5} (\psi^\dagger \gamma_a \psi)^2$$



O(3) tensor

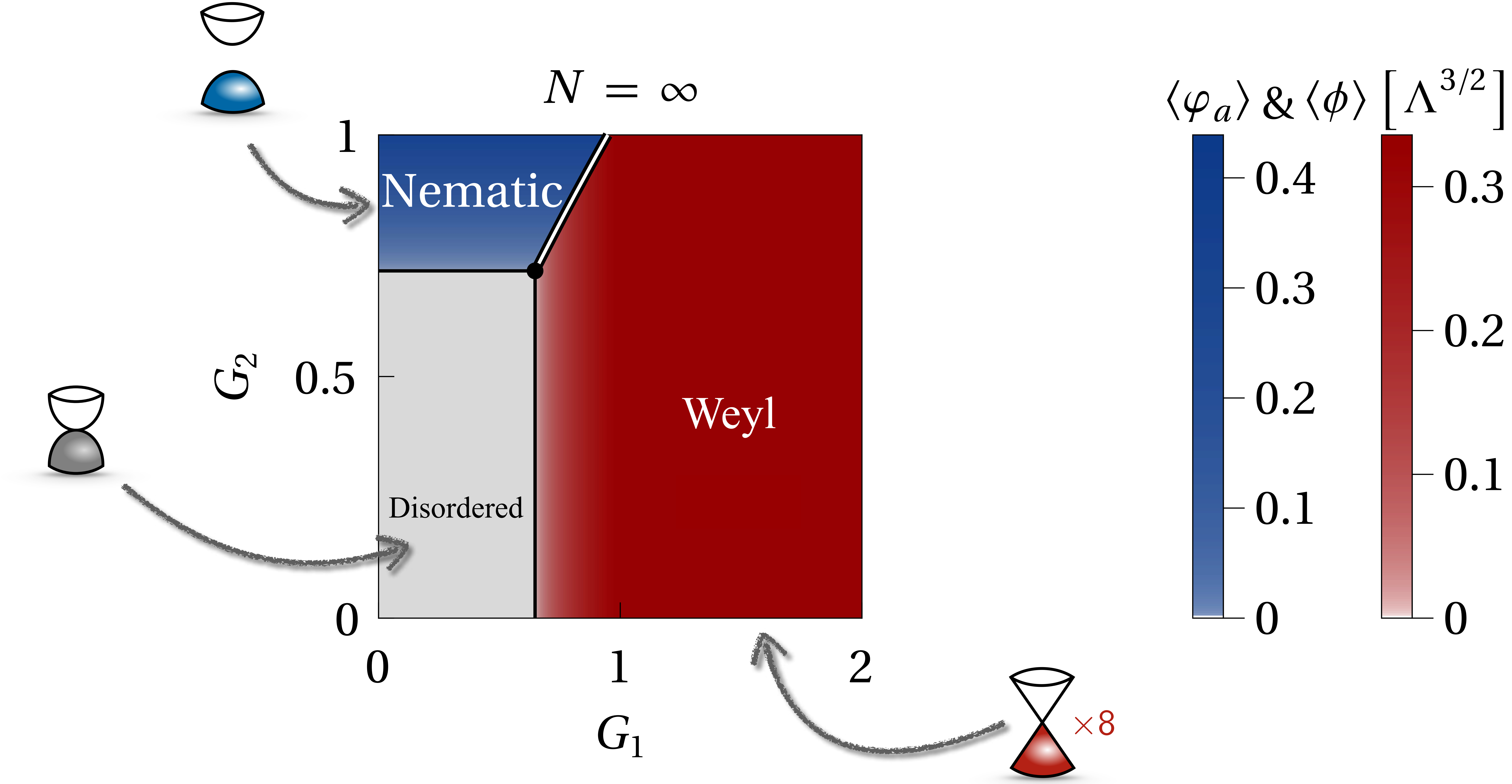
$$+ \frac{e^2}{8\pi N} \int \mathrm{d}^3\vec{y}\, \psi^\dagger(\vec{x})\psi(\vec{x}) \frac{1}{|\vec{x} - \vec{y}|} \psi^\dagger(\vec{y})\psi(\vec{y})$$



Coulomb

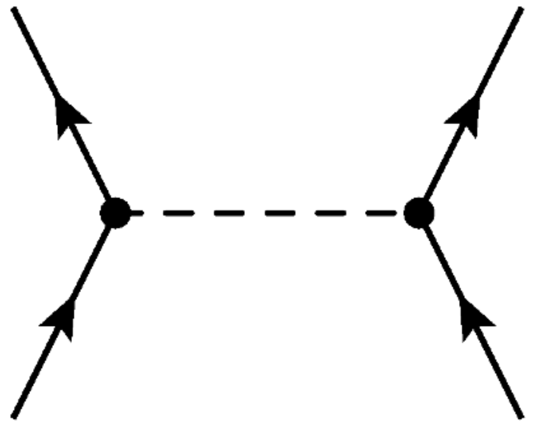
[Moser, LJ, *in preparation*]

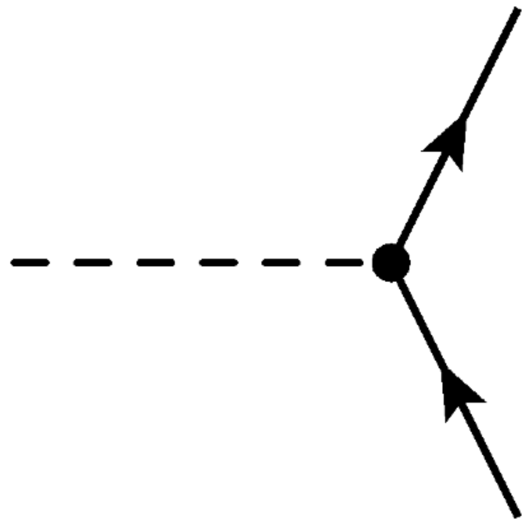
Mean-field theory



Partial bosonization

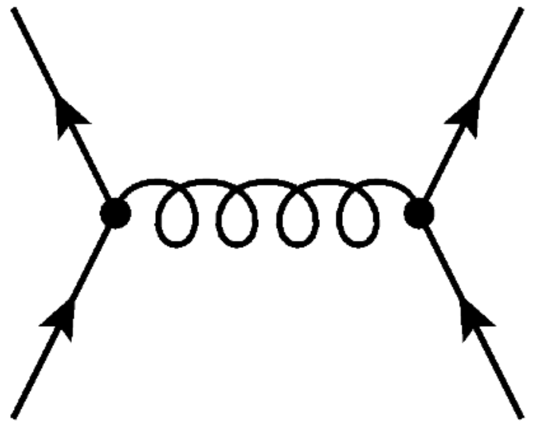
Weyl channel:

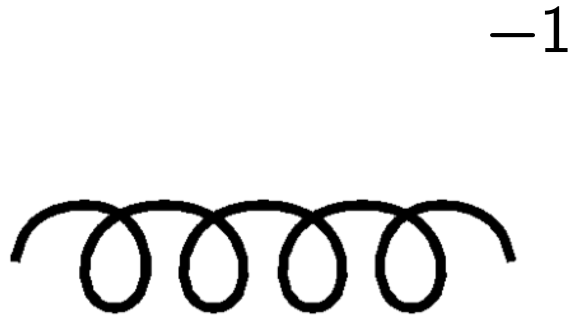


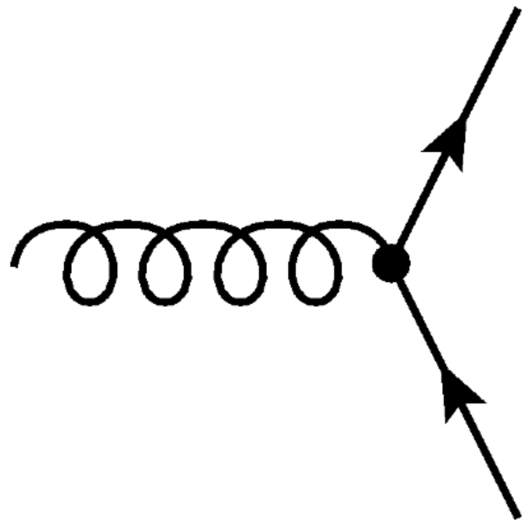


$$G_1(\psi^\dagger\gamma_{45}\psi)^2 \quad \mapsto \quad \frac{r_1}{2}\phi^2 \quad + \quad g_1\phi\psi^\dagger\gamma_{45}\psi$$

Nematic channel:



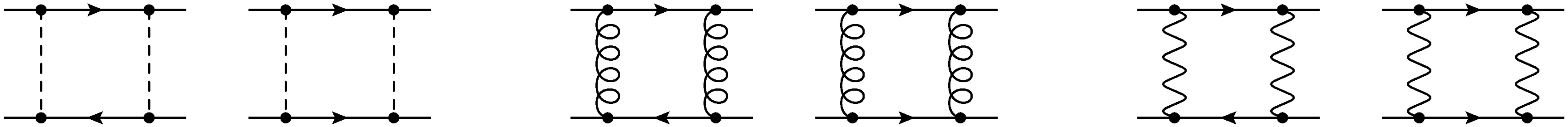




$$G_2(\psi^\dagger\gamma_a\psi)^2 \quad \mapsto \quad \frac{r_2}{2}\varphi_a^2 \quad + \quad g_2\varphi_a\psi^\dagger\gamma_a\psi$$

Dynamical bosonization

Fermion box diagrams:



Nematic channel:

New terms!

$$S_< = \int_{\vec{k},\omega} \frac{1}{2}(r_2 + \delta r_2)\varphi_a^2 + \int_{\vec{k}_1,\vec{k}_2,\omega_1,\omega_2} (g_2 + \delta g_2)\varphi_a \psi^\dagger \gamma_a \psi + \int_{\vec{k}_1,\vec{k}_2,\vec{k}_3,\omega_1,\omega_2,\omega_3} \delta G_2 (\psi^\dagger \gamma_a \psi)^2$$

Scale-dependent Hubbard-Stratonovich: $$\varphi_a \mapsto \varphi_a - \frac{\delta G_2}{g_2}(\psi^\dagger \gamma_a \psi)$$

... cancels 4-fermion term

Modified Yukawa-coupling flow: $$\left.\frac{\mathrm{d} g_2}{\mathrm{d} \ln b}\right|_{\text{dyn. bos.}} = -r_2 \frac{\partial \delta G_2}{\partial \ln b}$$

[Gies, Wetterich, PRD '02]
[Pawlowski, Ann. Phys. '07]
[Floerchinger, Wetterich, PLB '09]
...

Tree-level scaling

Charge: $[e^2] = 4 - d > 0$ relevant

4-fermion coupling: $[G_{1,2}] = 2 - d < 0$ irrelevant

Yukawa coupling: $[g_{1,2}] = 4 - d > 0$ relevant

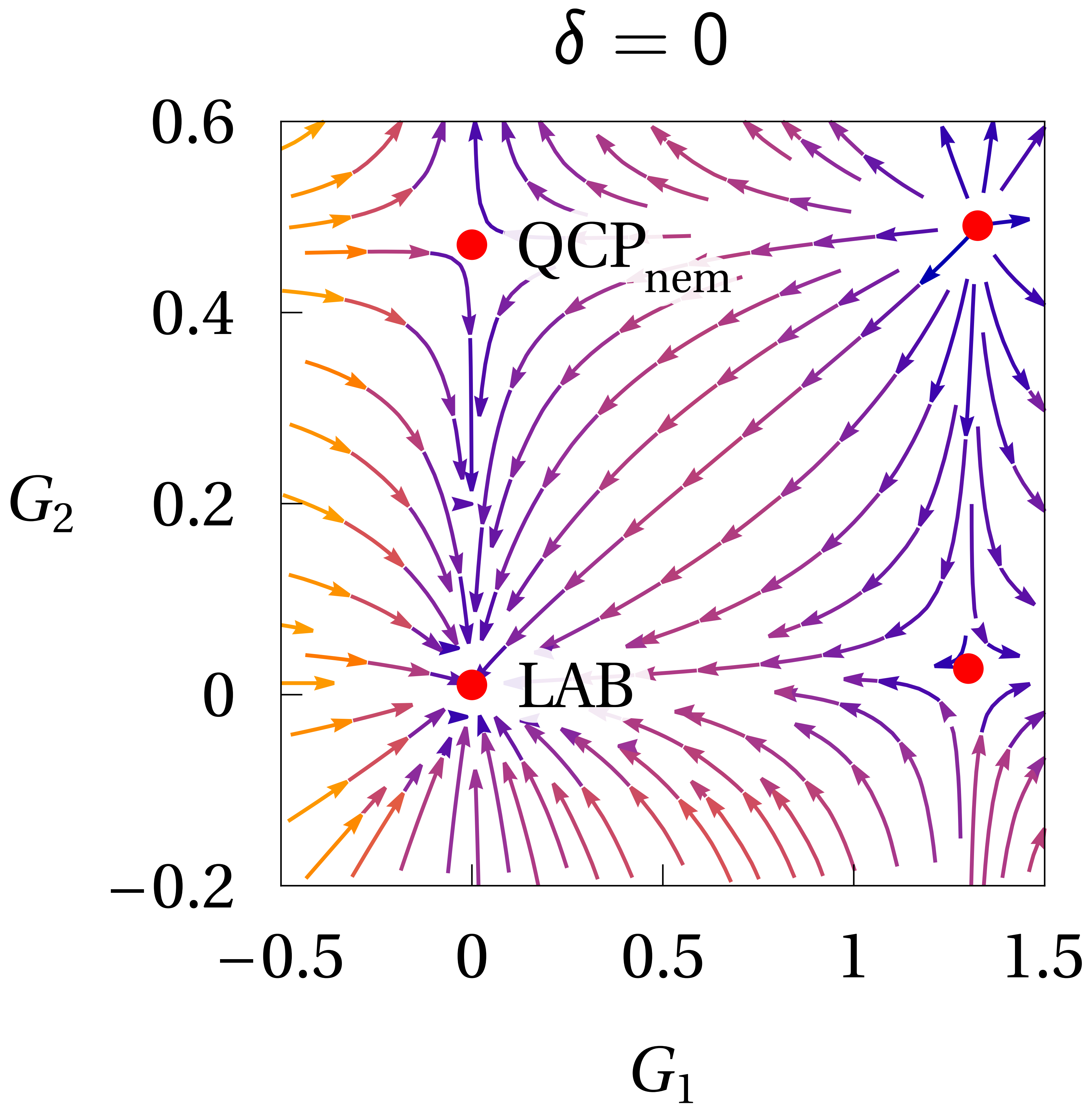
Order-parameter mass: $[r_{1,2}] = 2 > 0$ relevant

