TECHNISCHE UNIVERSITÄT DRESDEN

INTRODUCTION TO MATLAB

Vectors and matrices

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01 Useful commands to know

- Defining variables
- Operations
- Built-in functions
- Defining vectors
- Indexing of vectors
- Evenly space vectors

- help
- clc
- clear /clear all
- format short/long
- who, whos
- 6.022e23 (scientific notation)
- exp, sin, cos, ..., log, log10
- ‘ (transpose)
- linspace, 1:10:100
- size, length, numel

02 Concatenation of vectors and the fantabulous world of matrices

For two vectors, A = [1, 2, 3, 4, 5] and B = [7, 9, 10, 11, 12], concatenation means:

- D1 = [A, B] = [1, 2, 3, 4, 5, 7, 9, 10, 11, 12].
- D2 = [A; B] = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 7 & 9 & 10 & 11 & 12 \end{pmatrix}$
- D3 = [A',B'] = $\begin{pmatrix} 1 & 7 \\ 2 & 9 \\ 3 & 10 \\ 4 & 11 \\ 5 & 12 \end{pmatrix}$
- D4 = [A,B'] = ?

02 Matrix indexing

Matrices' elements are addressed with two ordered indices (row, column).

$$\begin{pmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ (3,1) & (3,2) & (3,3) \end{pmatrix}$$

For a matrix A = $\begin{pmatrix} 1 & 2 & 3 \\ 11 & 12 & 13 \\ 100 & 200 & 300 \end{pmatrix}$

A(1,2) = 2, A(3,3) = 300, etc.

You can use : as a wildcard to access all the elements in a row or column

A(1,:) displays all elements of row 1

A(:,2) displays all elements of column 2

A(:, 2:3) = A(:,[2,3]) displays elements 2 and 3 of all rows

02 Exercises with matrices

Define the three vectors A = [2, 4, 6,...,20], B = [-21, -20, ..., -12], C = zeros(1,10);

1. Create a matrix MatX whose rows are A, B and C, in that order.
2. Read out all the elements of the second row of MatX.
3. Read out the first five elements of rows one and two.
4. Replace the second column of MatX with zeroes using the command zeros(a,b).
5. Replace the element in the second row, third column, with $-\infty$.
6. Create a matrix A = magic(5). Obtain the sum of the elements of each column and row separately.
7. Create a matrix MatY that is MatX with an extra column at the end. This extra column should be populated with the sum of each corresponding row.

03 Operations between numbers, vectors and matrices

- scalar * vector
- scalar * matrix
- vector * vector
- vector * matrix
- matrix * matrix

03 Addition and substraction

For a scalar α, a vector VecX and a matrix MatX

$$\mathrm{VecX} = (\mathrm{u}, \mathrm{v}, \mathrm{w})$$

$$\mathrm{VecY} = (\mathrm{x}, \mathrm{y}, \mathrm{z})$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{pmatrix}$$

$$\mathrm{MatY} = \begin{pmatrix} \mathrm{e} & \mathrm{f} \\ \mathrm{g} & \mathrm{h} \end{pmatrix}$$

$$\mathrm{VecX} \pm \mathrm{VecY} = (\mathrm{u} \pm \mathrm{x}, \mathrm{v} \pm \mathrm{y}, \mathrm{w} \pm \mathrm{z})$$

$$\mathrm{MatX} \pm \mathrm{MatY} = \begin{pmatrix} \mathrm{a} \pm \mathrm{e} & \mathrm{b} \pm \mathrm{f} \\ \mathrm{c} \pm \mathrm{g} & \mathrm{d} \pm \mathrm{h} \end{pmatrix}$$

03 Multiplication with scalars

For a scalar α, a vector VecX and a matrix MatX

$$\mathrm{VecX} = (\mathrm{a}, \mathrm{b}, \mathrm{c})$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{pmatrix}$$

$$\alpha * \mathrm{VecX} = (\alpha \mathrm{a}, \alpha \mathrm{b}, \alpha \mathrm{c})$$

$$\alpha * \mathrm{MatX} = \begin{pmatrix} \alpha \mathrm{a} & \alpha \mathrm{b} \\ \alpha \mathrm{c} & \alpha \mathrm{d} \end{pmatrix}$$

03 Multiplication between vectors and matrices

For two vectors and a matrix
$\mathrm{VecX} = (\mathrm{u}, \mathrm{v}, \mathrm{w})$

$$\mathrm{VecY} = \begin{pmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{pmatrix}$$

$$\mathrm{MatX} = \begin{pmatrix} \mathrm{a} & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \\ \mathrm{e} & \mathrm{f} \end{pmatrix} \; \mathrm{MatX}' = \begin{pmatrix} \mathrm{a} & \mathrm{c} & \mathrm{e} \\ \mathrm{b} & \mathrm{d} & \mathrm{f} \end{pmatrix}$$

Their product (*) is:
$\mathrm{VecX} * \mathrm{VecY} = \mathrm{ux} + \mathrm{vy} + \mathrm{wz}$; a scalar.
$\mathrm{VecX} * \mathrm{MatX} = (\mathrm{ua} + \mathrm{vc} + \mathrm{we}, \mathrm{ub} + \mathrm{vd} + \mathrm{wf})$

$$(\mathrm{MatX}') * \mathrm{VecY} = \begin{pmatrix} \mathrm{ax} + \mathrm{cy} + \mathrm{ez} \\ \mathrm{bx} + \mathrm{dy} + \mathrm{fz} \end{pmatrix}$$

03 Multiplication between matrices

$$\text{MatX} = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$$

$$\text{MatY} = \begin{pmatrix} u & v & w \\ x & y & z \end{pmatrix}$$

Their products are:

$$\text{MatX} * \text{MatY} = \begin{pmatrix} au + bx & av + by & aw + bz \\ cu + dx & cv + dy & cw + dz \\ eu + fx & ev + fy & ew + fz \end{pmatrix}$$

$$\text{MatY} * \text{MatX} = \begin{pmatrix} ua + vc + we & ub + vd + wf \\ xa + yc + ze & xb + yd + zf \end{pmatrix} = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} \\ \sigma_{2,1} & \sigma_{2,2} \end{pmatrix}$$

$\sigma_{1,1}$ = First row of MatY multiplied (*) by first column of MatX
$\sigma_{1,2}$ = First row of MatY multiplied (*) by second column of MatX
$\sigma_{2,1}$ = Second row of MatY multiplied (*) by first column of MatX
$\sigma_{2,2}$ = Second row of MatY multiplied (*) by second column of MatX

03 Matix operation exercises

Define the matrix

$$\text{MatX} = \begin{pmatrix} 3 & 1 & -5 \\ 10 & -1.2 & 0 \end{pmatrix}$$

and the vector

$\text{VecX} = (-1, 100, 3)$

1. Create a matrix MatY, whose first row is the first of MatX multiplied by 5, and whose second row is the second row of MatX multiplied by 7
2. Multiply VecX and MatX
3. Multiply MatX with itself (using the transpose operator) to obtain a 3x3 matrix.
4. Add MatX to a matrix whose rows are copies of VecX