



MAX-PLANCK-GESELLSCHAFT

# Modeling Cancer Ecology

examples of the public goods game

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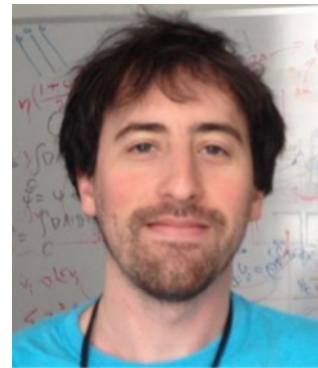
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@evlosci

# thanks



Meghan Ferrall-Fairbanks, PhD  
(University of Florida)



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Philip Gerlee, PhD  
(Chalmers)



Brian Johnson, BSc  
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Noemi Andor, PhD  
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# COIs

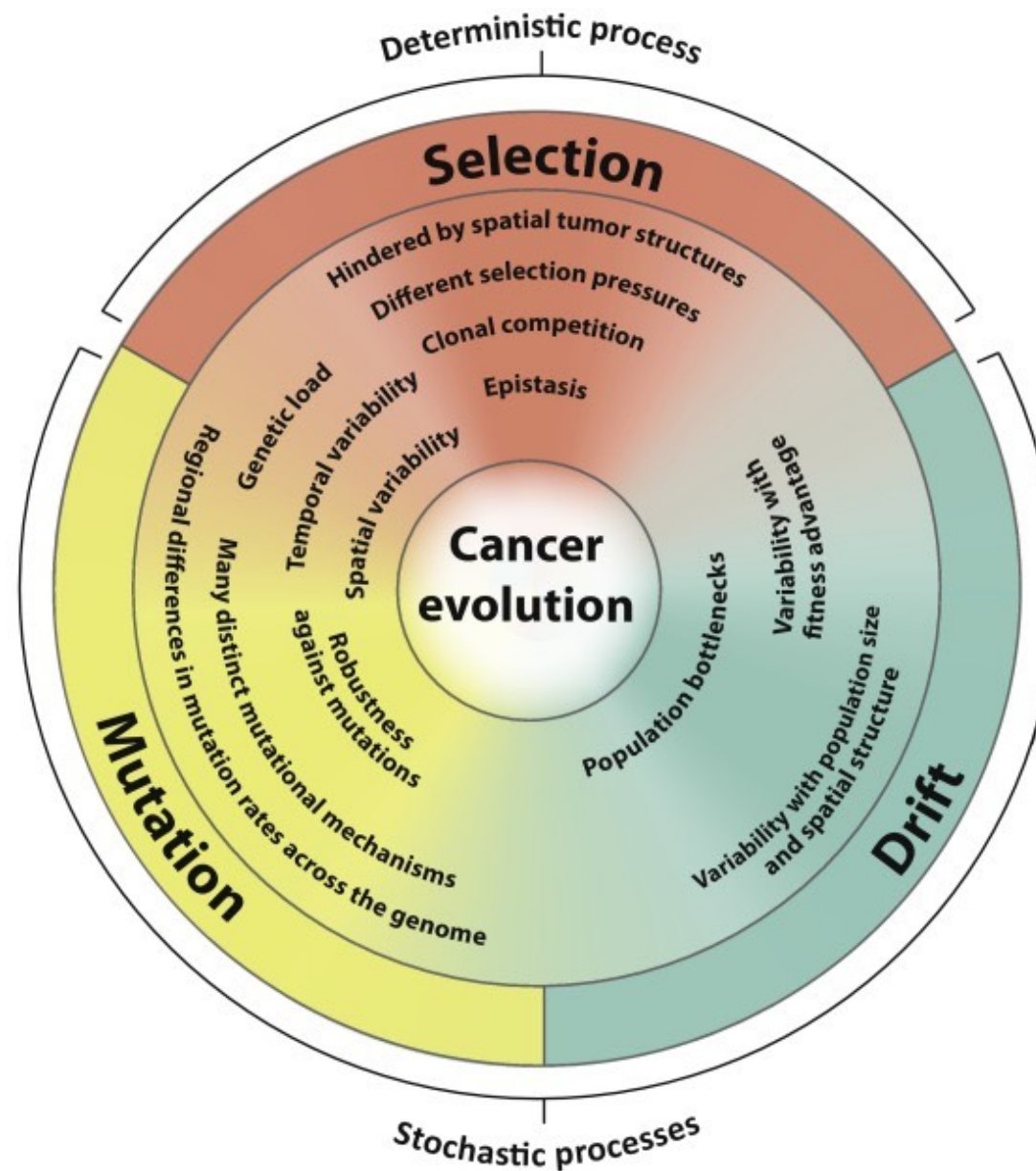
KITE Pharma (Gilead): research funding  
CRISPR Therapeutics: consulting

# outline

- Evolution and Ecology of Cancer
- Clonal interactions and the heterogeneity
- Public Goods Game among Cells
- Stochasticity, Assortment, Space
- Conclusions and Outlook