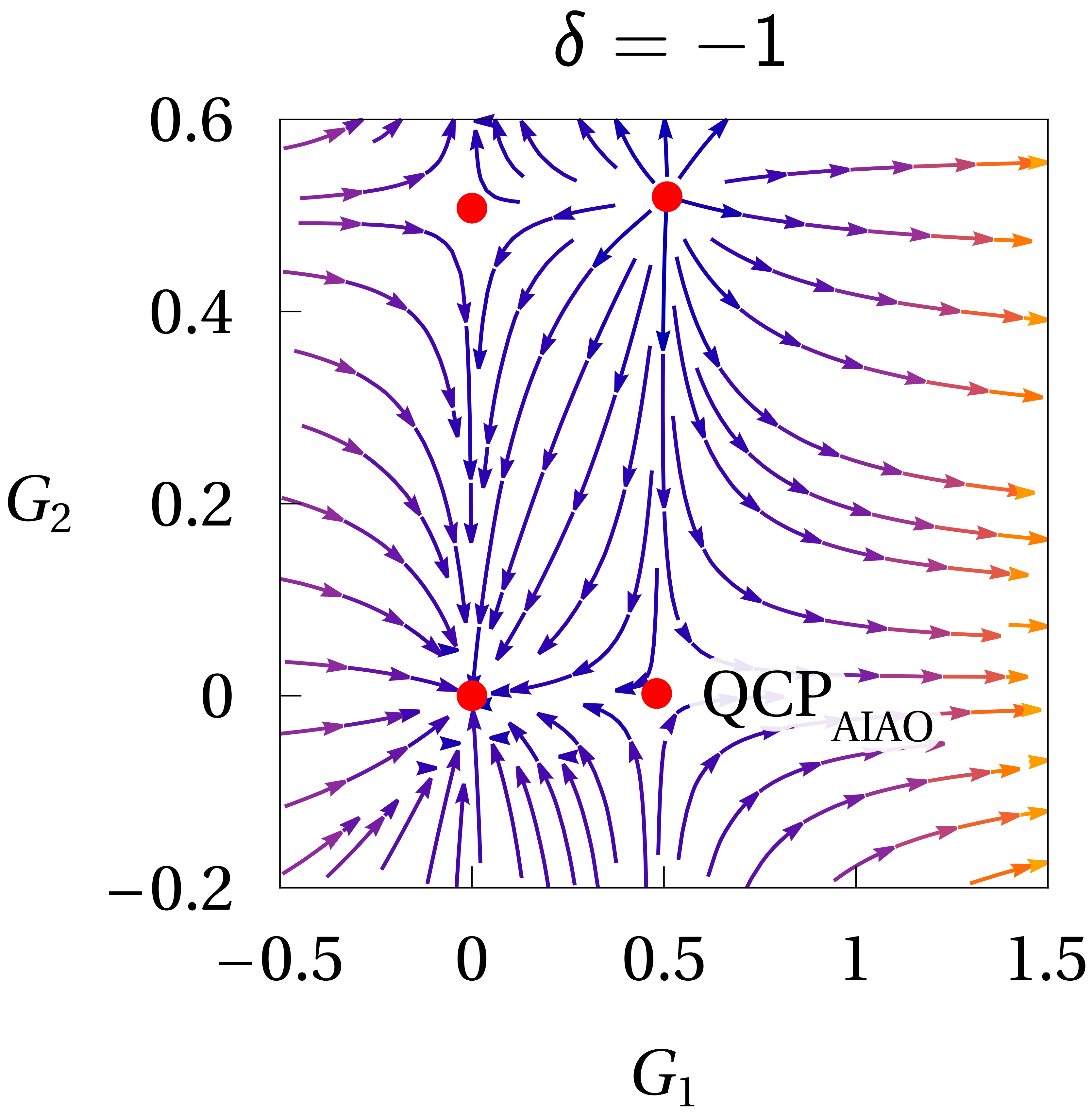
Order-parameter couplings:
$$[\lambda_n] = \frac{4 + (2-n)d}{2} = \begin{cases} (4-d)/2 > 0, & n = 3 \quad \text{relevant} \\ 2 - d < 0, & n = 4 \quad \text{irrelevant} \end{cases}$$

