TECHNISCHE UNIVERSITÄT DRESDEN

INTRODUCTION TO MATLAB

Data handling: vectors, matrices, and variables

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01 Review of previous session

- Defining variables
- Operations
- Built-in functions
- Defining vectors
- Indexing of vectors
- Evenly space vectors

- help
- clc
- clear /clear all
- format short/long
- who, whos
- 6.022e23 (scientific notation)
- exp, sin, cos, ..., log, log10
- ‘ (transpose)
- linspace, 1:10:100
- size, length, numel

02 Concatenation of vectors and the fantabulous world of matrices

For two vectors, A = [1, 2, 3, 4, 5] and B = [7, 9, 10, 11, 12], concatenation means:

- D1 = [A, B] = [1, 2, 3, 4, 5, 7, 9, 10, 11, 12].
- D2 = [A; B] = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 7 & 9 & 10 & 11 & 12 \end{pmatrix}$
- D3 = [A',B'] = $\begin{pmatrix} 1 & 7 \\ 2 & 9 \\ 3 & 10 \\ 4 & 11 \\ 5 & 12 \end{pmatrix}$
- D4 = [A,B'] = ?

02 Matrix indexing

Matrices' elements are addressed with two ordered indices (row, column).

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{pmatrix}$$

For a matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 11 & 12 & 13 \\ 100 & 200 & 300 \end{pmatrix}$
A(1,2) = 2, A(3,3) = 300, etc.
You can use : as a wildcard to access all the elements in a row or column
A(1,:) displays all elements of row 1
A(:,2) displays all elements of column 2
A(:, 2:3) = A(:,[2,3]) displays elements 2 and 3 of all rows

02 Exercises with matrices

Define the three vectors A = [2, 4, 6,...,20], B = [-21, -20, ..., -12], C = zeros(1,10);

1. Create a matrix MatX whose rows are A, B and C, in that order.
2. Read out all the elements of the second row of MatX.
3. Read out the first five elements of rows one and two.
4. Replace the second column of MatX with zeroes using the command zeros(a,b).
5. Replace the element in the second row, third column, with $-\infty$.
6. Create a matrix A = magic(5). Obtain the sum of the elements of each column and row separately.
7. Create a matrix MatY that is MatX with an extra column at the end. This extra column should be populated with the sum of each corresponding row.

03 Operations between numbers, vectors and matrices

- scalar * vector
- scalar * matrix
- vector * vector
- vector * matrix
- matrix * matrix

03 Addition and substraction

For a scalar α, a vector VecX and a matrix MatX

$$\mathrm{VecX} = (\mathrm{a}, \mathrm{b}, \mathrm{c})$$

$$\mathrm{VecY} = (\mathrm{x}, \mathrm{y}, \mathrm{z})$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{pmatrix}$$

$$\mathrm{MatY} = \begin{pmatrix} \mathrm{w} & \mathrm{x} \\ \mathrm{y} & \mathrm{z} \end{pmatrix}$$

$$\mathrm{VecX} \pm \mathrm{VecY} = (\mathrm{a} \pm \mathrm{x}, \mathrm{b} \pm \mathrm{y}, \mathrm{c} \pm \mathrm{z})$$

$$\mathrm{MatX} \pm \mathrm{MatY} = \begin{pmatrix} \mathrm{a} \pm \mathrm{w} & \mathrm{b} \pm \mathrm{x} \\ \mathrm{c} \pm \mathrm{y} & \mathrm{d} \pm \mathrm{z} \end{pmatrix}$$

03 Multiplication with scalars

For a scalar α, a vector VecX and a matrix MatX

$$\mathrm{VecX} = (\mathrm{a}, \mathrm{b}, \mathrm{c})$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{pmatrix}$$

$$\alpha * \mathrm{VecX} = (\alpha\mathrm{a}, \alpha\mathrm{b}, \alpha\mathrm{c})$$

$$\alpha * \mathrm{MatX} = \begin{pmatrix} \alpha\mathrm{a} & \alpha\mathrm{b} \\ \alpha\mathrm{c} & \alpha\mathrm{d} \end{pmatrix}$$