# cancer is a complex *evolving* system



Trends in Cancer

**We seek to understand selection in cancer cell populations.**

# modeling cancer dynamics

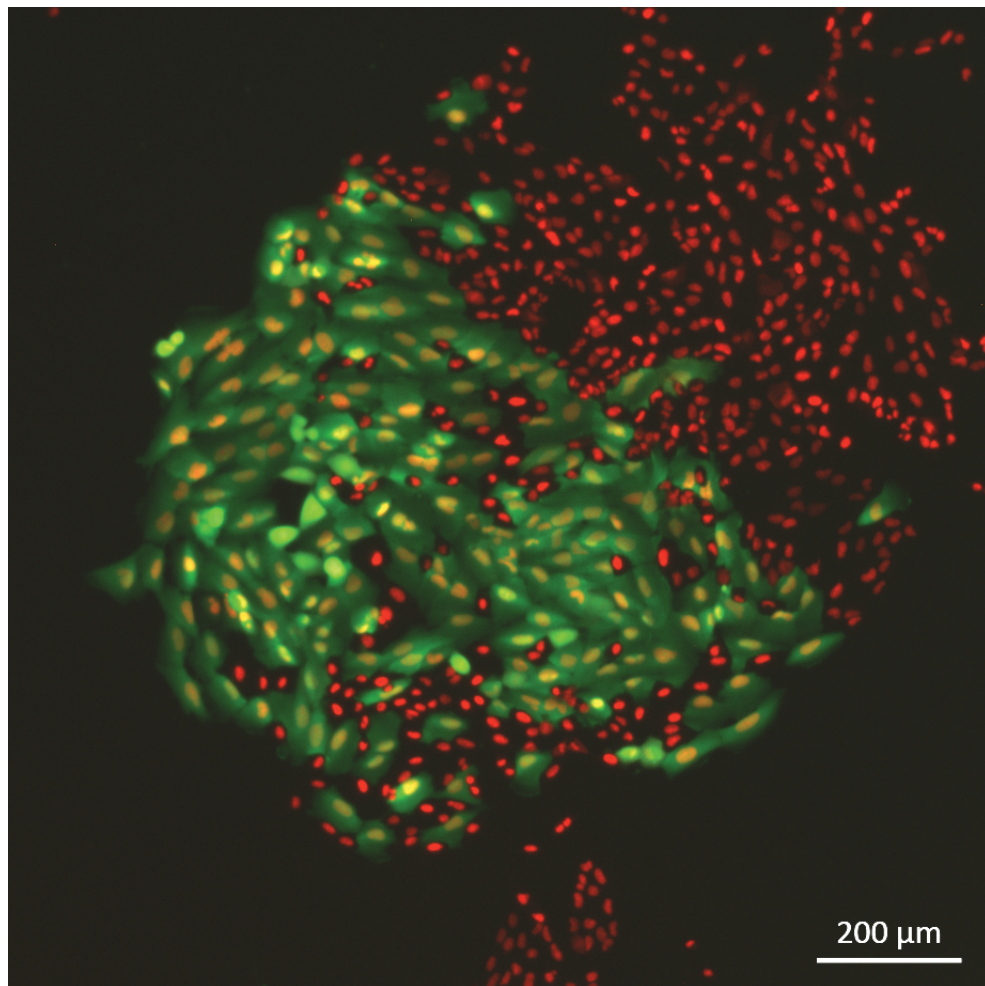
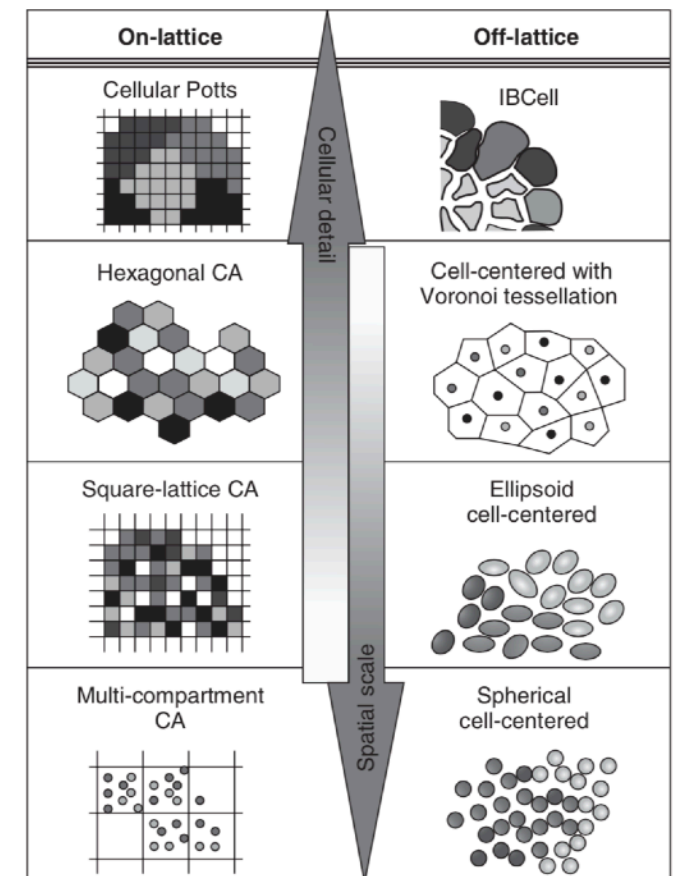
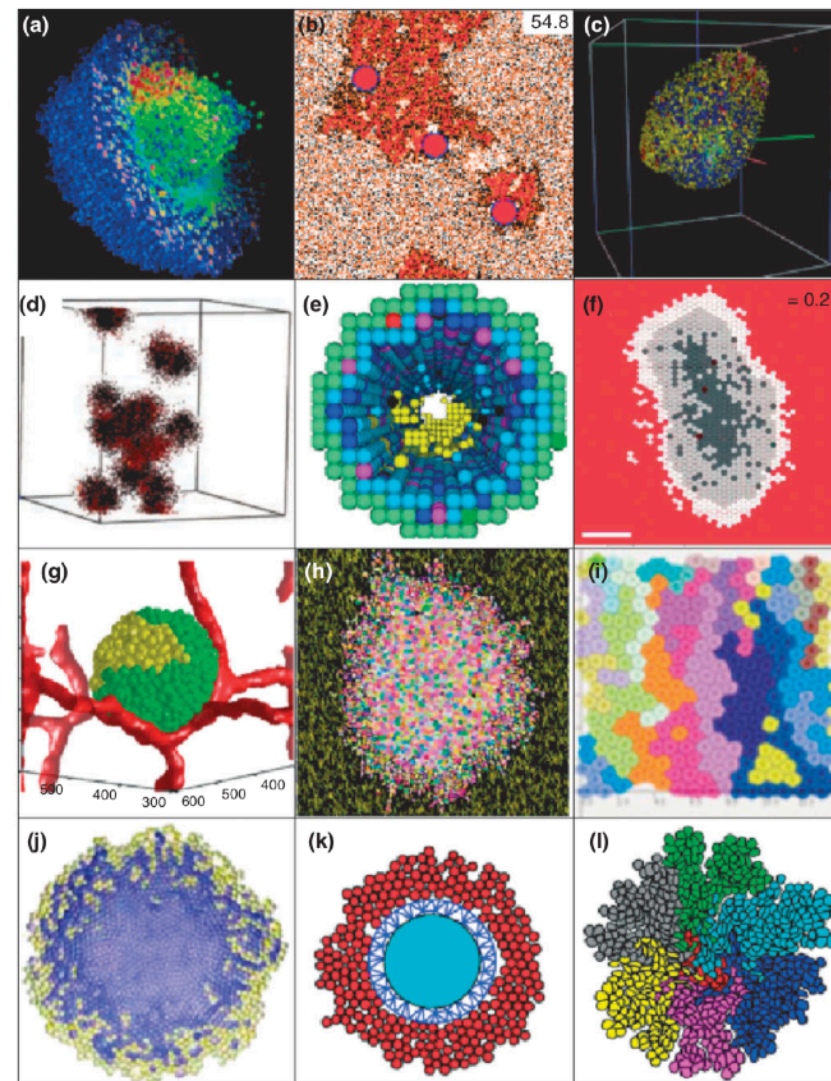


Image of live fluorescent colony formed after a selection experiment  
Human breast cancer cell lines: SUM159/MDA231



**FIGURE 1** | Reciprocal relation between the number of cells handled by the models and the level of included cellular details. In each class (on-lattice and off-lattice), the models complexity rises from cells represented by single points to fully deformable bodies.

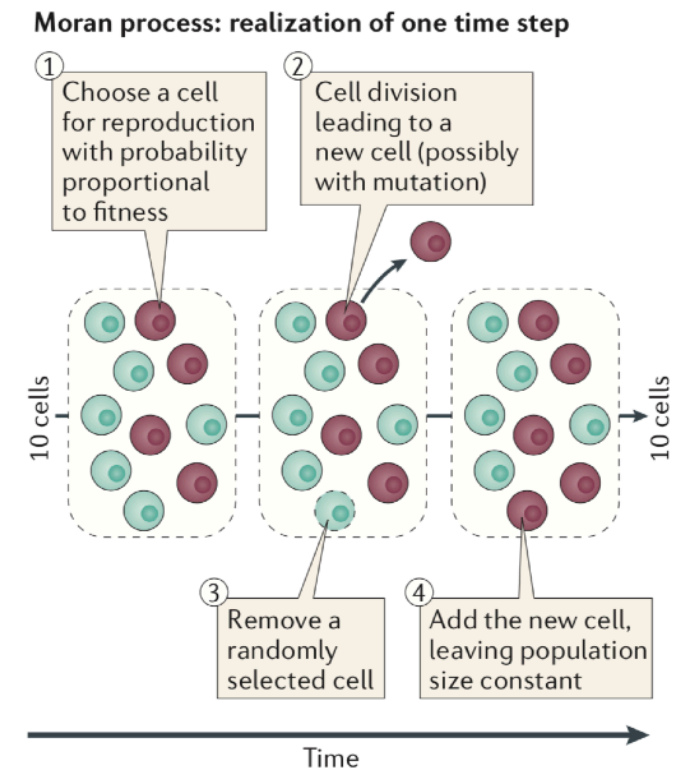
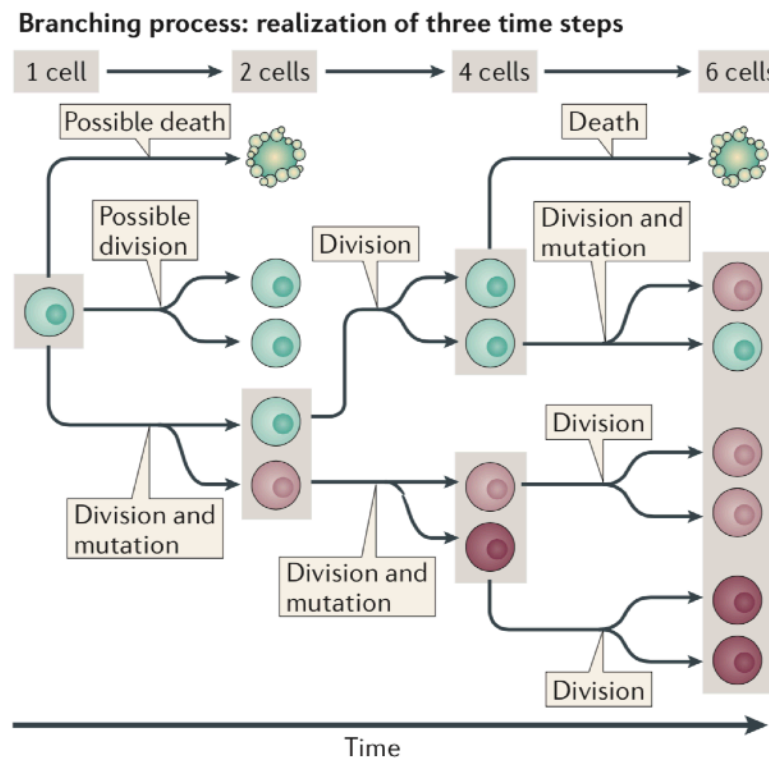