Anisotropy parameter: $[\delta] = 0$ marginal

Point-like limit $(r_{1,2} \to \infty)$

$N = 10$

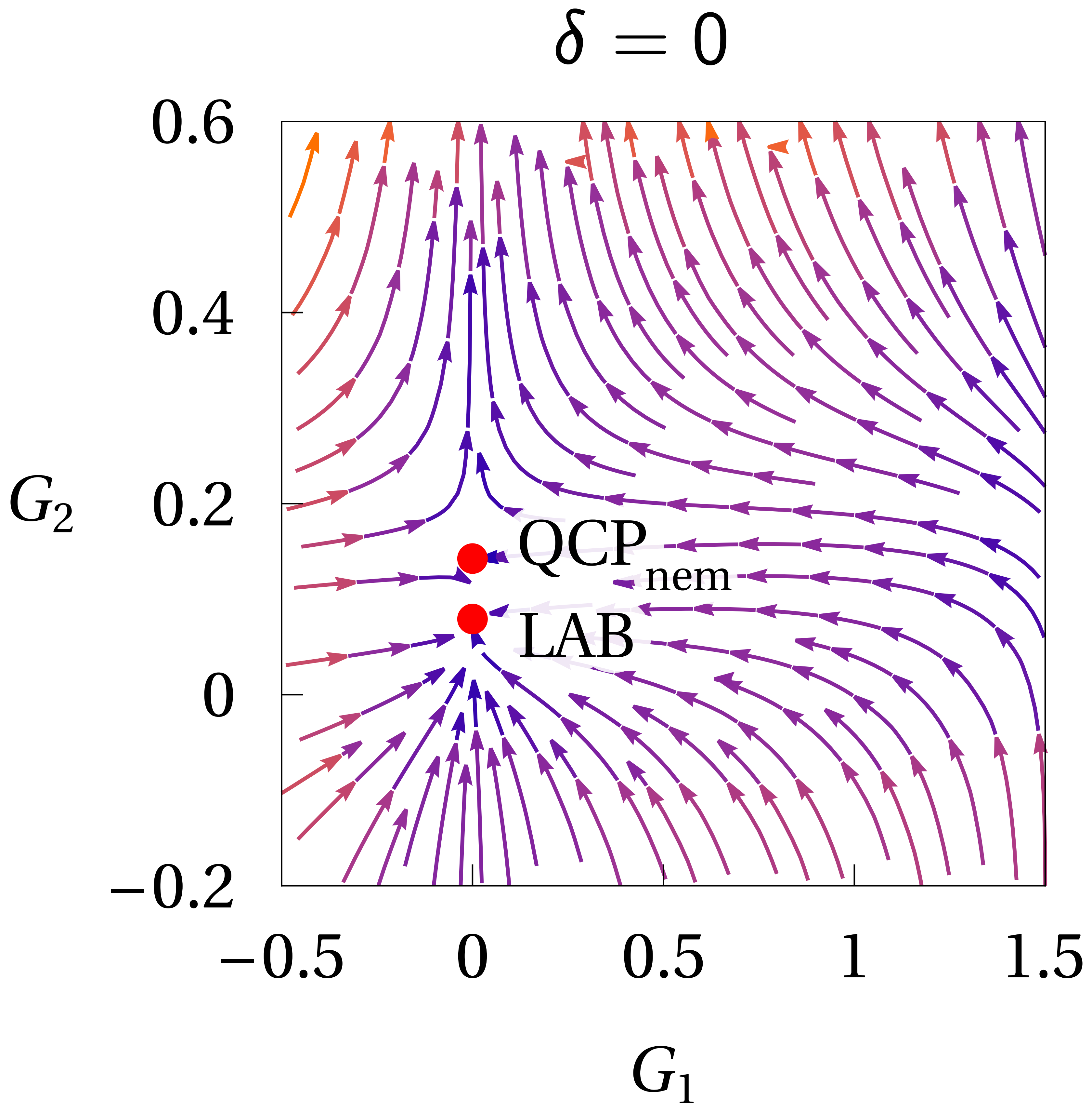


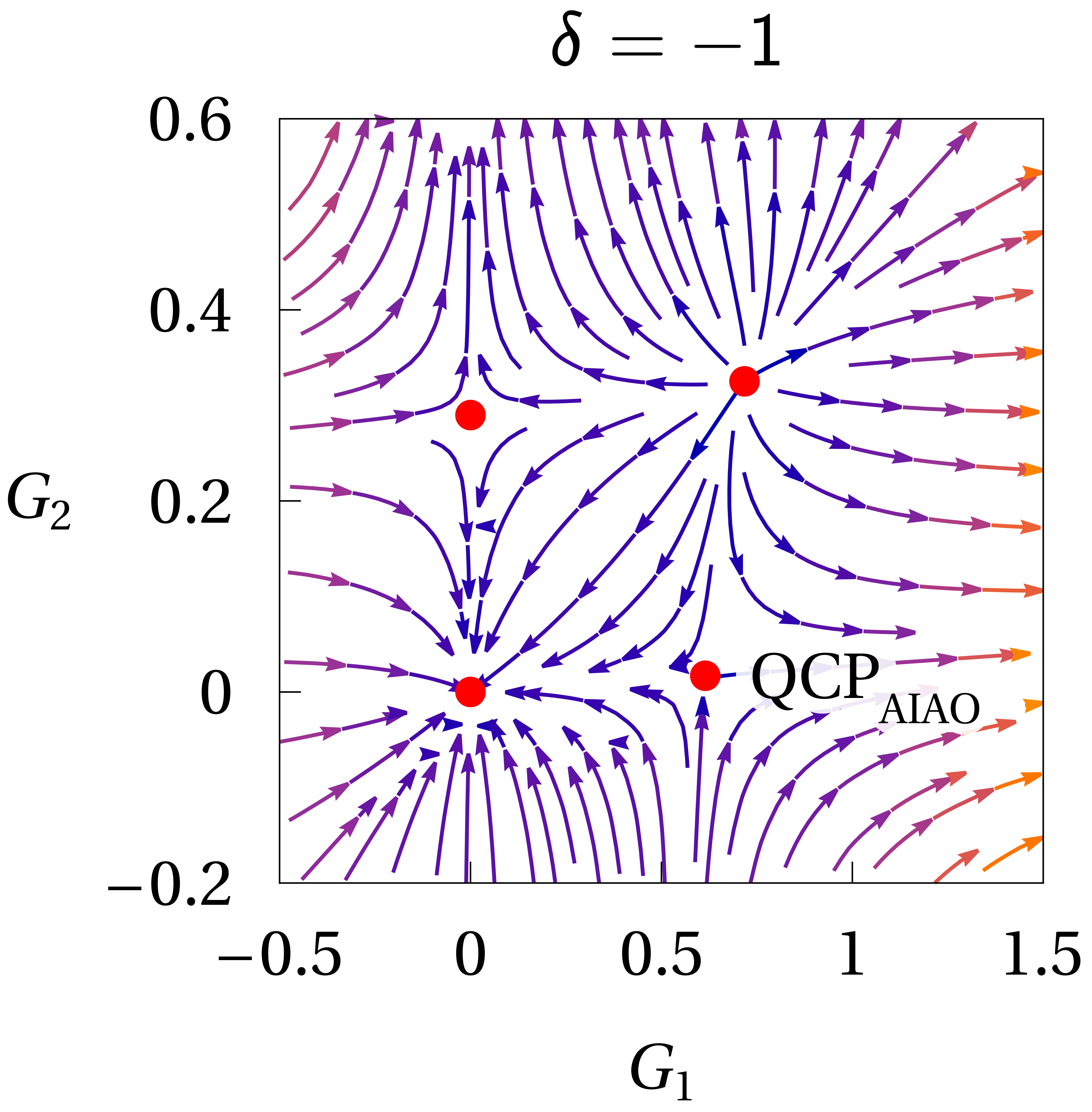


… for charge $e^2 = e_\star^2 > 0$

Point-like limit $(r_{1,2} \to \infty)$

$N = 2$

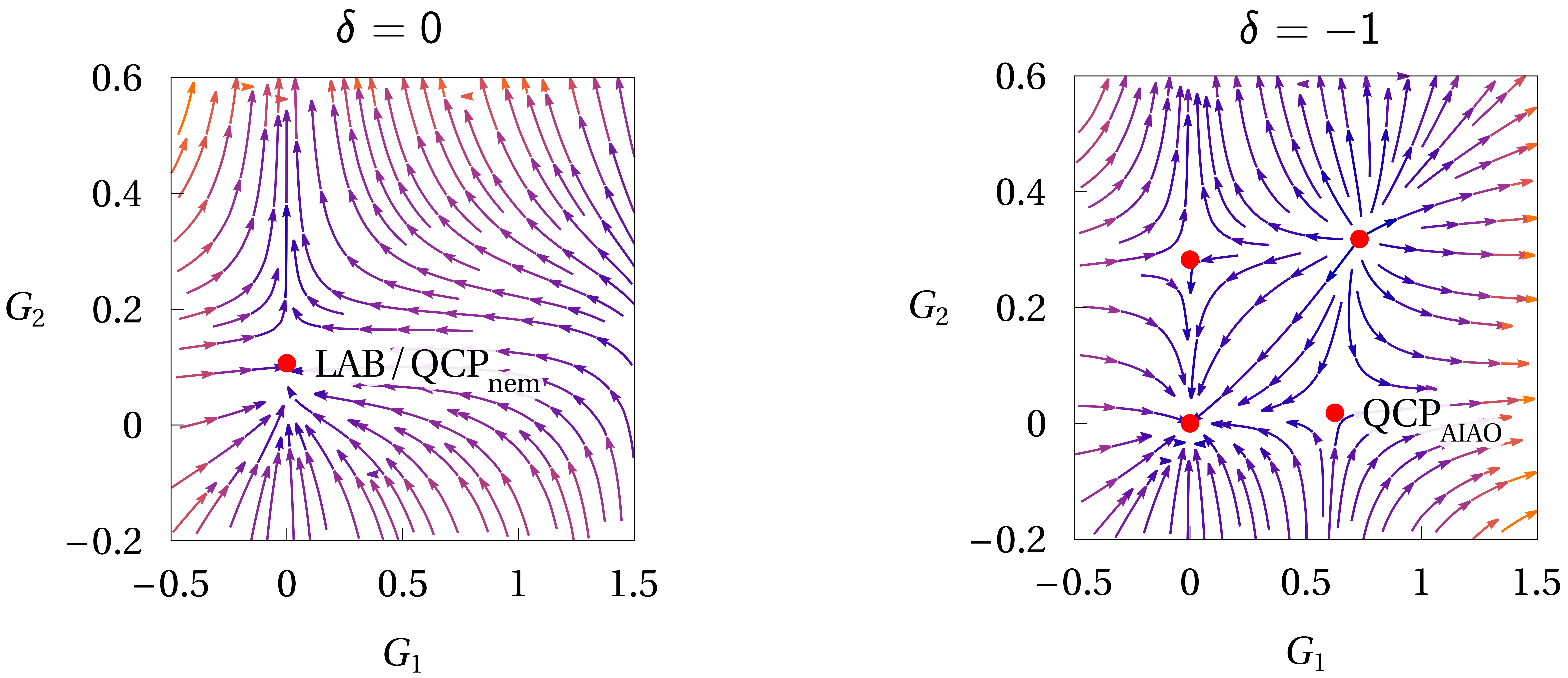




... for charge $e^2 = e_\star^2 > 0$

Point-like limit $(r_{1,2} \to \infty)$

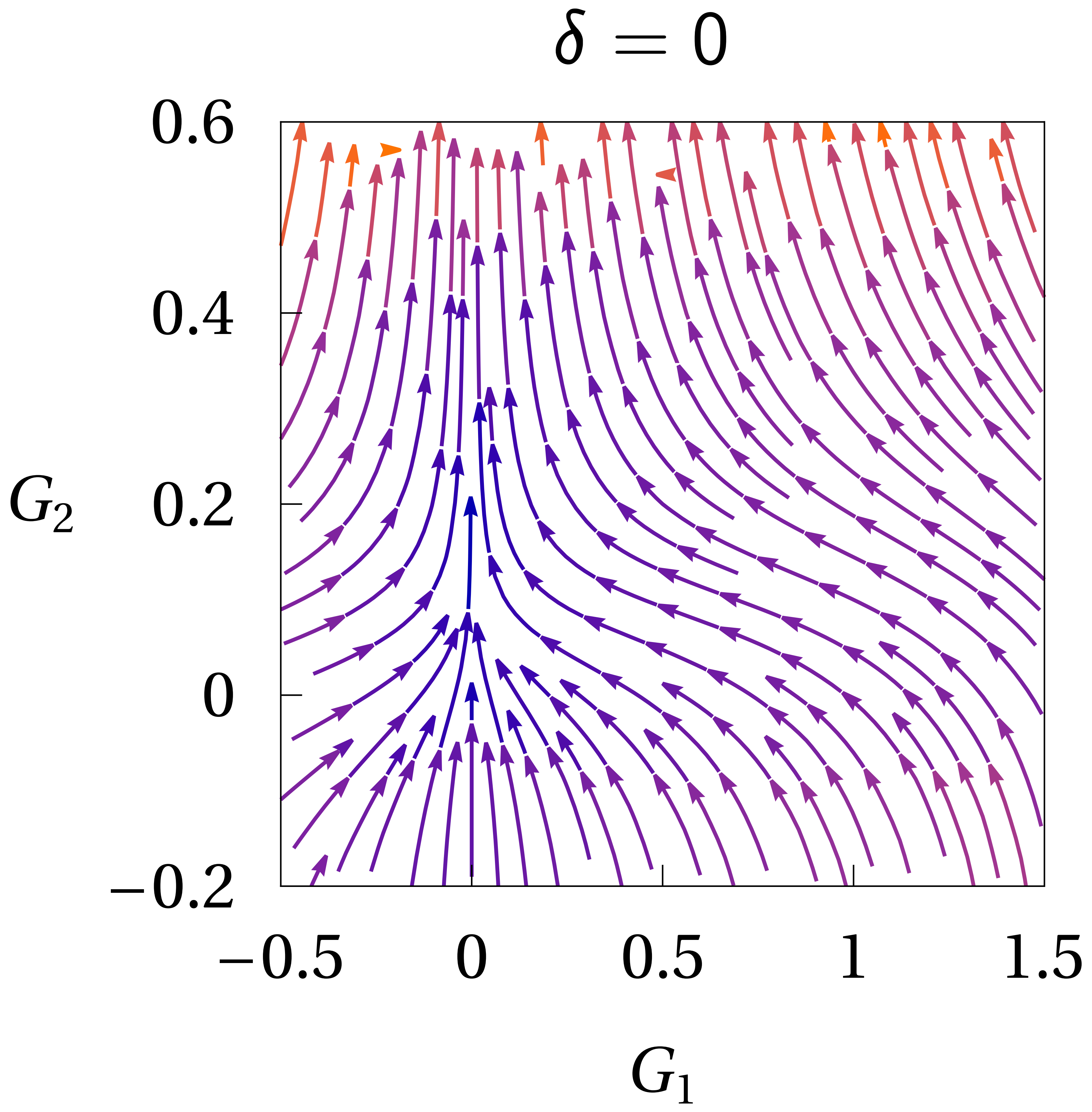
$N = 1.91$

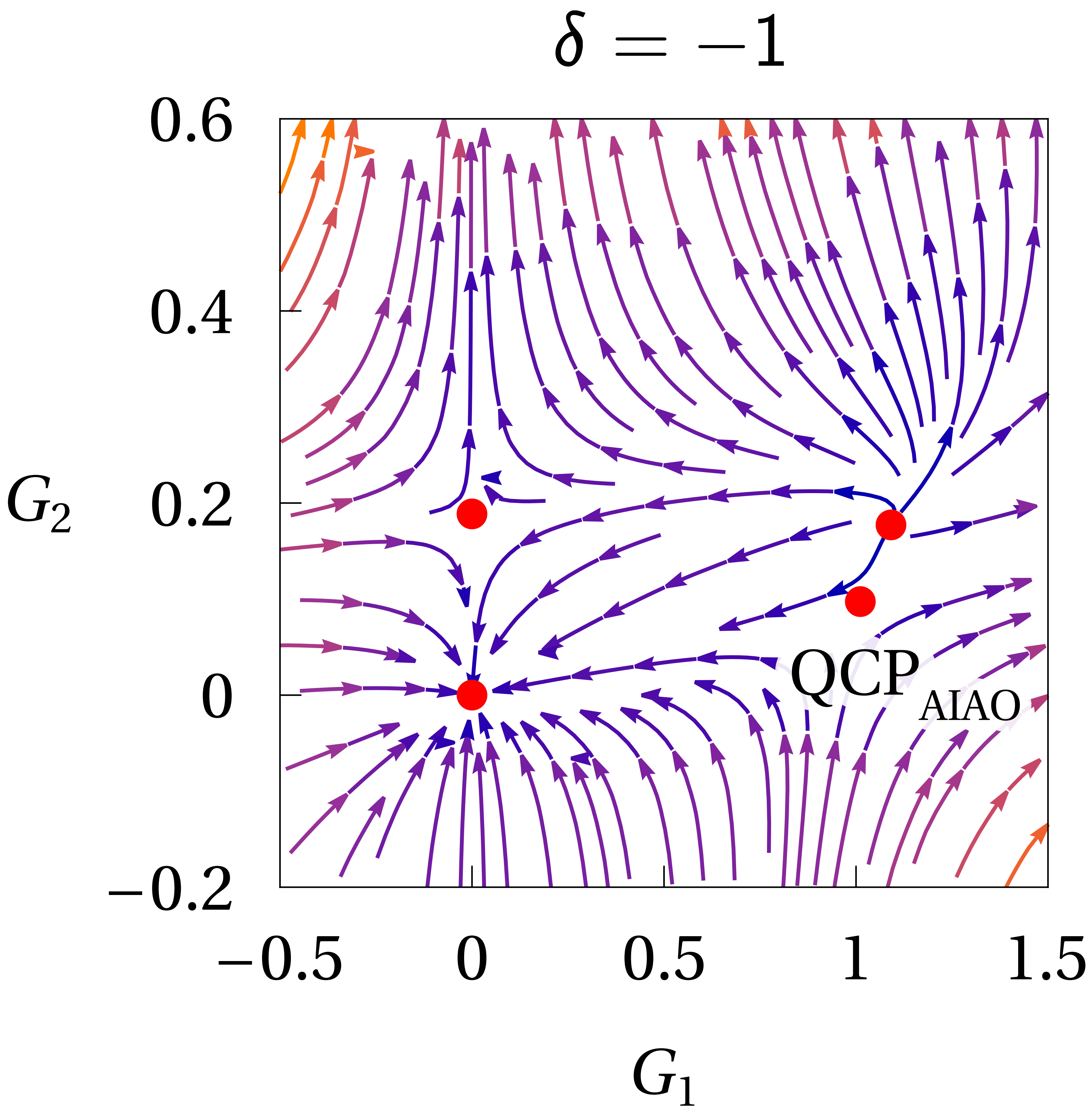


... for charge $e^2 = e_\star^2 > 0$

Point-like limit $(r_{1,2} \to \infty)$

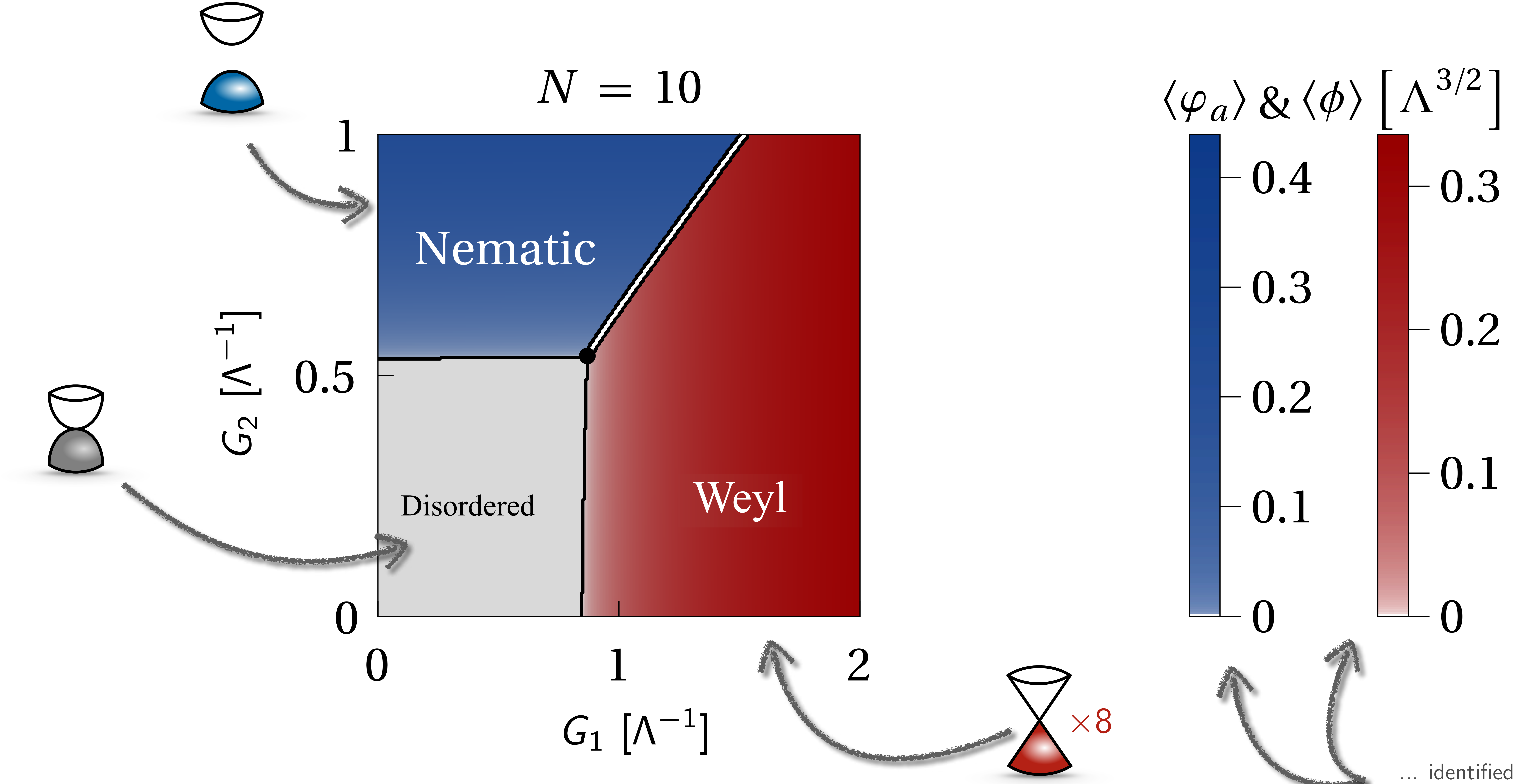
$N = 1$





... for charge $e^2 = e_\star^2 > 0$

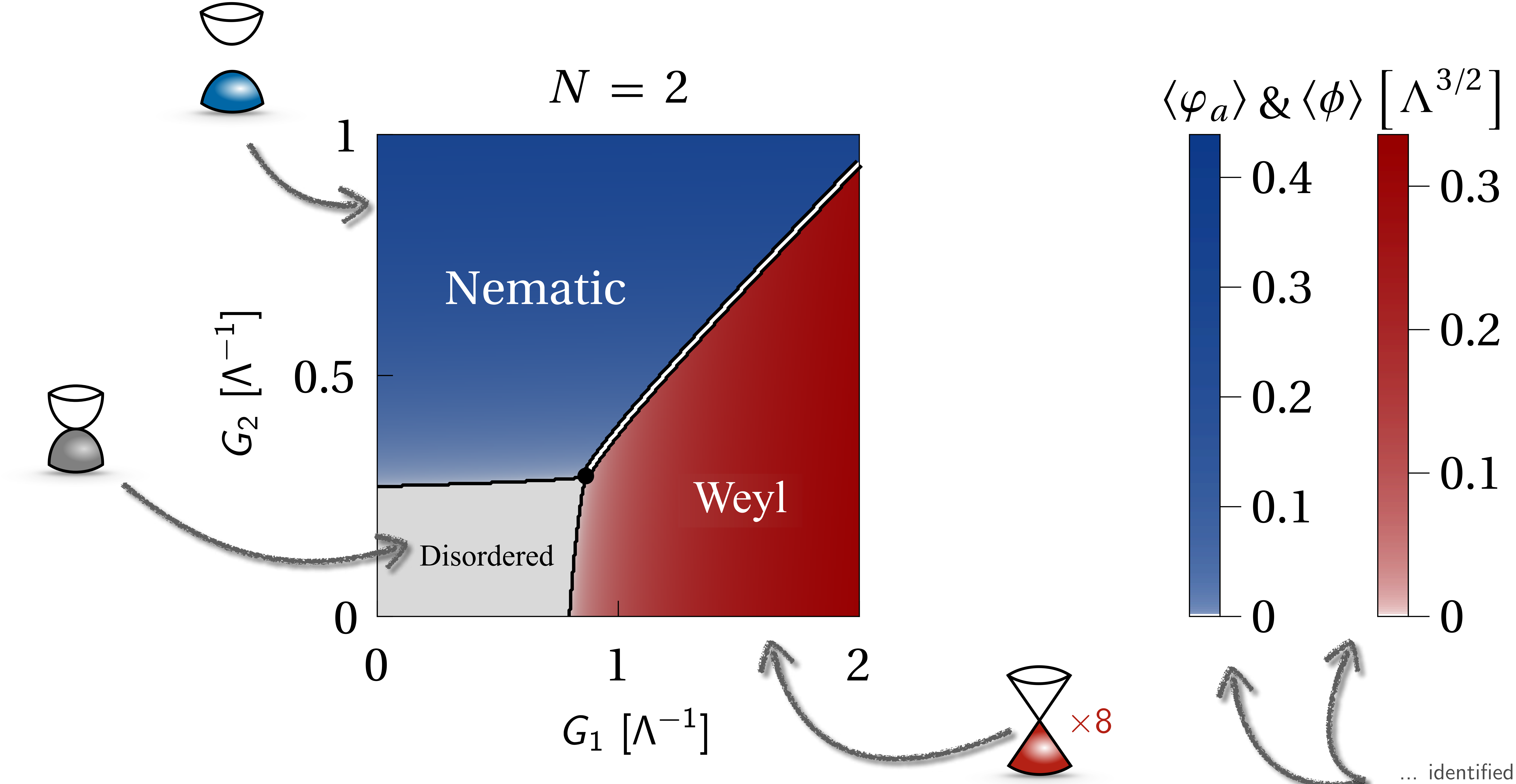
Dynamically bosonized RG flow



... identified by scale t_{IR}, at which r_1 or r_2 vanishes, $\langle\phi\rangle \sim \exp(-2t_{\mathrm{IR}})$