03 Multiplication between vectors and matrices

For two vectors and a matrix
$\mathrm{VecX} = (\mathrm{a}, \mathrm{b}, \mathrm{c})$

$$\mathrm{VecY} = \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{pmatrix}$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \\ \mathrm{e} & \mathrm{f} \end{pmatrix} \ \mathrm{MatX}' = \begin{pmatrix} \mathrm{a} & \mathrm{c} & \mathrm{e} \\ \mathrm{b} & \mathrm{d} & \mathrm{f} \end{pmatrix}$$

Their product (*) is:
$\mathrm{VecX} * \mathrm{VecY} = \mathrm{ax} + \mathrm{by} + \mathrm{cz}$ a scalar.
$\mathrm{VecX} * \mathrm{MatX} = (\mathrm{aa} + \mathrm{bc} + \mathrm{ce}, \mathrm{ab} + \mathrm{bd} + \mathrm{cf})$

$$(\mathrm{MatX}') * \mathrm{VecY} = \begin{pmatrix} \mathrm{ax} + \mathrm{cy} + \mathrm{ez} \\ \mathrm{bx} + \mathrm{dy} + \mathrm{fz} \end{pmatrix}$$

03 Multiplication between matrices

$$\text{MatX} = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$$

$$\text{MatY} = \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix}$$

Their product is:

$$\text{MatY} * \text{MatX} = \begin{pmatrix} aa + cc + ee & ab + cd + ef \\ ba + dc + fe & bb + dd + ff \end{pmatrix} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2} \end{pmatrix}$$

$\sigma_{1,1}$ = First row of MatY multiplied (*) by first column of MatX
$\sigma_{1,2}$ = First row of MatY multiplied (*) by second column of MatX
$\sigma_{2,1}$ = Second row of MatY multiplied (*) by first column of MatX
$\sigma_{2,2}$ = Second row of MatY multiplied (*) by second column of MatX

03 Matix operation exercises

Define the matrix
$\text{MatX} = \begin{pmatrix} 3 & 1 & -5 \\ 10 & -1.2 & 0 \end{pmatrix}$ and the vector
$\text{VecX} = (-1, 100, 3)$

1. Create a matrix MatY, whose first row is the first of MatX multiplied by 5, and whose second row is the second row of MatX multiplied by 7
2. Multiply VecX and MatX
3. Multiply MatX with itself
4. Add MatX to a matrix whose rows are copies of VecX

04 Exercises with matrices

Create the following matrices using one line of code:

1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

0	1	1	1	1	10
2	1	1	1	1	9
4	1	1	1	1	8
6	1	1	1	1	7
8	1	1	1	1	6
10	1	1	1	1	5
12	1	1	1	1	4
14	1	1	1	1	3
16	1	1	1	1	2
18	1	1	1	1	1
20	1	1	1	1	0

0	2	4	6	8	10
0	1	1	1	1	10
2	1	1	1	1	9
4	1	1	1	1	8
6	1	1	1	1	7
8	1	1	1	1	6
10	1	1	1	1	5
12	1	1	1	1	4
14	1	1	1	1	3
16	1	1	1	1	2
18	1	1	1	1	1
20	1	1	1	1	0
20	16	12	8	4	0

04 Matrix multiplication

For two matrices $A_{n \times m}$ and $B_{m \times l}$,
Then $C = A * B$ is of size $n \times l$
The number of columns of A must be the same as the number of rows in B.
For example:

$$A = \texttt{ones(3,4)} = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}, B = \texttt{zeros(4,2)} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$A * B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

`size(A)`$= 3 \times 4$, `size(B)`$= 4 \times 2$, `size(A*B)`$= 3 \times 2$
Try the command: `size(ones(3,4)*zeros(4,2))`

04 Exercises

1. Define the matrix MatA = $\begin{pmatrix} 1 & 2 & \cdots & 10 \\ 10 & 20 & \cdots & 100 \\ 100 & 200 & \cdots & 1000 \end{pmatrix}$
2. Create the matrix MatI = `eye(4)`
3. Create a matrix MatB with columns of MatA such that you can do MatB*MatI
4. Add rows to MatA so that you can multiply MatI*MatA. The new rows must follow the pattern in MatA
5. Create the vector VecA with the second row of MatA. Then delete the values 20 and 90 from it by assigning them to the empty vector "[ ]".
6. Delete the extra rows created in MatA by assigning an empty vector "[ ]" to these rows.
7. Select the appropiate operations that are possible: A□A′□`eye(3)` =

05 Operations

Matrix times matrix:

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \quad B = \begin{pmatrix} w & x \\ y & z \end{pmatrix}$$

$$A. * B = \begin{pmatrix} aw & bx \\ cy & dz \end{pmatrix} \neq A * B$$

$$A./B = \begin{pmatrix} a/w & b/x \\ c/y & d/z \end{pmatrix} \neq A/B$$

$$A. \pm B = A \pm B = \begin{pmatrix} a \pm w & b \pm x \\ c \pm y & d \pm z \end{pmatrix}$$

$$A.\hat{}\,2 = \begin{pmatrix} a^2 & b^2 \\ c^2 & d^2 \end{pmatrix} \neq A\,\hat{}\,2$$

Note: the sizes of the two matrices in element-wise operations must be exactly the same.

05 Exceptions

- `2+ones(2,3)`
- `2*ones(2,3)`
- `2./ones(2,3)`
- `2.^ones(2,3)`

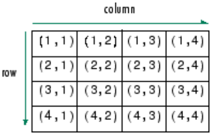
05 Exercises

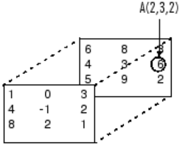
1. Compute $S(N) = \sum_{n=1}^{N} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{N}$, for $N = 100$
2. Compute $G(N) = \sum_{n=1}^{N} x^n = x + x^2 + x^3 + \cdots + x^N$, $x = 0.5$, for $N = 100$

06 Variable types

- Multidimensional arrays
- Cell
- Structures
- Strings

06 Multidimensional arrays



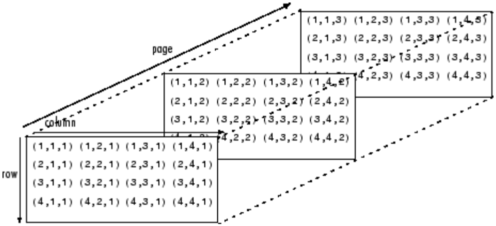



```
A(:,:,1) =

     1     0     3
     4    -1     2
     8     2     1


A(:,:,2) =

     6     8     3
     4     3     6
     5     9     2
```



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06 Multidimensional examples

Example 1:

```
A(:,:,1) = magic(5);
A(:,:,2) = zeros(5);
A(:,:,3) = ones(5);
```

Example 2:

```
A = zeros(2,2,4);
```

Example 3:

```
A = ones(3,6,5);
```

Exercise:

- Create a matrix 4x4x3, such that the first layer has 1s in the diagonal, the second has 2s, the third has 3s.
- Create a 6x6x10, such that the first five layers have just 1s, layers from 6 to 9 have just 0s, the 10th layer is:

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

06 Cells and structures

Cells:
They are similar to arrays, but each element can have a different size
Example:
To initialize a cell array:

```
A = cell(3,2)
```

To index, use curly brackets:

```
A{1,1} = magic(5);
A{3,2} = zeros(2,1);
```

To index a cell's element's elements: `A{1,1}(1,1)`

Structures:
Like Cells, but indexed with names:
Example:
For a structure named "subject",

```
subject.age = 30;
subject.country = 'Mexico';
subject.height = 1.83;
subject.results = [1, 0, 1, 1, 0];
```

To index the element's element, `subject.results(5)`

06 Cells and structures exercises

1. Create a vector-cell CellA whose first element is `[1]`, the second `[1, 2]`, then `[1,2,3]`, etc., until 5. The 6th element is `magic(7)`. The 7th one is empty.
2. Create a structure called MyStruct with elements: NoOfClassmates, CurrentYear, MyCell and Magia. The value of MyCell should be CellA from the previous exercise. The value of Magia should be the 6th element of CellA.
3. From MyStruct, change the 7th element of MyCell (that is, `MyStruct.MyCell{7}`) to `rand(2,10)`

06 Strings

Strings are arrays of letters.

`A = 'I am a Vahid';`

They are indexed like an array:

`A(1)` gives I , `A(2)` gives (empty space) ;

To create two-dimensional arrays of chars:

`B = char(A, 'Yes I am');`

Note: `C = '52';` is NOT a number. `C+5` throws an error. Examples for indexing:

`A(8:end)` gives Vahid

`B(2,1:3)` gives Yes

Exercise: Substitute Vahid's name for your own in A. You might have to add or delete characters at the end.