# processes and mechanisms in cancer evolution: stochastic processes

## Branching processes, Moran process with selection

Branching process in cancer:  
work by Bozic, Durrett, Antal, Tomasetti

Evolutionary Moran process:  
in cancer: work by Michor, Foo

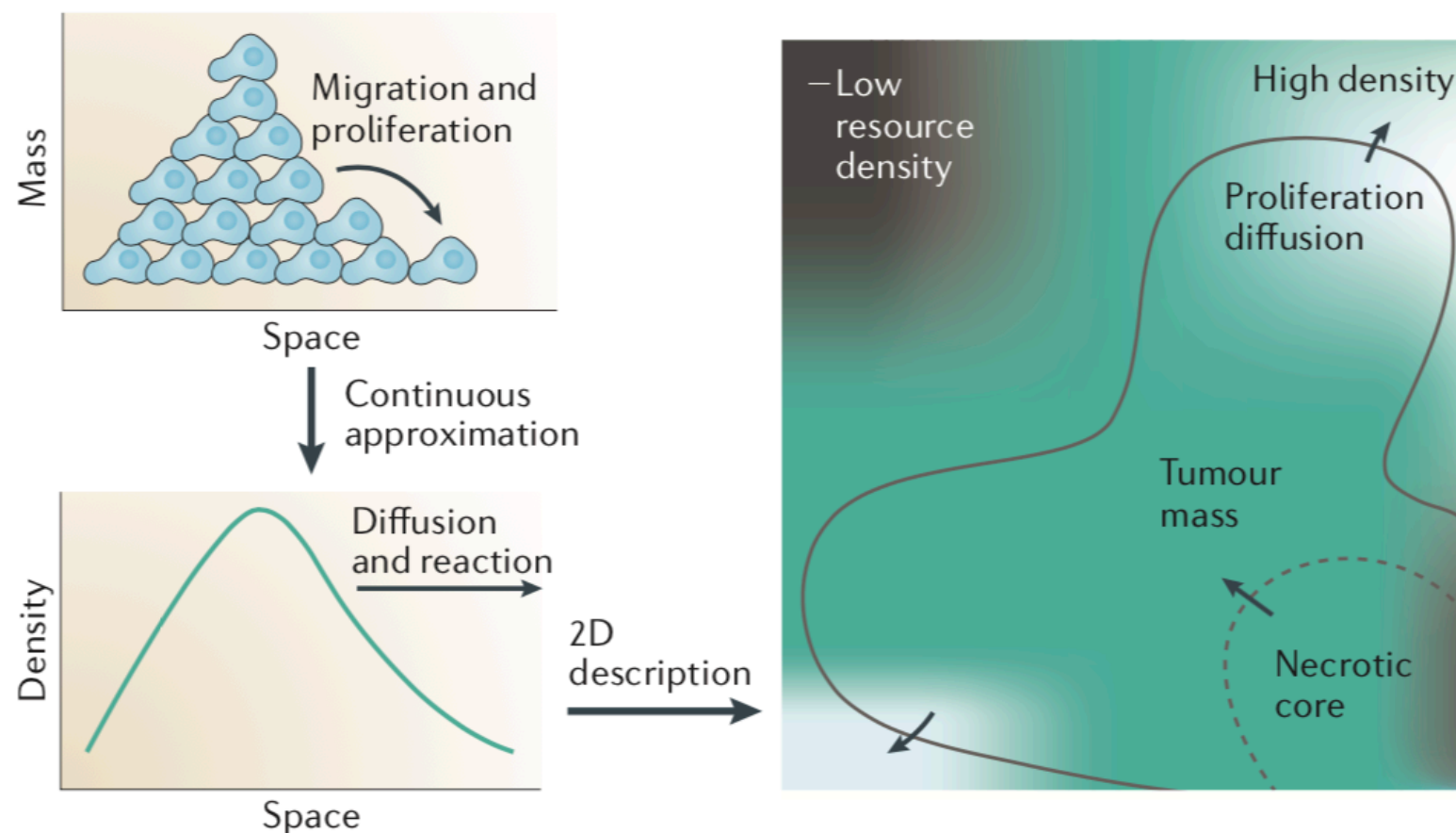


Altrock, Liu & Michor, Nature Reviews Cancer 2015

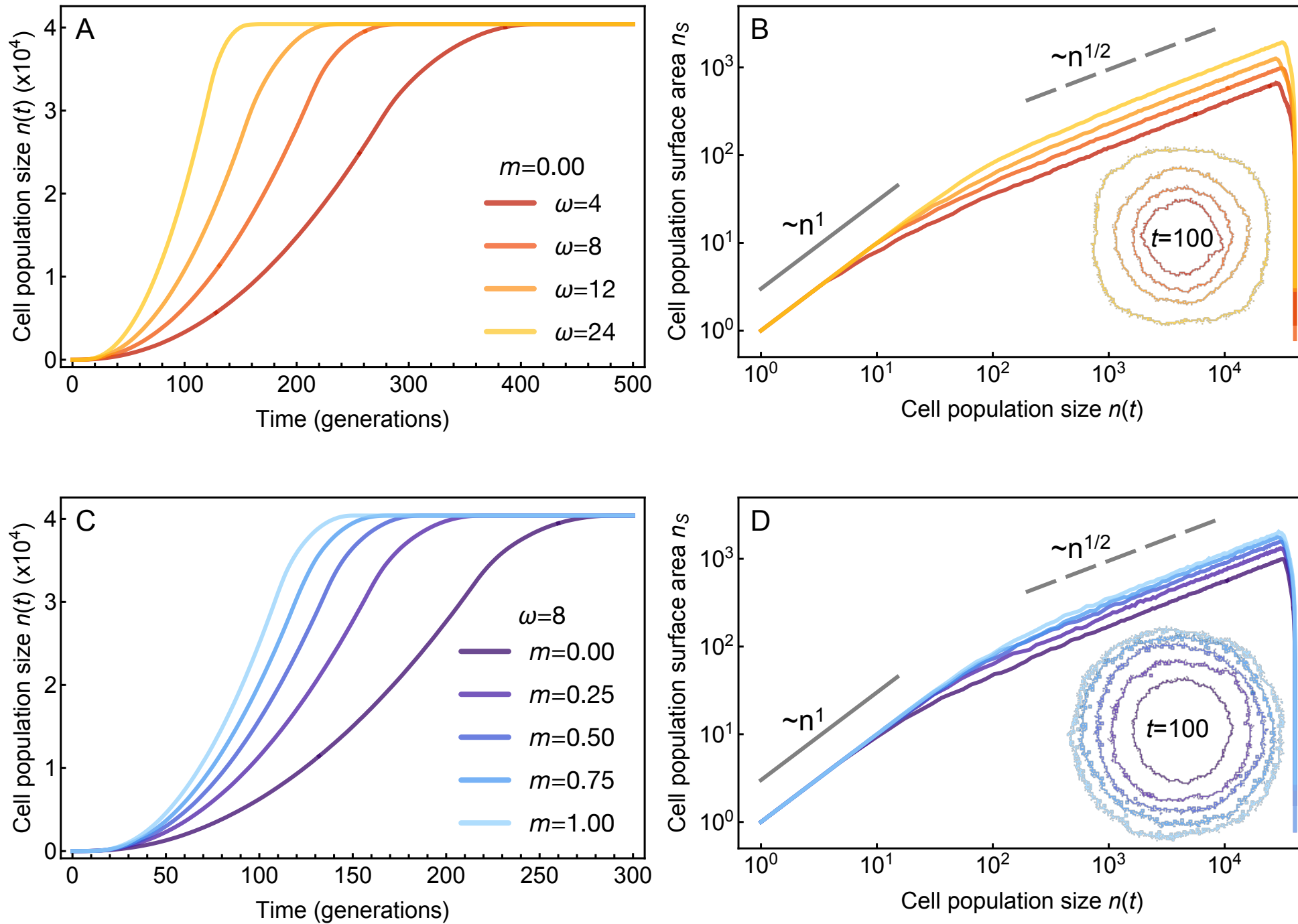
# processes and mechanisms in cancer evolution: nonlinear dynamics