... with $G_{1,2} = g_{1,2}^2/r_{1,2}$ in units of Λ^{-1}

Dynamically bosonized RG flow



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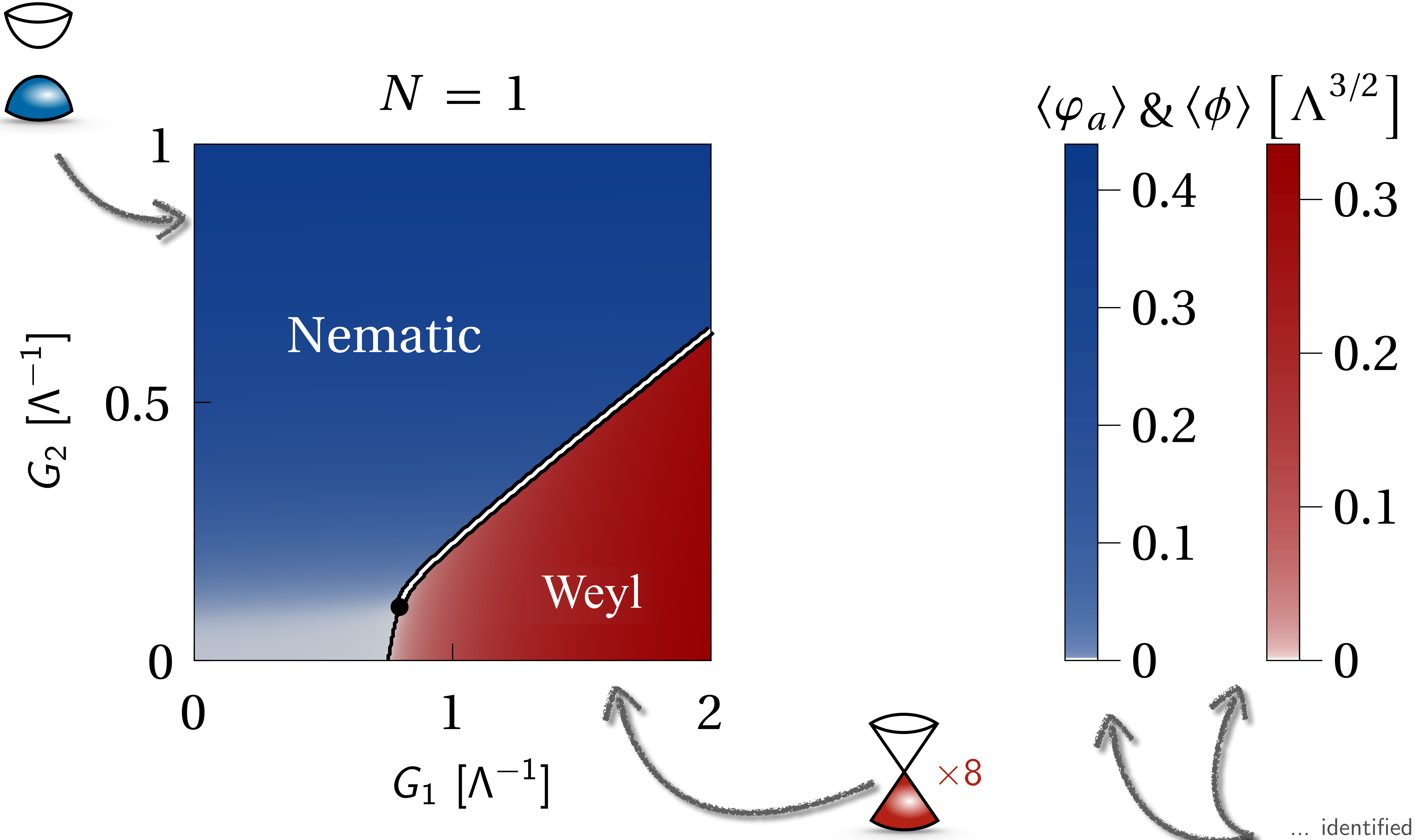
Dynamically bosonized RG flow

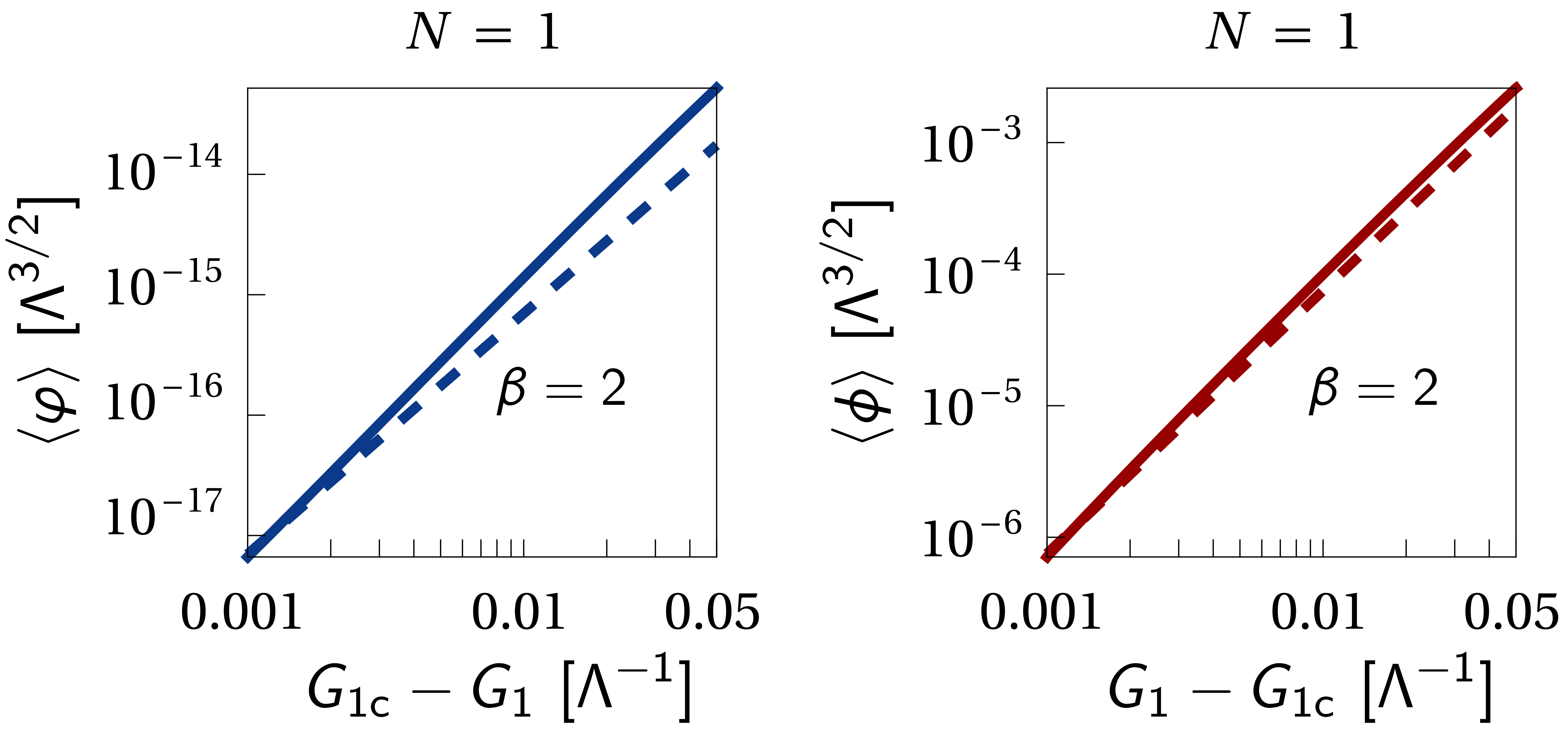


... identified by scale t_{IR}, at which r_1 or r_2 vanishes, $\langle\phi\rangle \sim \exp(-2t_{\mathrm{IR}})$

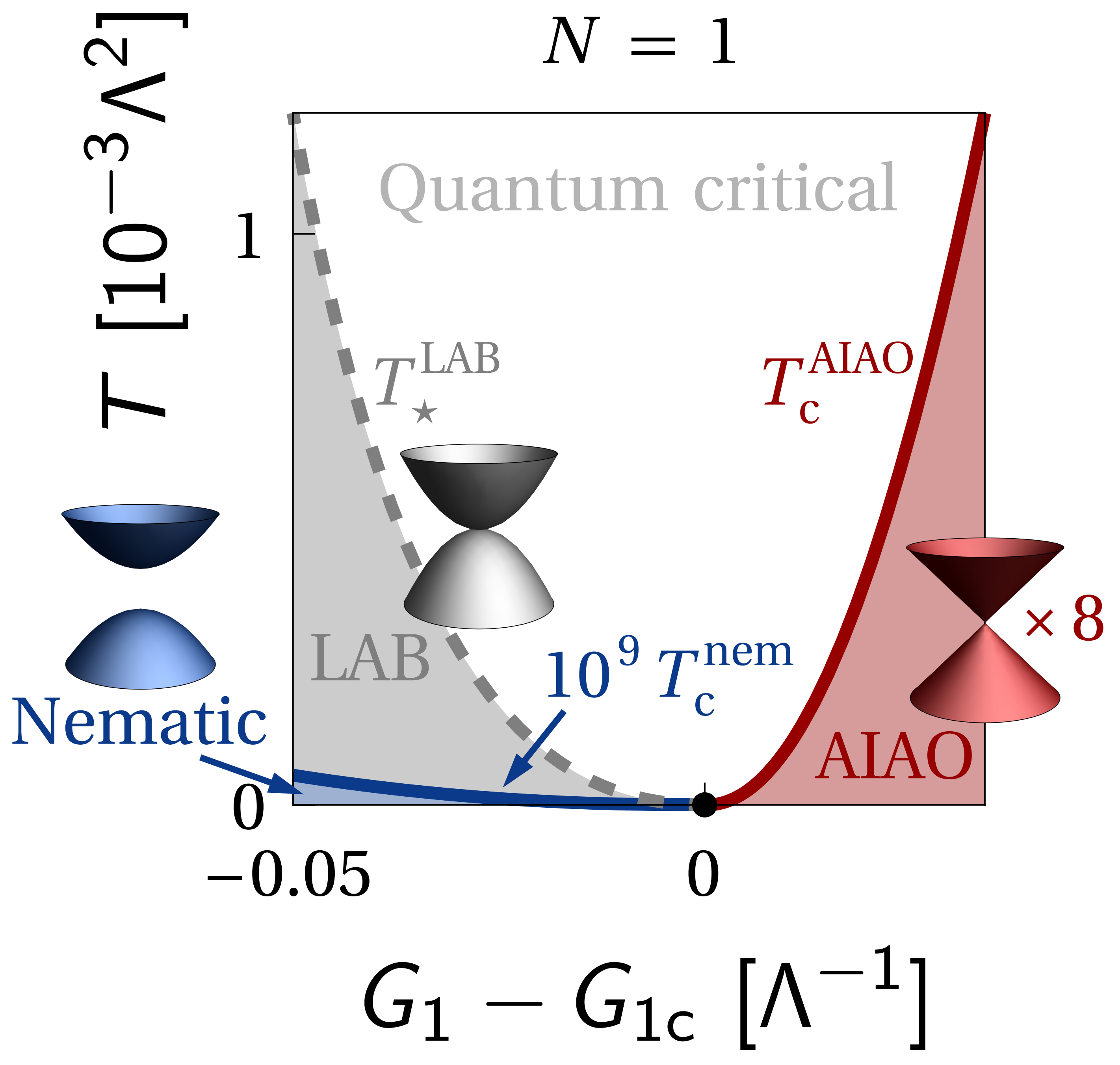
... with $G_{1,2} = g_{1,2}^2/r_{1,2}$ in units of Λ^{-1}

Critical behavior

Order parameters:



Finite-temperature phase diagram:



... asymmetry in energy scales!

[Moser, LJ, *in preparation*]

Outline

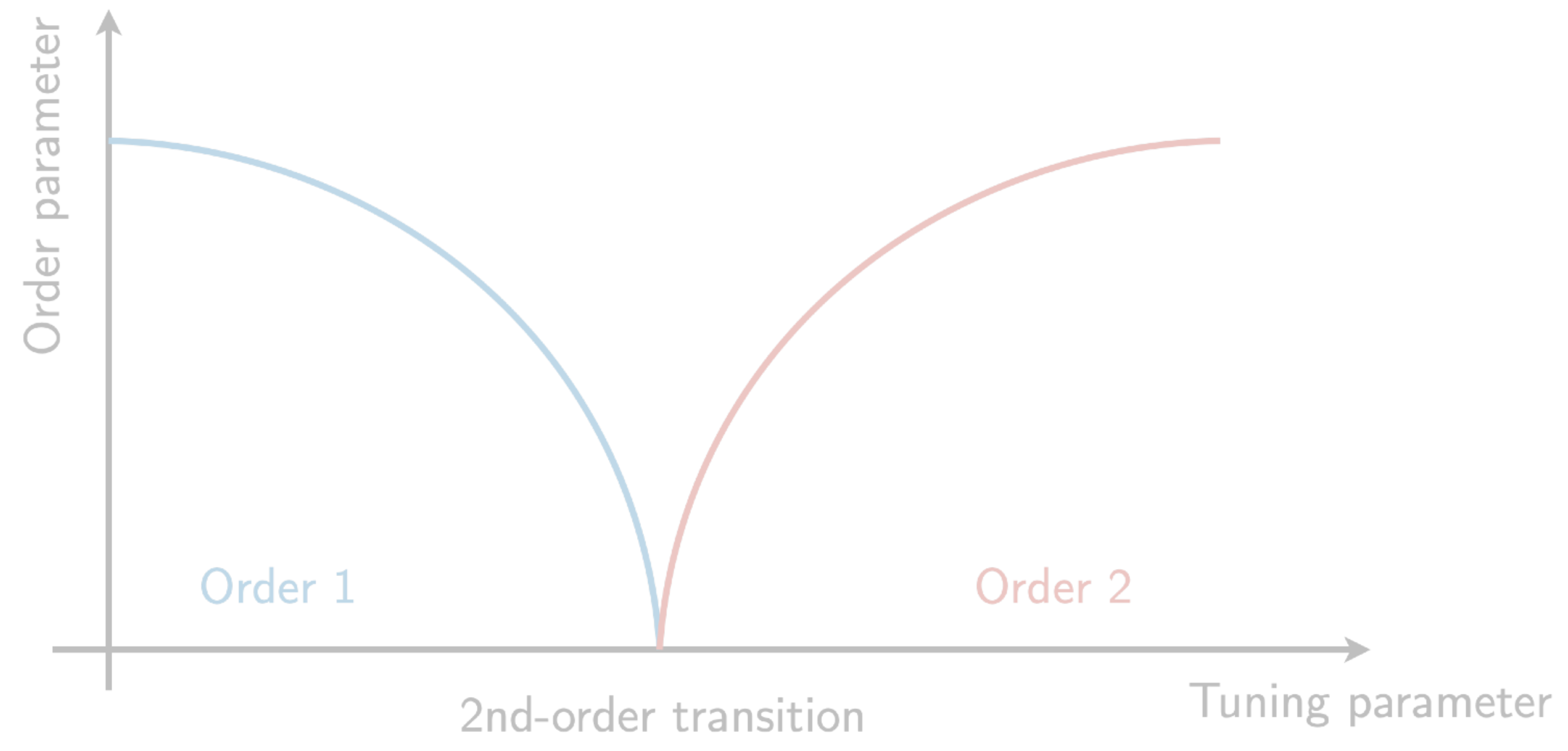
(1) Introduction

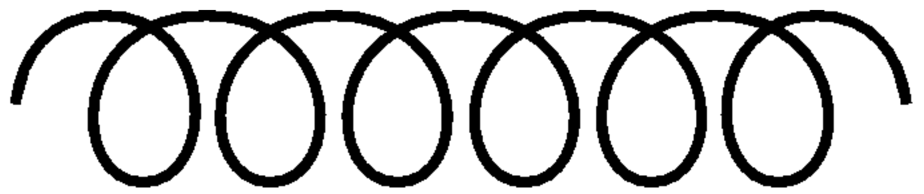
(2) Continuous order-to-order transition from fixed-point annihilation

(3) Examples

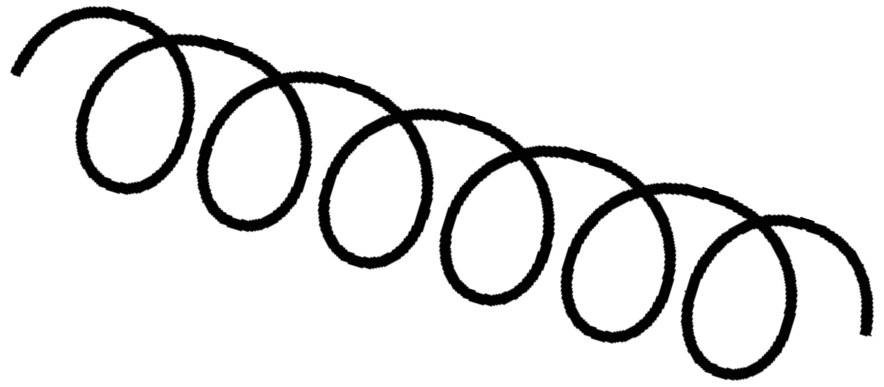
- Luttinger semimetals
- QCD$_4$ plus 4-fermion interactions
- Spin-boson models

(4) Conclusions





QCD$_4$ plus 4-fermion interactions



Lagrangian:

$$i\bar{\psi}\gamma^\mu D_\mu \psi + \frac{1}{4} F_z^{\mu\nu} F_{\mu\nu}^z + \frac{1}{2} \sum_{\alpha=1}^{2} G_\alpha \mathcal{O}_\alpha$$

in Veneziano limit $N_{\text{color}}, N_{\text{flavor}} \to \infty$ with fixed $x = N_{\text{flavor}}/N_{\text{color}}$

$$\mathcal{O}_1 = \left(\bar{\psi}^a \gamma_\mu \psi^b\right)^2 + \left(\bar{\psi}^a \gamma_\mu \gamma_5 \psi^b\right)^2$$

$$\mathcal{O}_2 = \left(\bar{\psi}^a \psi^b\right)^2 - \left(\bar{\psi}^a \gamma_5 \psi^b\right)^2$$

[Gies, Jaeckel, Wetterich, PRD '04]
[Gies, Jaeckel, EPJC '06]
[Braun, JPG '12]
[Gukov, NPB '17]
...

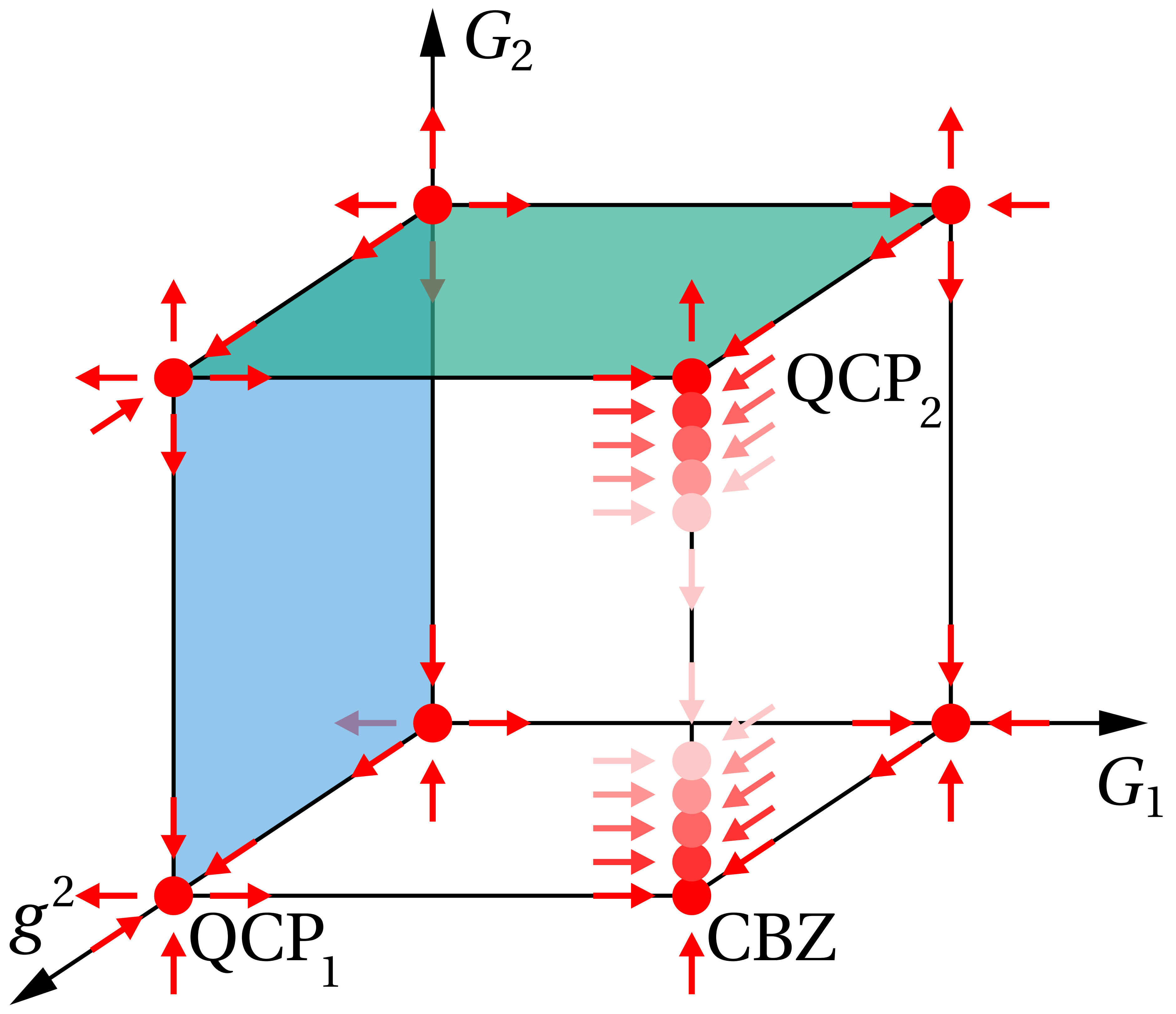
Tree-level scaling:

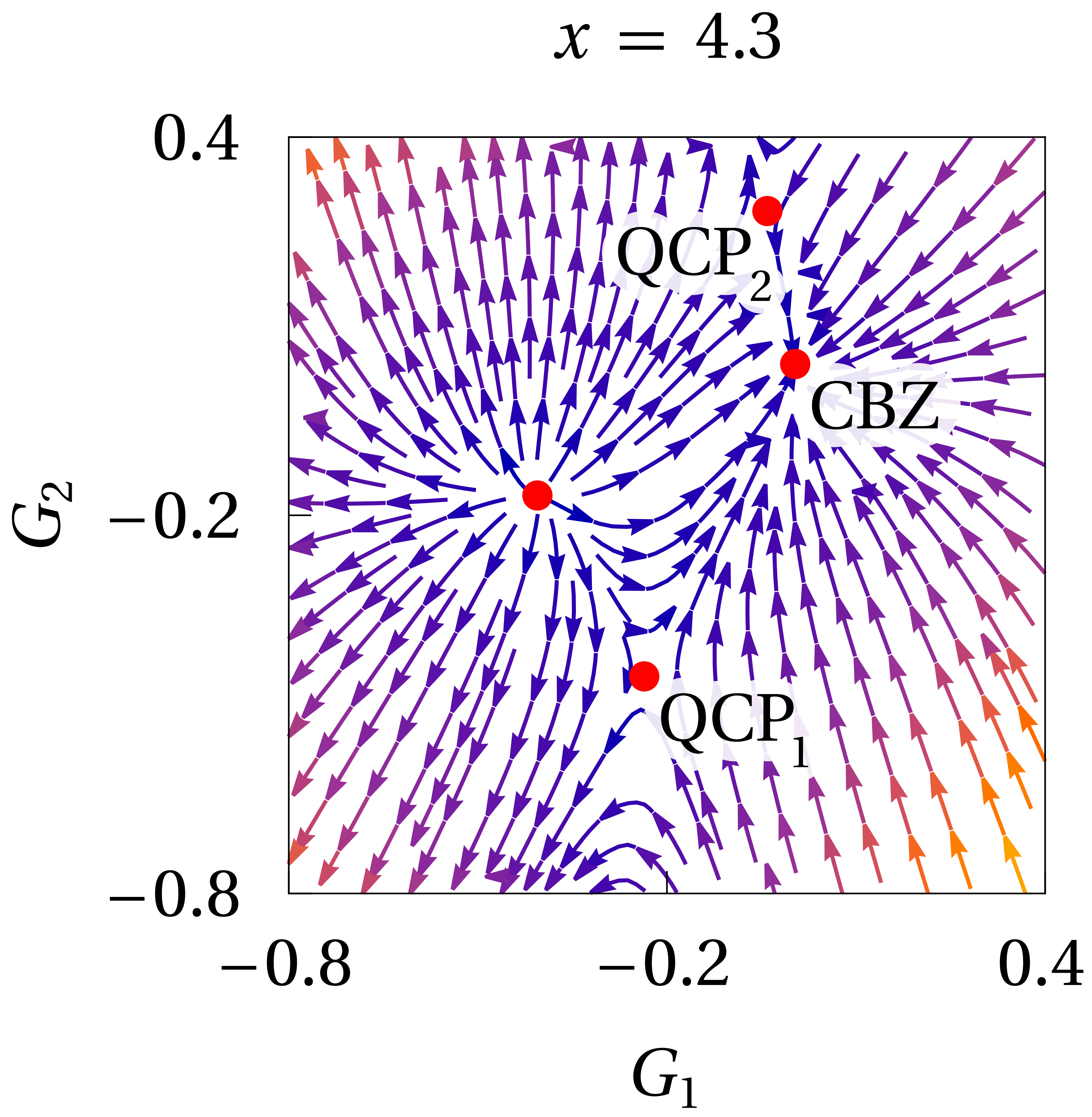
Gauge coupling $[g] = 0$ marginal

... marginally relevant (asymptotic freedom)