## Nonlinear dynamics, reaction-advection-diffusion systems

Work by Anderson, Maini & many others  
Kimmel, Dane, Heiser, Alrock & Andor, Cancer Res. 2020

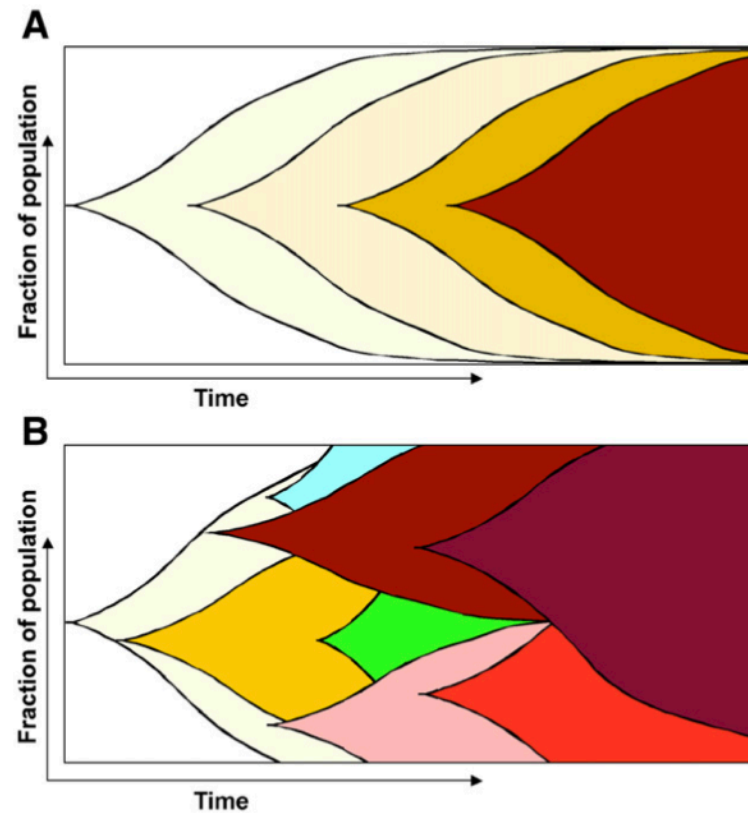


# cell movement ( $m$ ) and growth neighborhood ( $\omega$ ) can lead to different growth curves

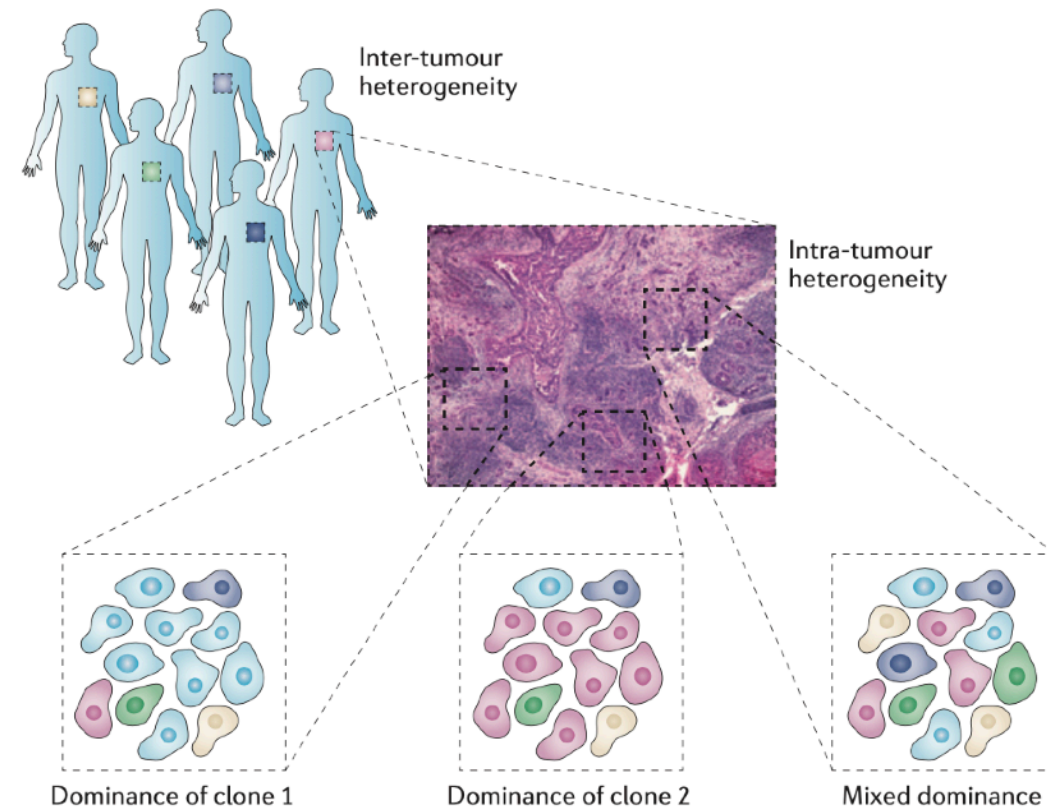


# cancer evolution leads to heterogeneity, e.g., due to clonal interference or ecological interactions

“at the time of clinical diagnosis, the majority of human tumors display startling heterogeneity in many morphological and physiological features, such as expression of cell surface receptors, proliferative and angiogenic potential”