4-fermion couplings $[G_{1,2}] = -2$ irrelevant

RG flow diagram





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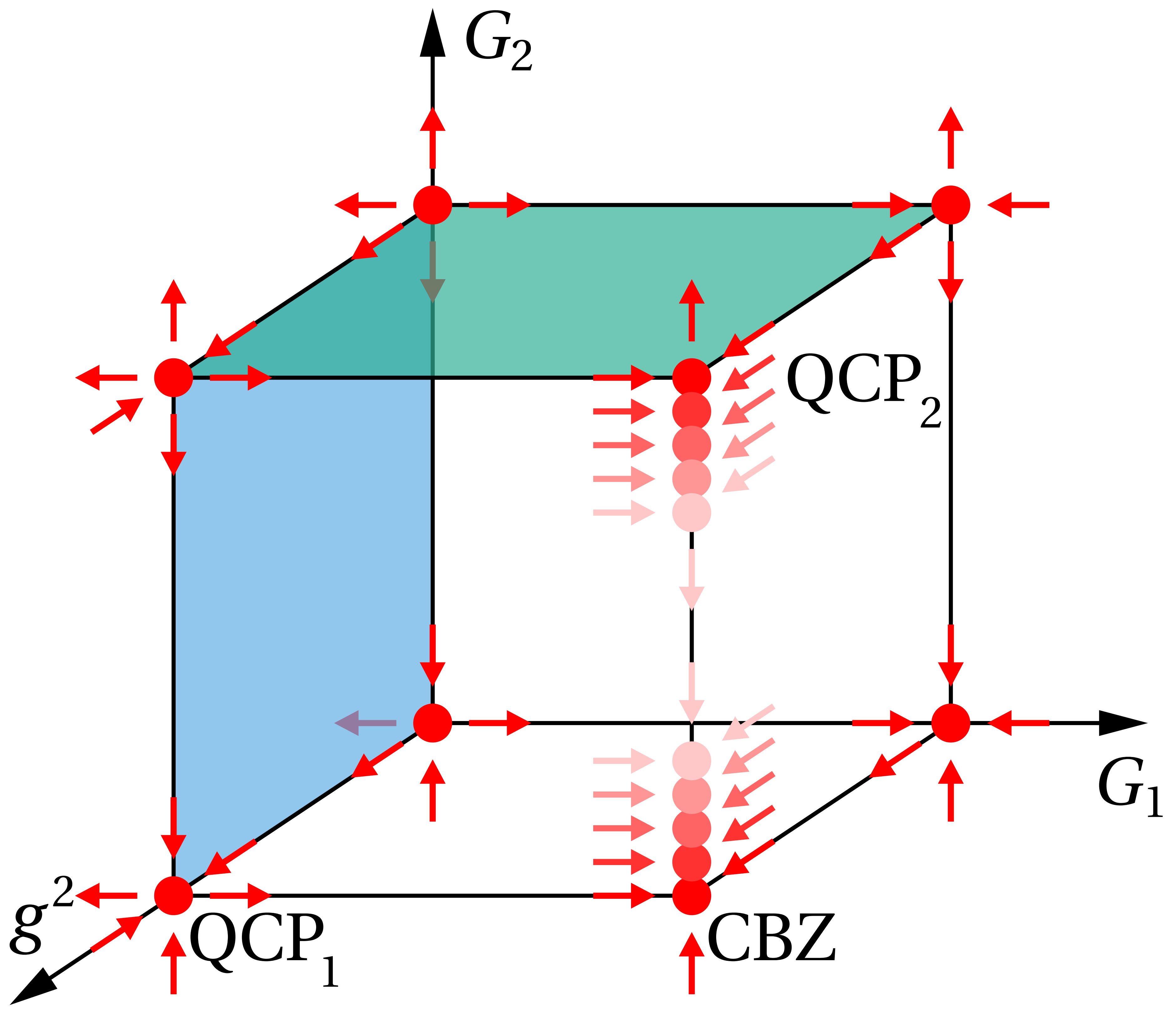
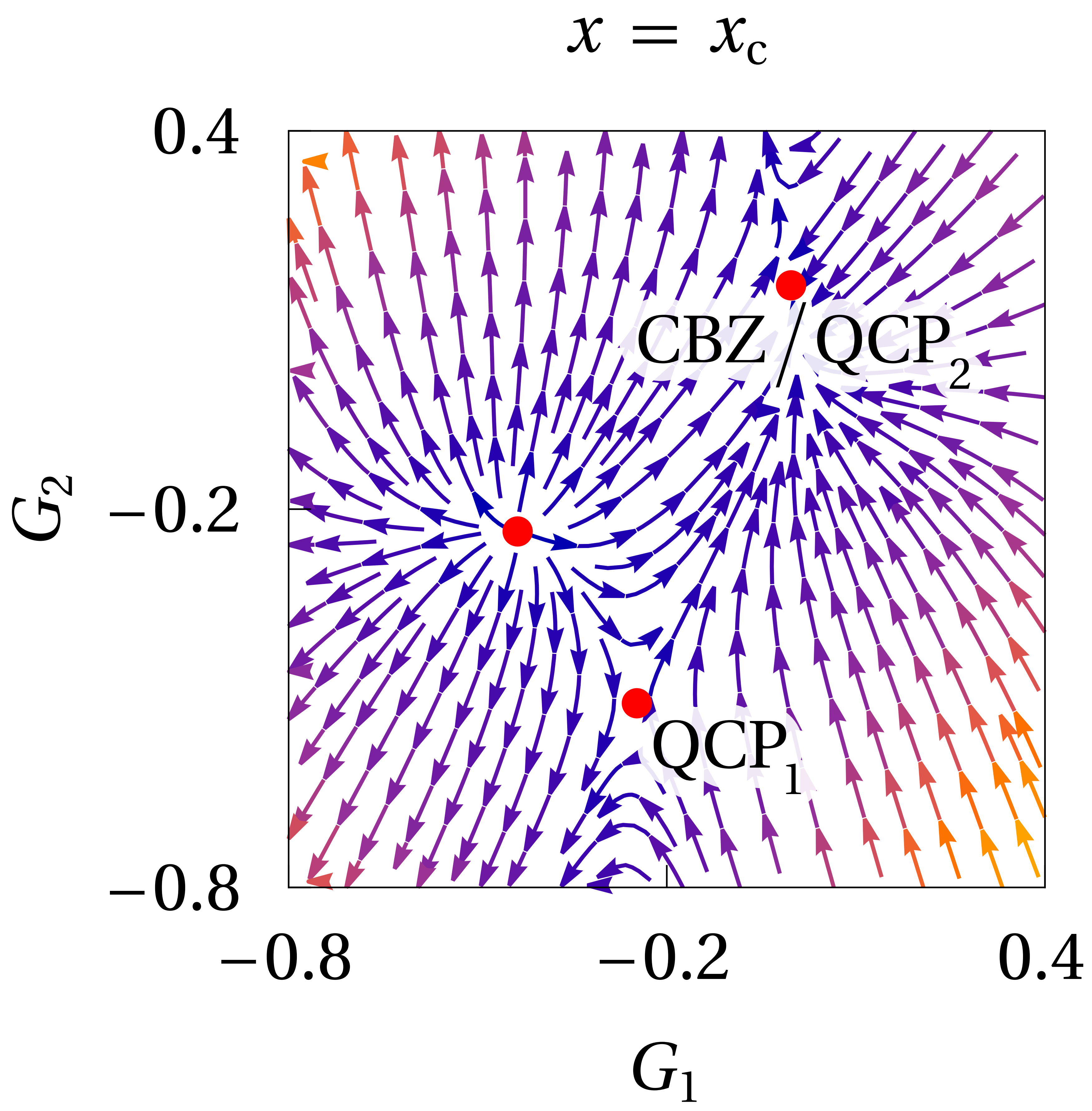
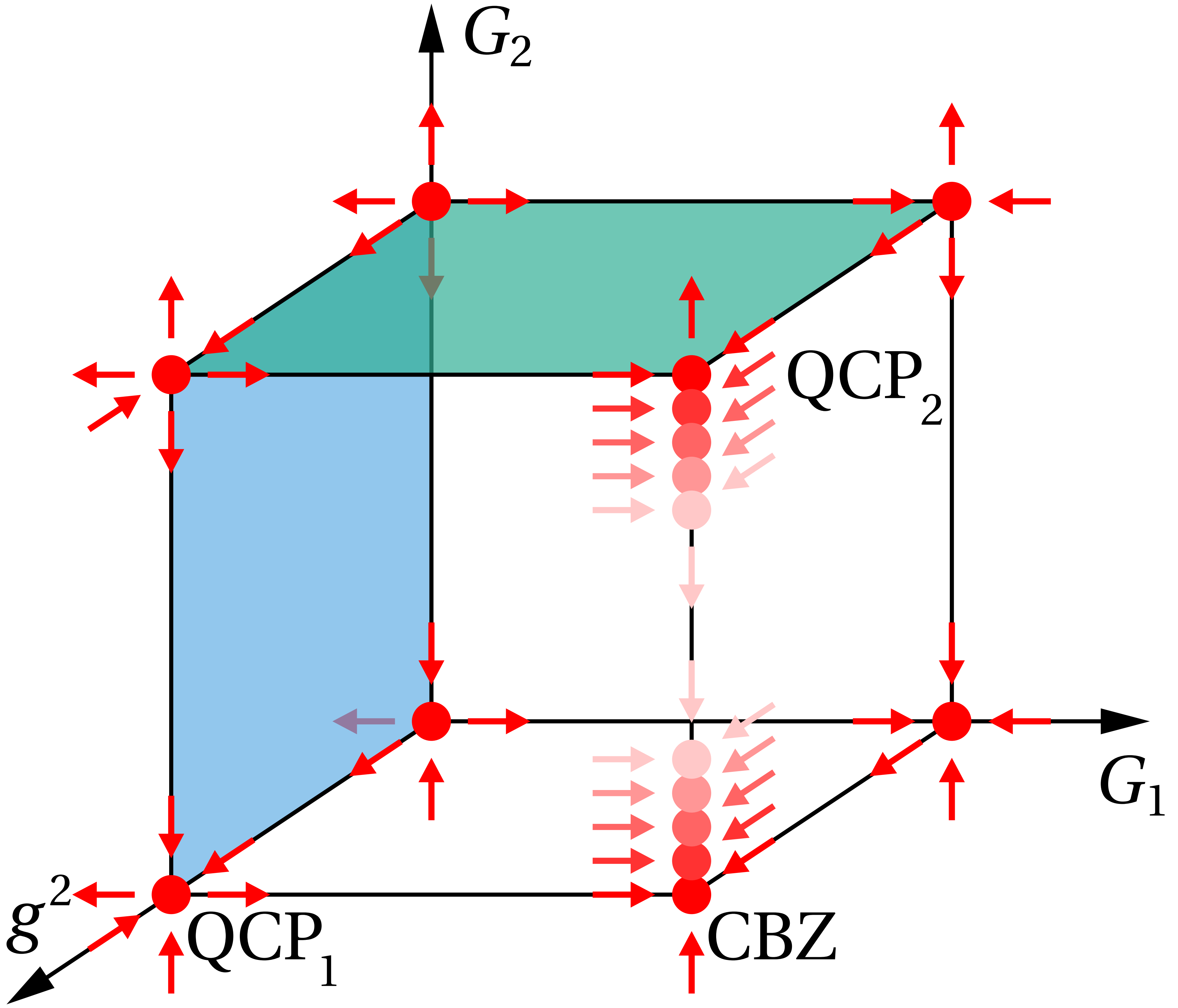
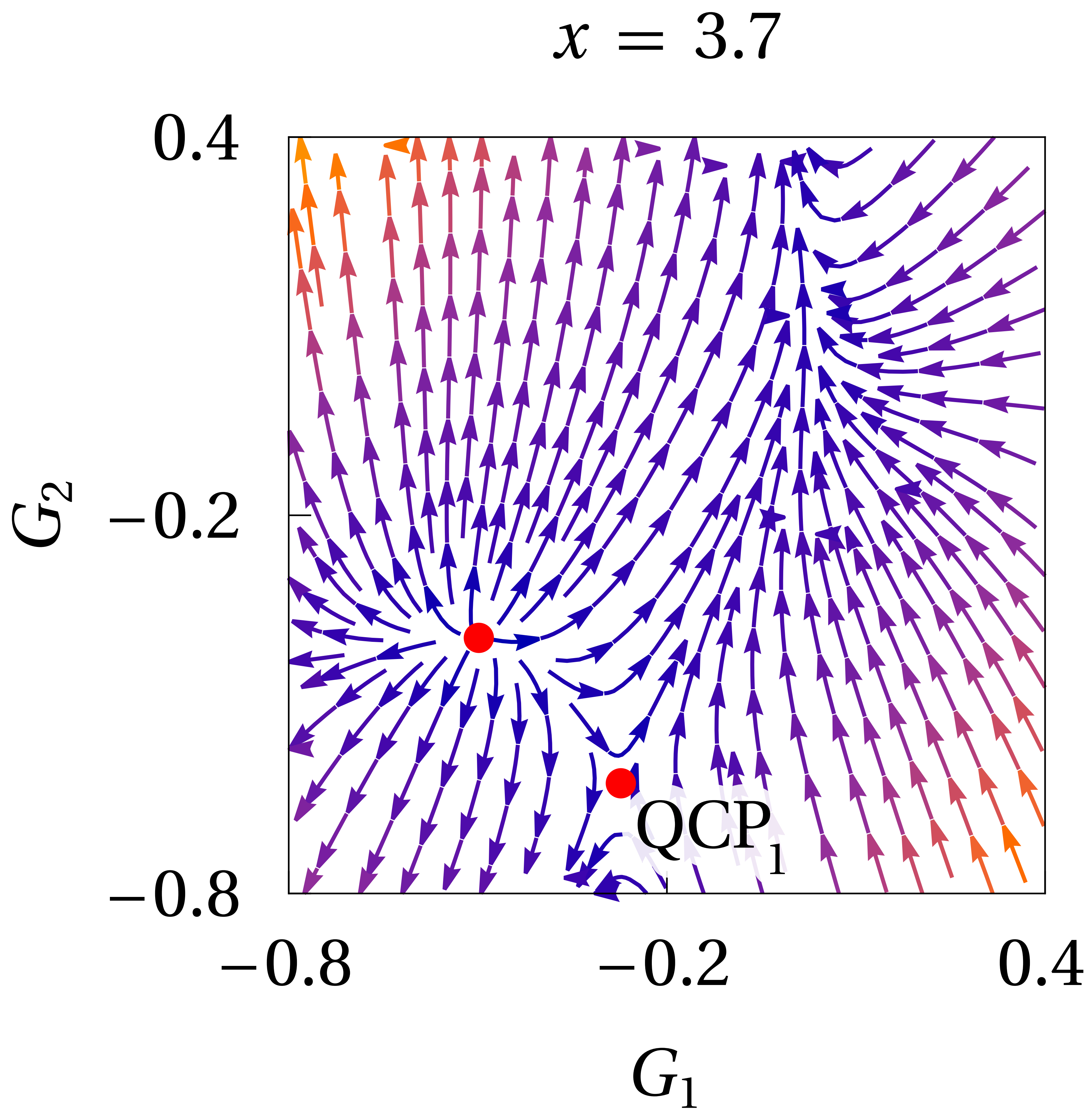




RG flow diagram





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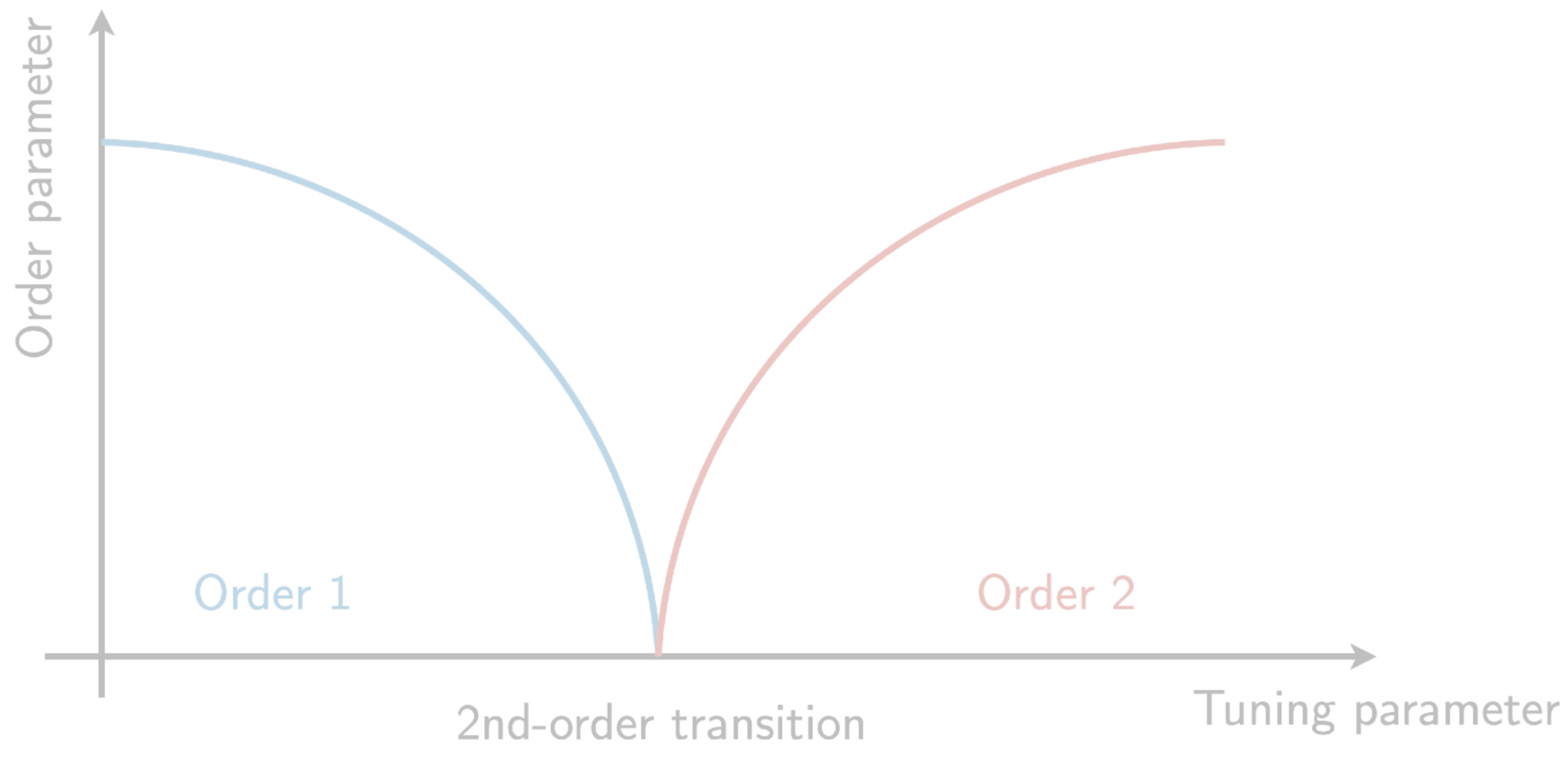
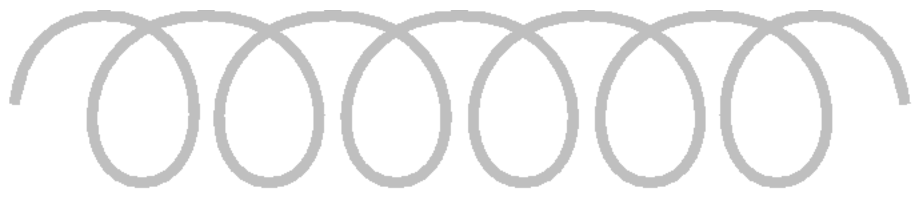
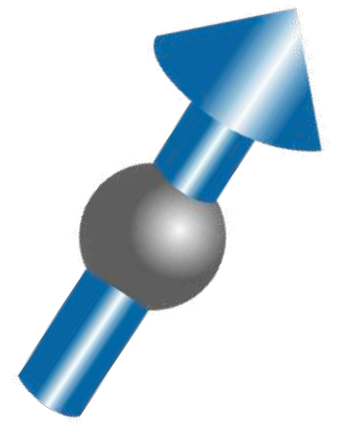
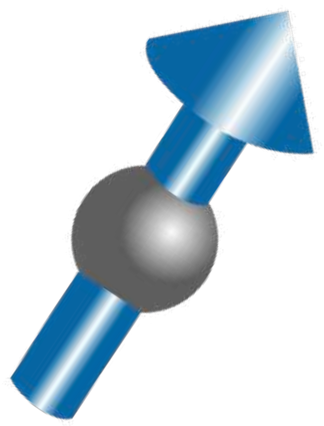






Spin-boson model



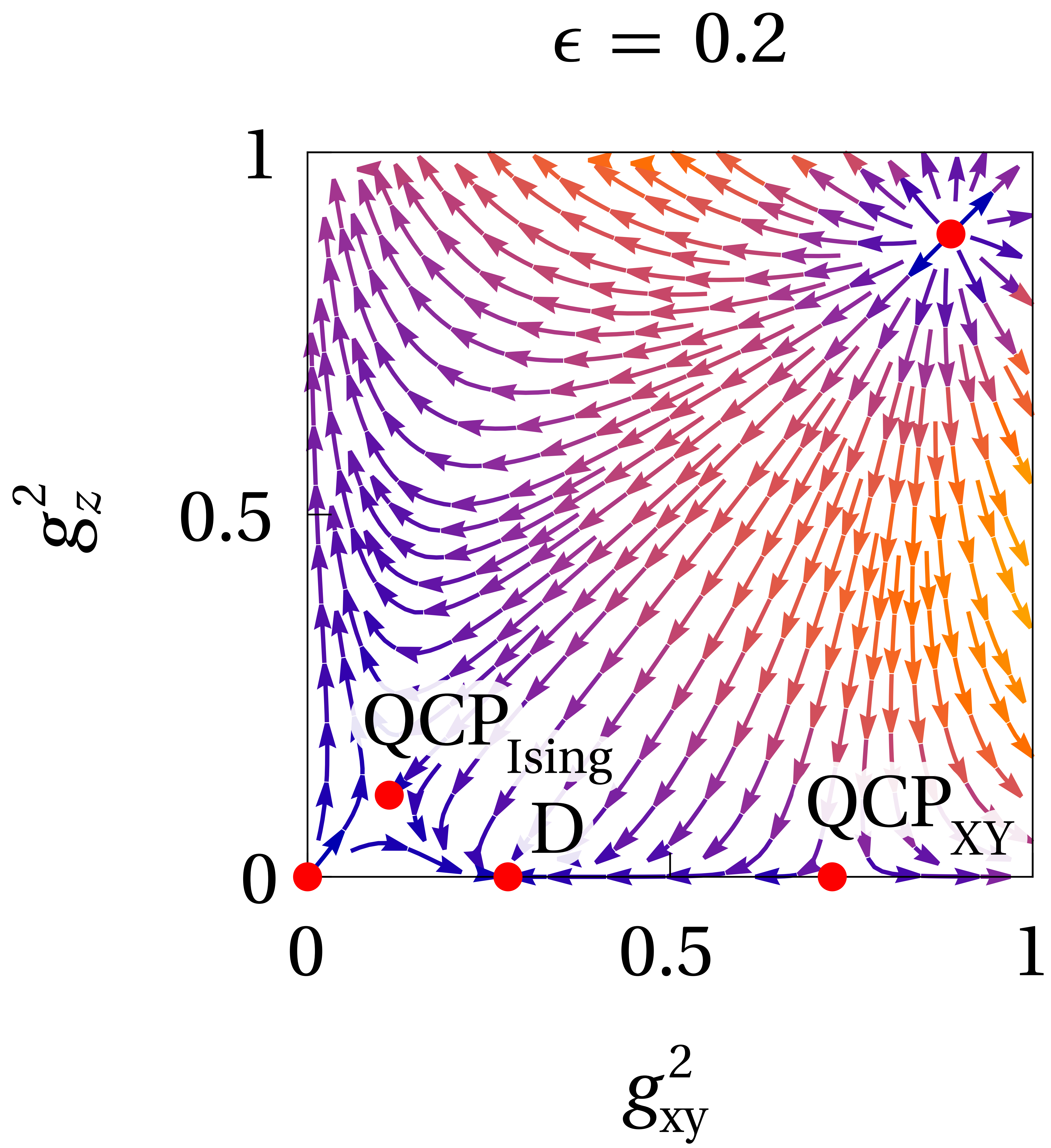
Spin-1/2 in fluctuating field:

$$\mathcal{H}_{\text{spin-boson}} = g_{xy}(h^x S^x + h^y S^y) + g_z h^z S^z + \mathcal{H}_{\text{bulk}}(\vec{h})$$

with $\langle \mathcal{T}_\tau h^a(\tau) h^b(0) \rangle \propto \dfrac{\delta^{ab}}{|\tau|^{2-\epsilon}}$

[Sengupta, PRB '00]
[Zhu, Si, PRB '02]
[Zaránd, Demler, PRB '02]
[Weber, Vojta, PRL '23]
[Weber, arXiv '24]
...

RG flow diagram



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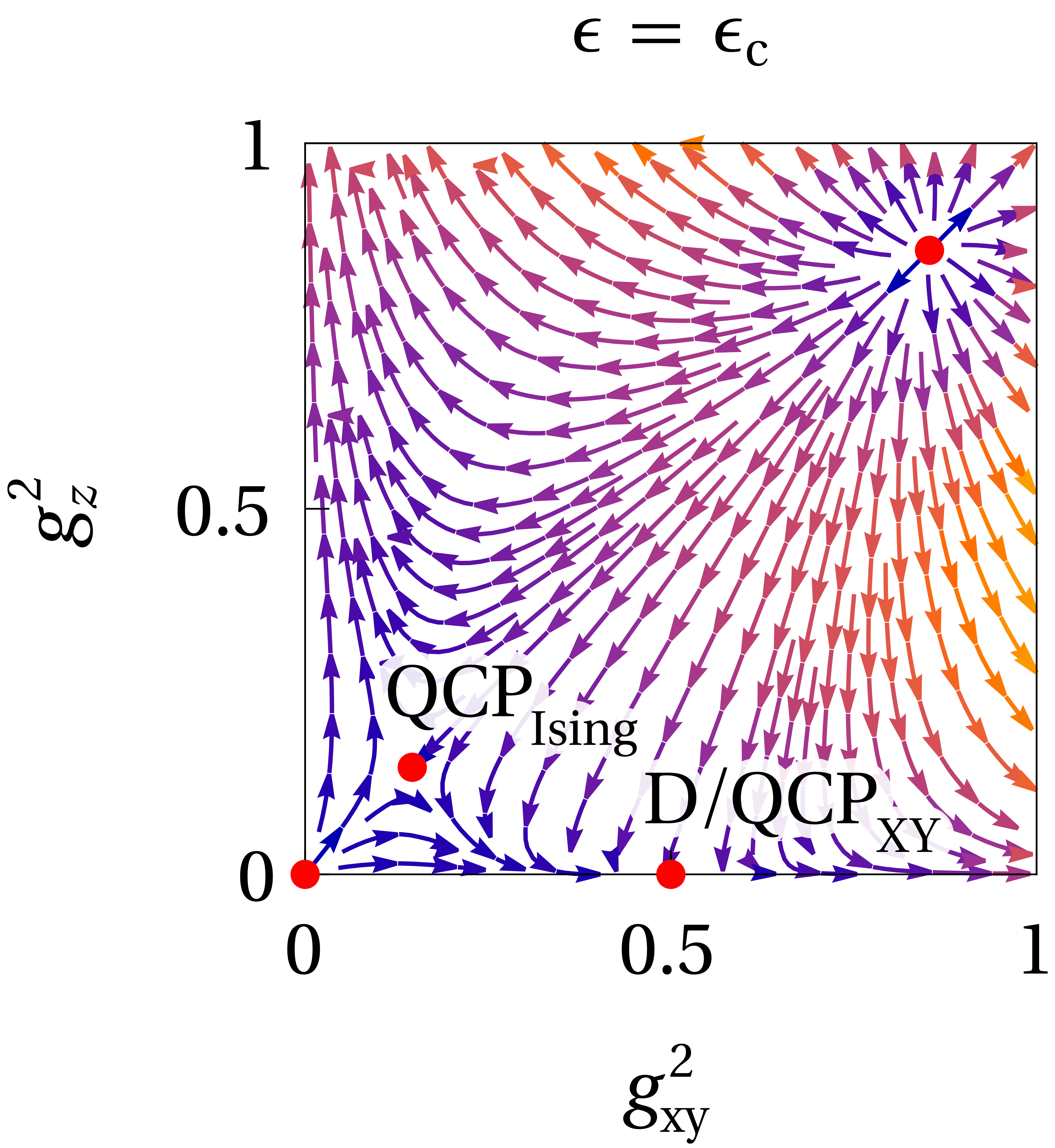

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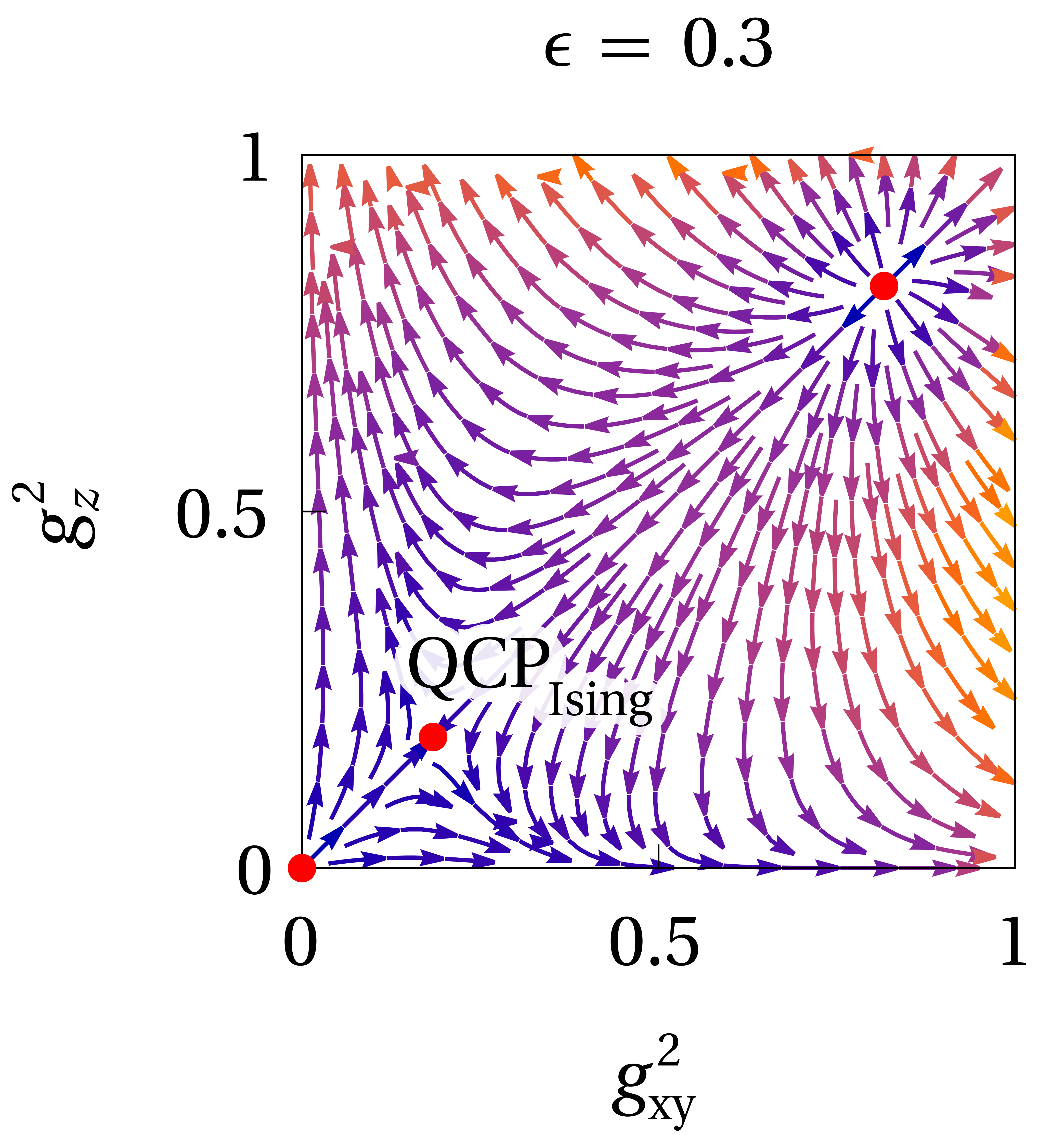
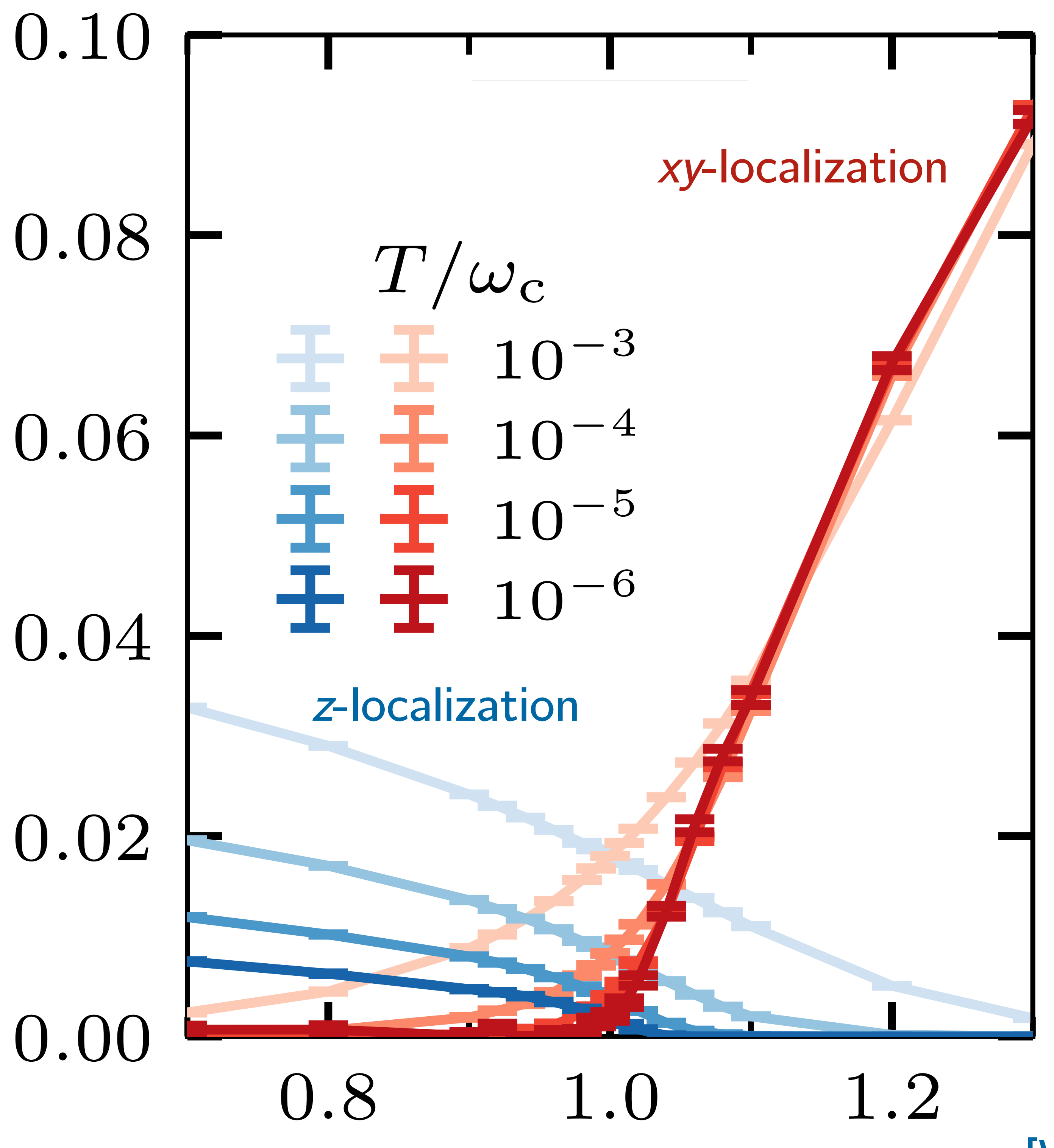


Quantum Monte Carlo:



[Weber, arXiv '24]

... consistent with predictions!