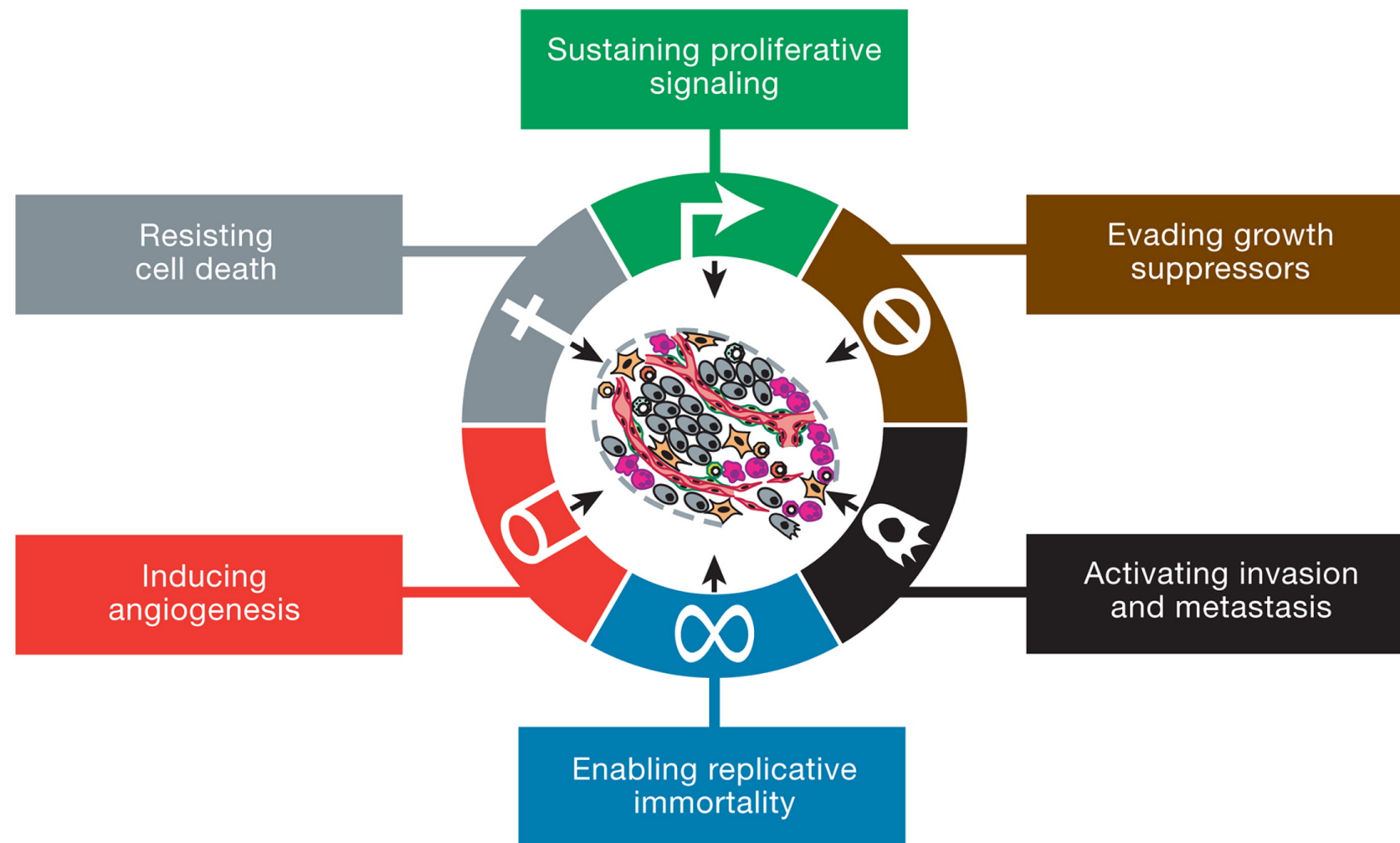


**Fig. 1.** Schematic view of monoclonal and multiclonal models of tumor progression. Increasing color intensity correlates with tumor progression, whereas different colors reflect different clones. (A) Traditional, linear model of clonal succession, where progressive mutations in oncogenes and tumor suppressor genes drive linear succession of rounds of clonal expansion, manifested as tumor progression. (B) Multi-clonal model of tumor progression: although all cells in tumors originate from a single initiated cell, the evolution of the tumor is more “messy”, with genetically divergent tumor clones co-existing within tumors for substantial periods of time. The population sizes and characteristics of clones change as tumors evolve, with some clone populations expanding in size and others remaining unchanged or becoming extinct. In advanced stages of tumor evolution, tumors might become dominated by single clones.



**Figure 4 | Tumour heterogeneity in diagnostics.** Similar to inter-tumour heterogeneity, intra-tumour heterogeneity of cellular phenotypes that result from genetic and non-genetic influences can complicate definitive diagnostics and can obstruct therapeutic decision-making. First, spatial phenotypic heterogeneity can lead to a situation in which a biopsy does not provide an adequate reflection of the phenotypic composition of the whole tumour. Second, decisions made based on scoring the dominant phenotype in a given sample might be misleading if they do not account for minor subpopulations with clinically and biologically important distinct features.

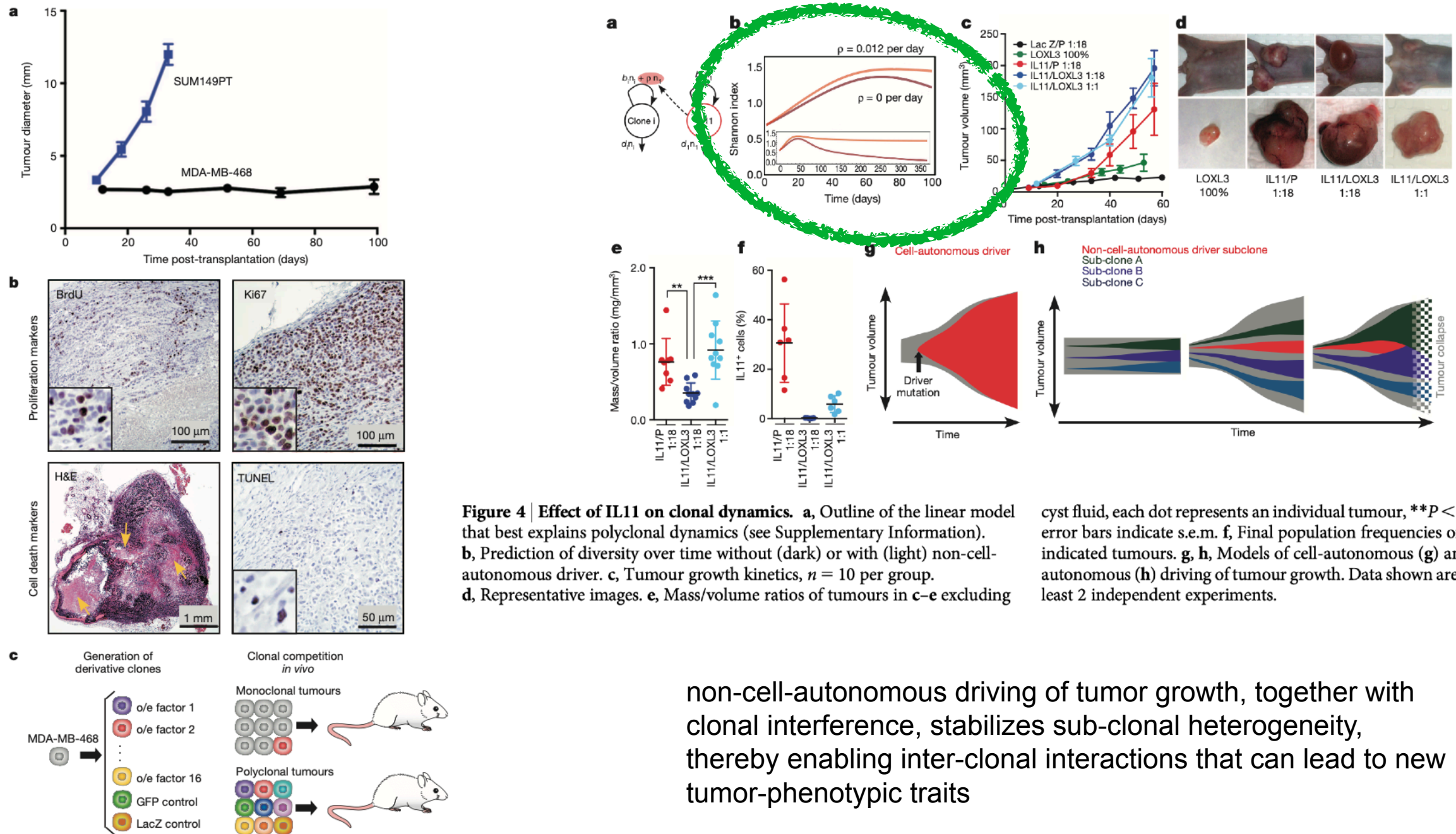
# proliferative signaling/outgrowth are hallmarks of cancer