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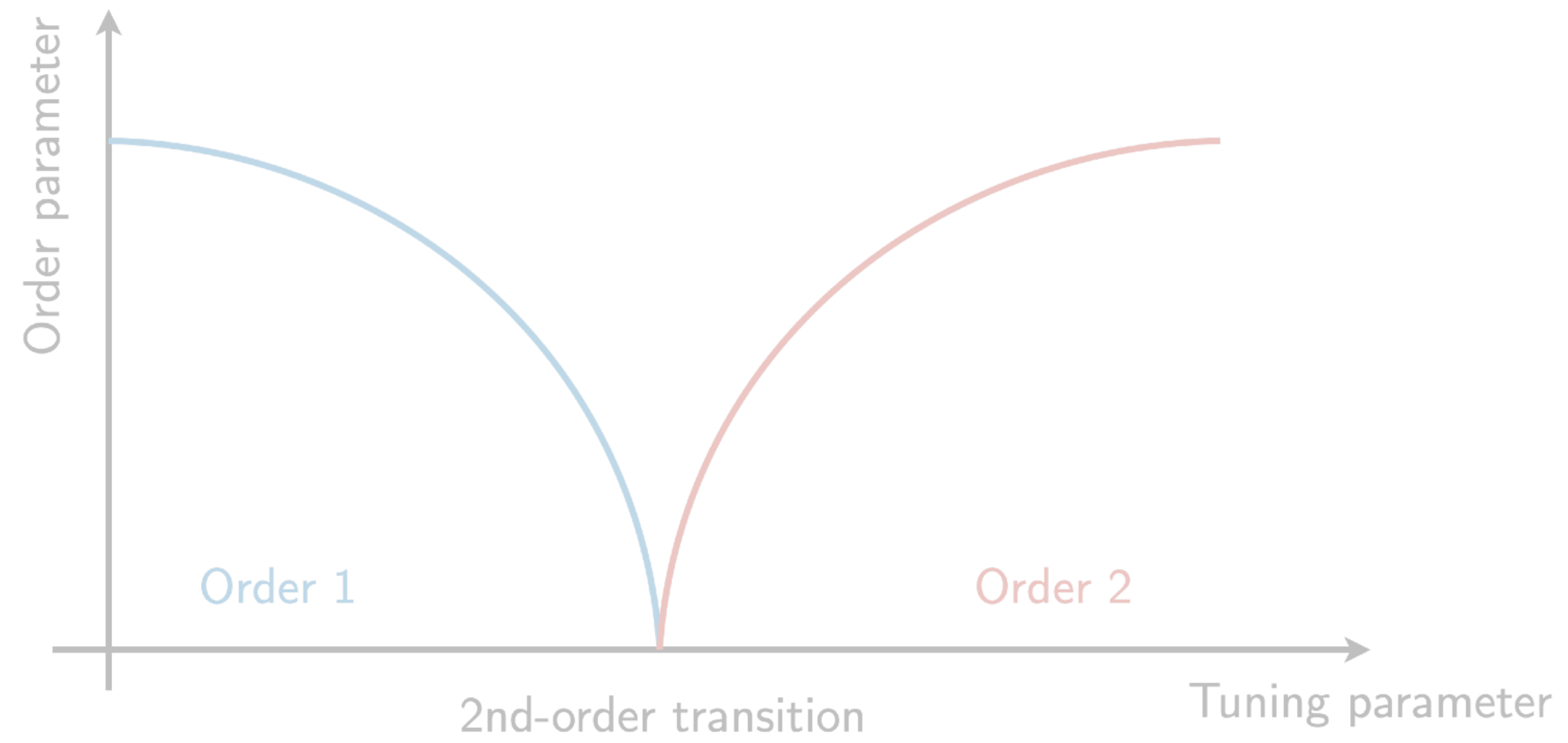
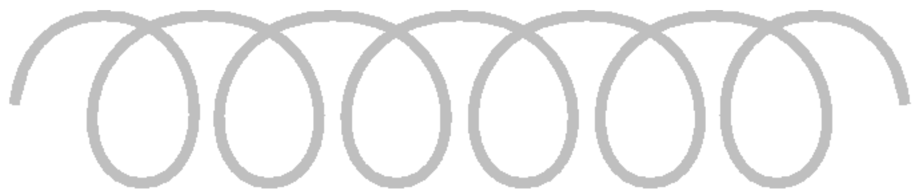
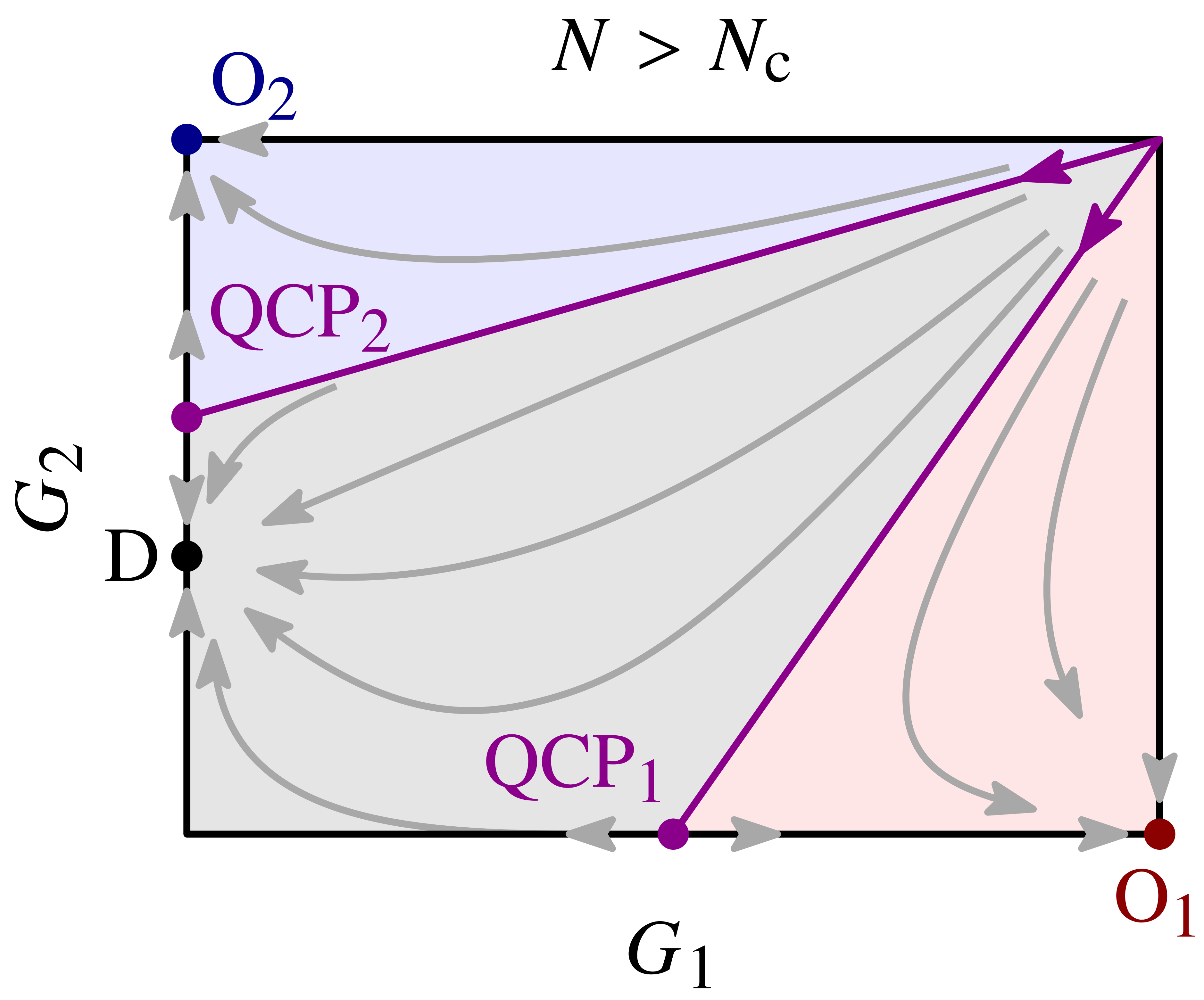
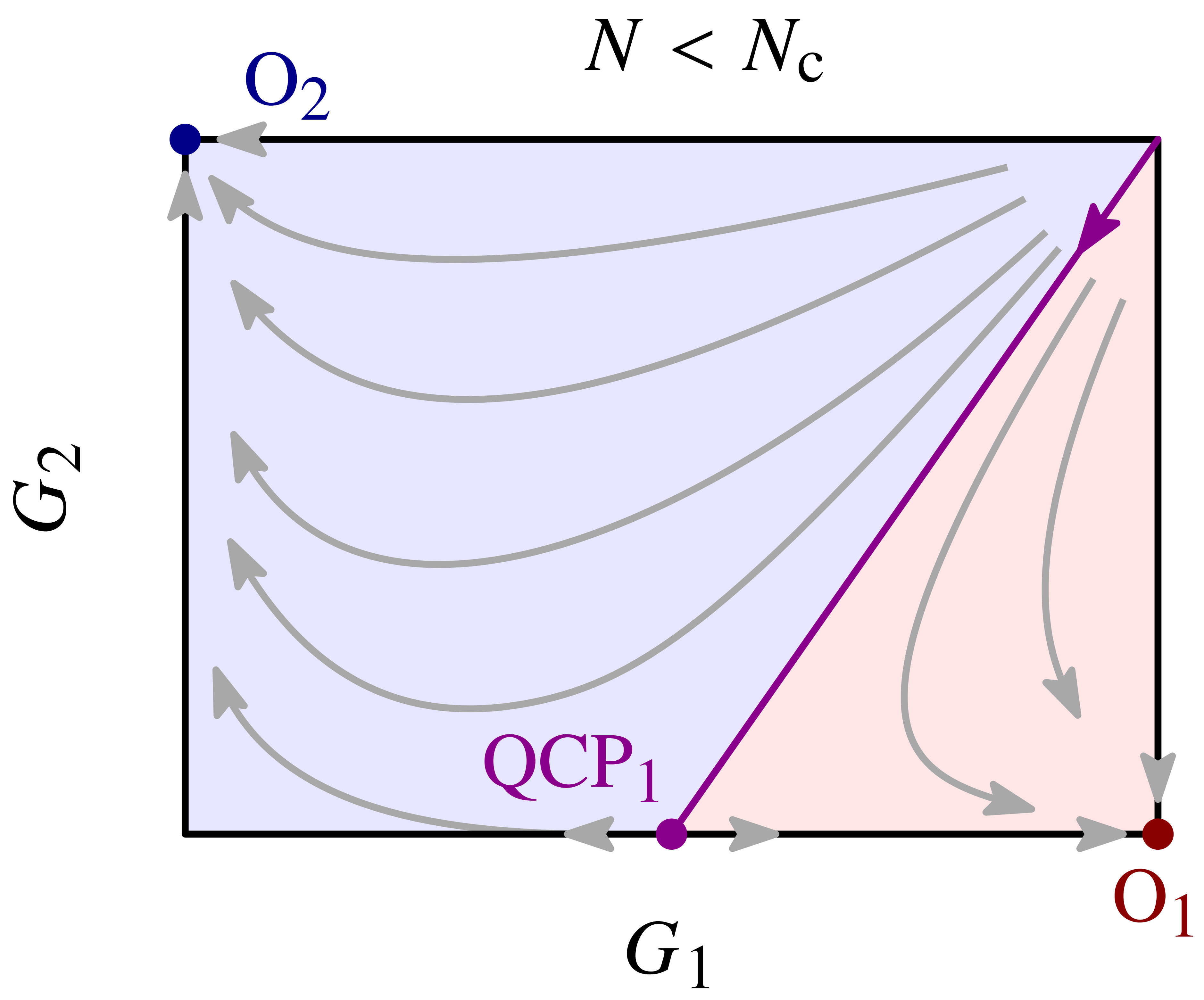
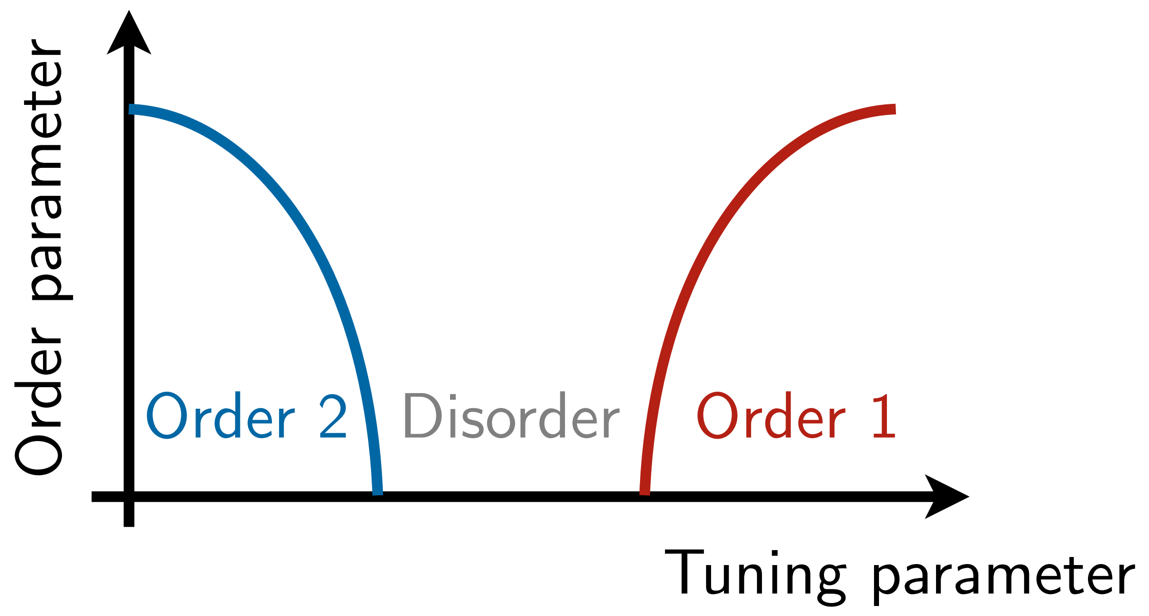


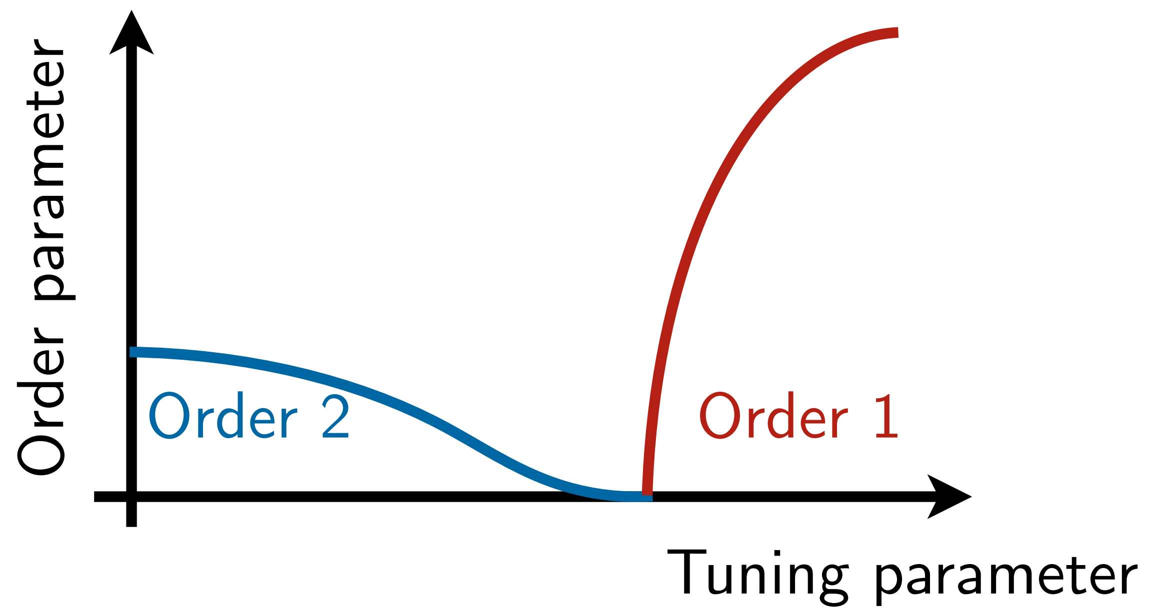


Conclusions









[Moser, LJ, *in preparation*]