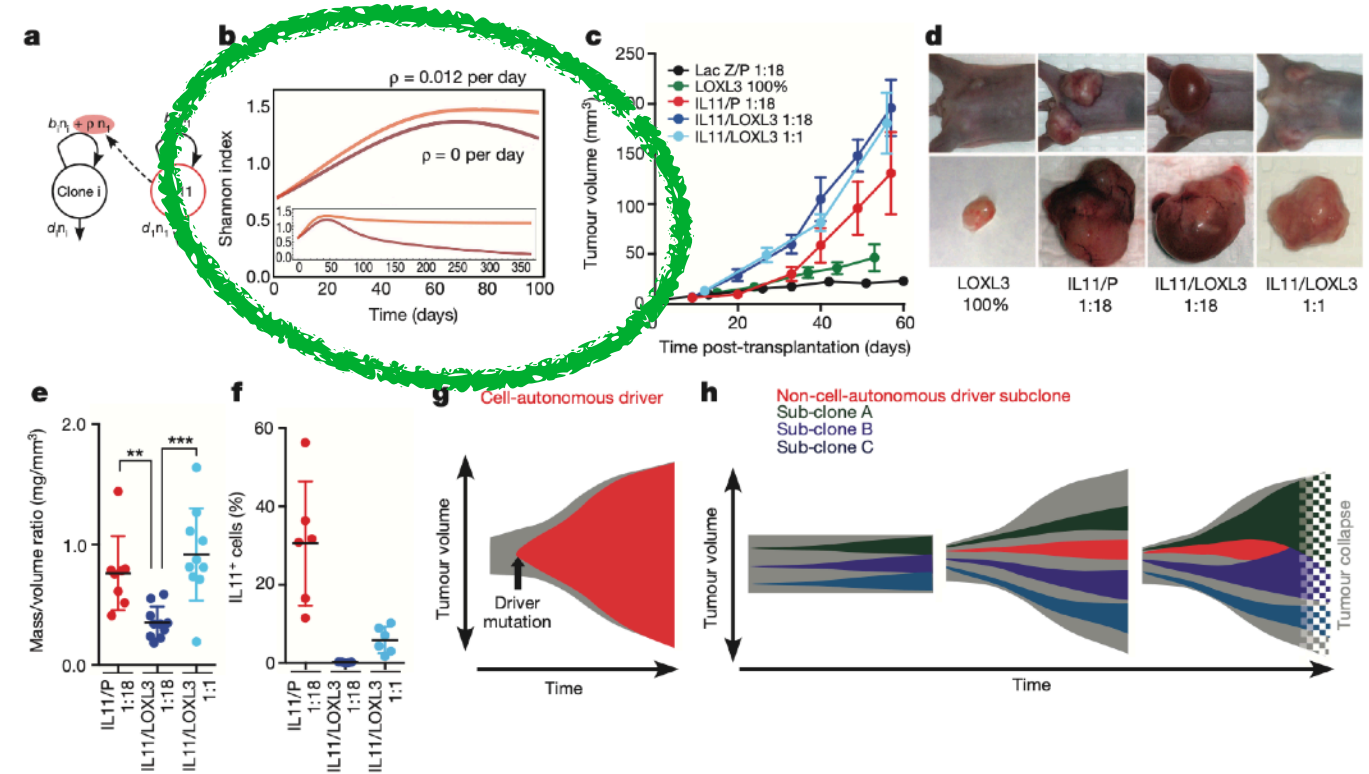
Hanahan & Weinberg, Hallmarks of Cancer: The Next Generation, Cell 2011



# Interactions among cancer clones can facilitate heterogeneity (vs. purifying selection)



**Figure 1 | Experimental system.** a, Growth of tumours upon mammary fat pad transplantation of indicated cell lines,  $n = 10$  per group, combined data from 2 independent experiments, error bars indicate s.e.m. b, Representative images of indicated staining. Arrows indicate necrotic areas. H&E, haematoxylin and eosin. c, Experimental scheme.



**Figure 4 | Effect of IL11 on clonal dynamics.** a, Outline of the linear model that best explains polyclonal dynamics (see Supplementary Information). b, Prediction of diversity over time without (dark) or with (light) non-cell-autonomous driver. c, Tumour growth kinetics,  $n = 10$  per group. d, Representative images. e, Mass/volume ratios of tumours in c–e excluding

cyst fluid, each dot represents an individual tumour, \*\* $P < 0.01$ , \*\*\* $P < 0.001$ ; error bars indicate s.e.m. f, Final population frequencies of IL11<sup>+</sup> cells in the indicated tumours. g, h, Models of cell-autonomous (g) and non-cell-autonomous (h) driving of tumour growth. Data shown are representative of at least 2 independent experiments.

non-cell-autonomous driving of tumor growth, together with clonal interference, stabilizes sub-clonal heterogeneity, thereby enabling inter-clonal interactions that can lead to new tumor-phenotypic traits

Marusyk *et al.*, Non-cell-autonomous driving of tumour growth supports sub-clonal heterogeneity, Nature 2014



# kinds of interactions in the tumor eco system

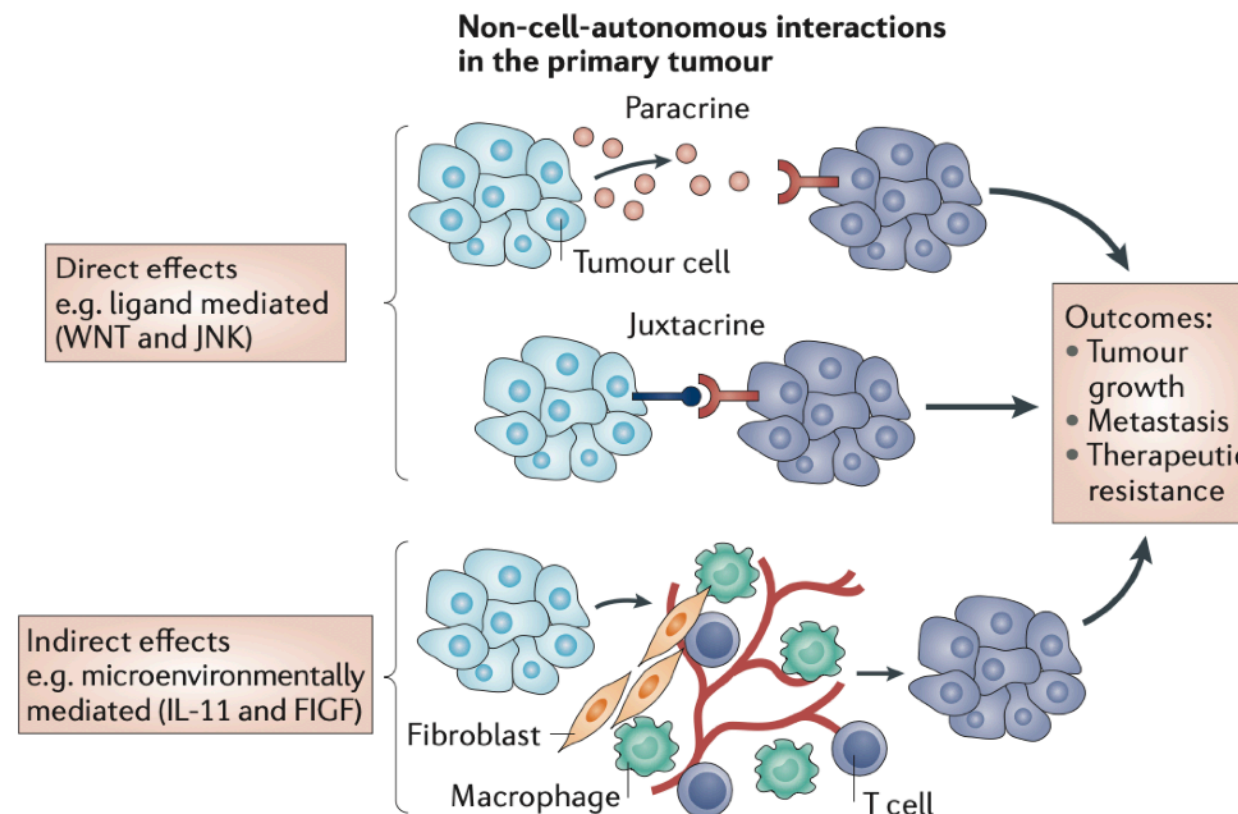
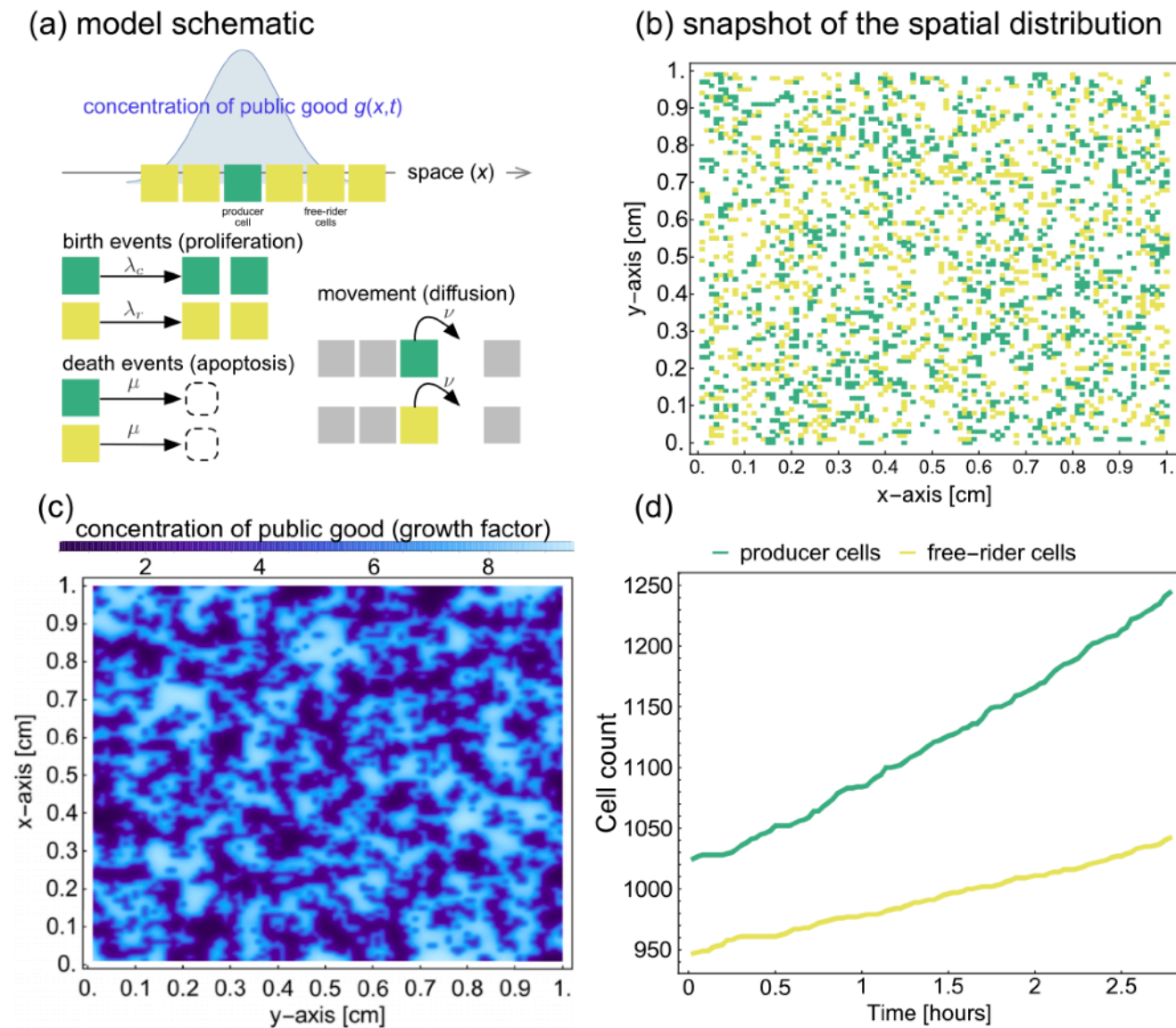


Figure 1 | **Non-cell-autonomous interactions between populations can affect tumorigenesis, metastasis and therapeutic resistance.** Non-cell-autonomous interactions may contribute to increasing the robustness of the tumour, leading to increased tumour growth, enhanced metastasis and the emergence of resistance. As exemplified by two distinct cellular populations communicating in a unidirectional manner, such interactions may occur directly through paracrine<sup>27-29</sup> or juxtacrine effects of ligands<sup>49-52,54</sup> that are produced by one cell and received by the second, or these interactions could also be indirectly mediated via components of the microenvironment, such as blood vessels, immune cells and fibroblasts<sup>28,30,49-51</sup>. FIGF, c-fos induced growth factor; IL-11, interleukin-11; JNK, JUN N-terminal kinase.

# cells grow, move, some produce, public good diffuses and decays

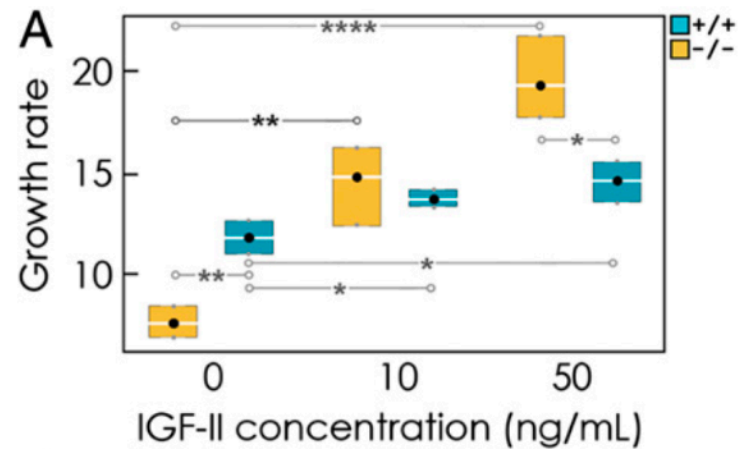


Spectral decomposition of cell densities to derive conditions of the expected growth rate advantage

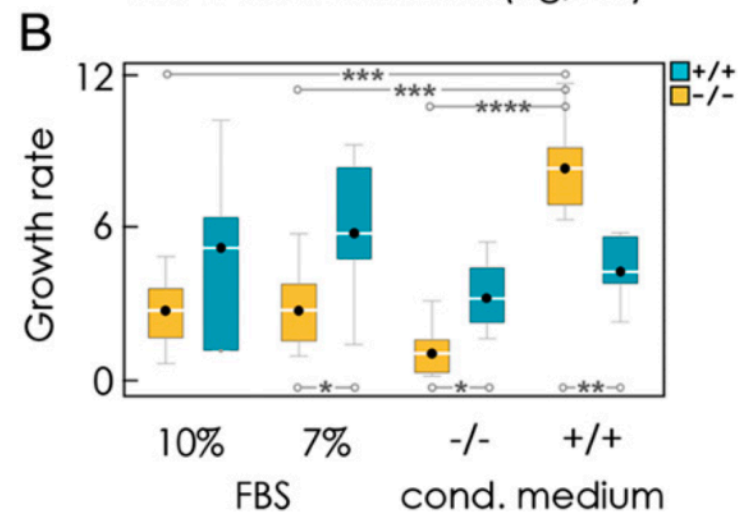
Exactly describes the dynamics of a randomly assorted population, and serves as good general approximation

Key assumptions: time scale separation, limited dispersal. Gerlee & Alrock, Physical Review E (2019)

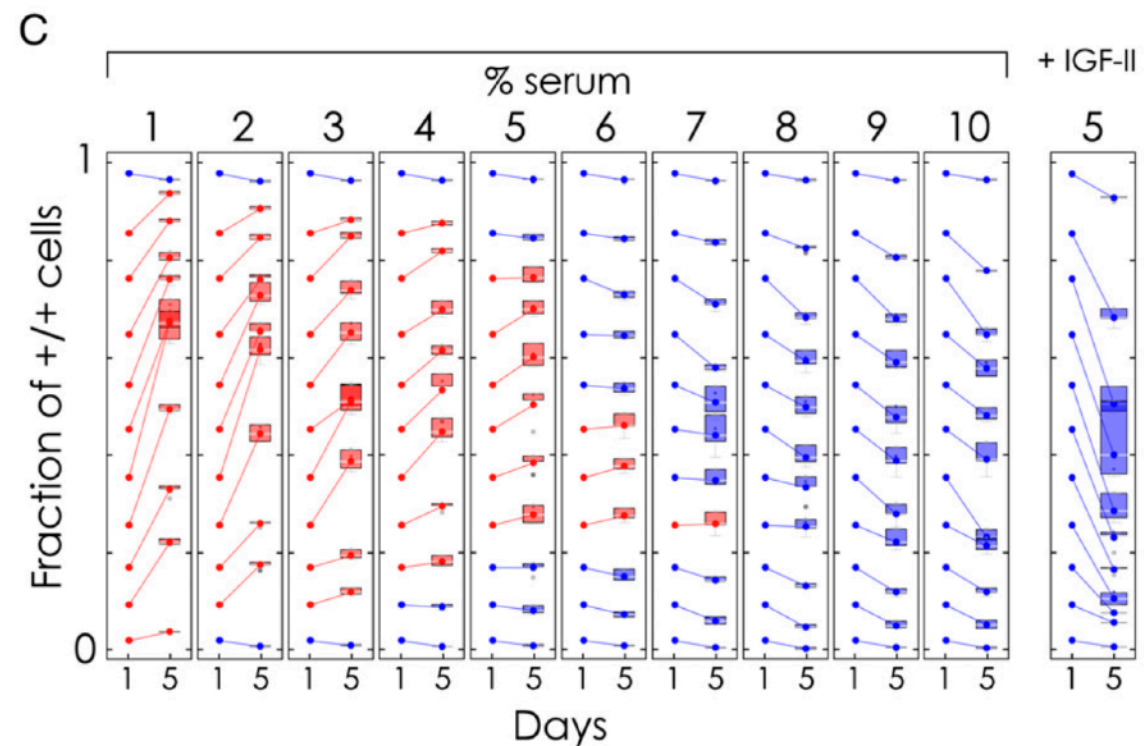
# growth factors influence competition between producers (C) and non-producers (D)



there can be a cost of production

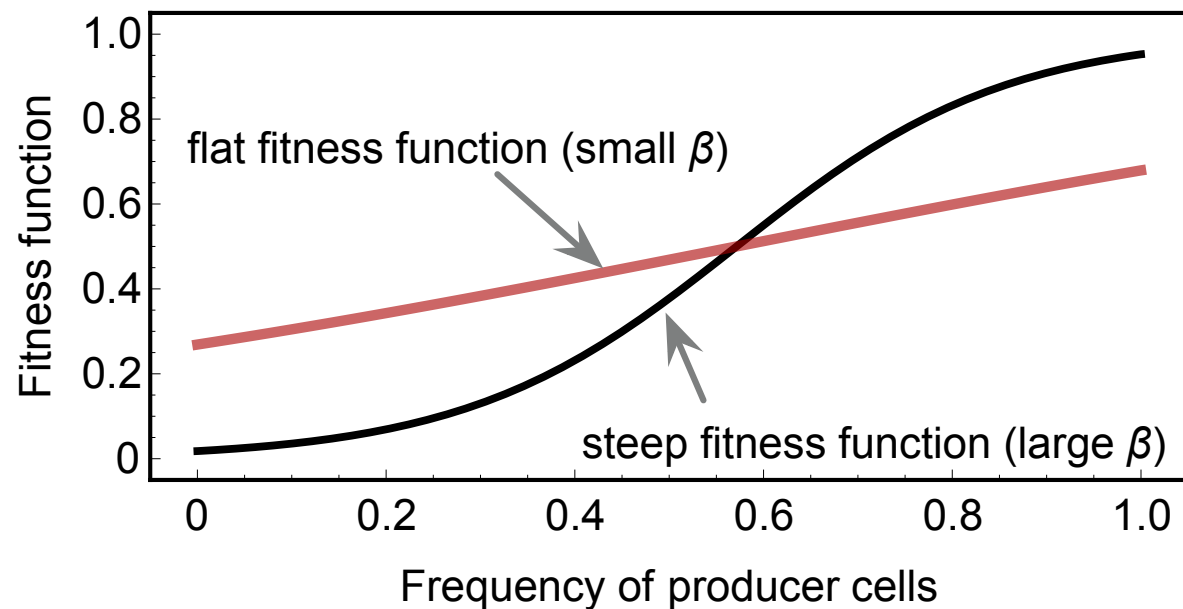
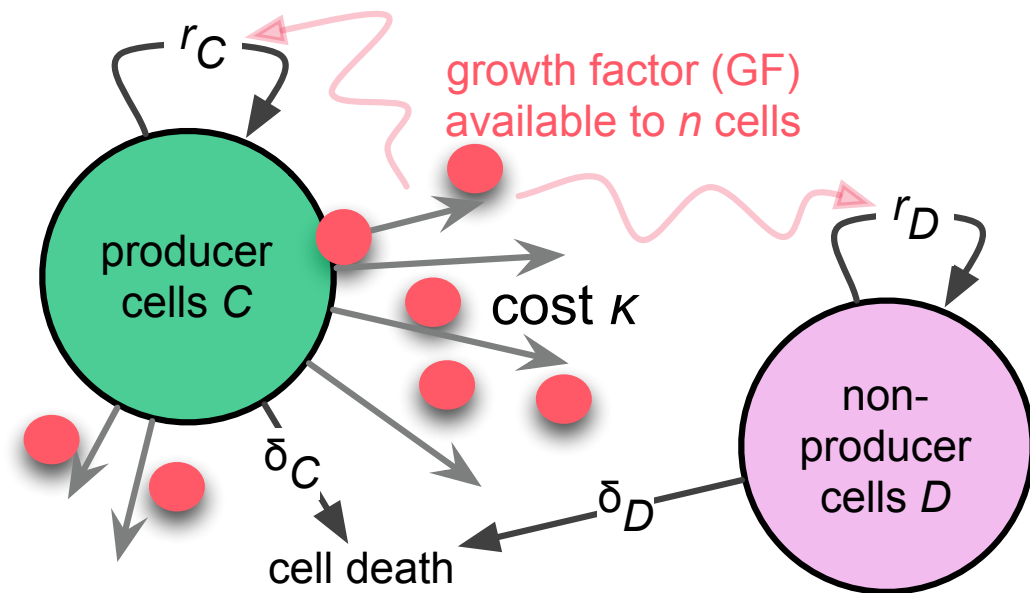


the system can have multiple equilibria



**Fig. 1.** IGF-II is a nonlinear public good. (A) Growth rates of producer (+/+) and nonproducer (-/-) cells in vitro (relative to day 1) at different concentrations of exogenous IGF-II in the growth medium. (B) Growth rates of +/+ and -/- cells in vitro (relative to the day with the minimum number of cells) with medium containing FBS (7% or 10%) or in conditioned (cond.) medium from -/- or +/+ cultures. Box plots show the median and the 25% and 75% quartiles (upper and lower fences, respectively). Asterisks show significant *P* values in a *t* test: \**P* < 0.05; \*\**P* < 0.005; \*\*\**P* < 0.0005; \*\*\*\**P* < 0.00005.

# nonlinear growth as a function of (locally) available public good



$$\lambda(G) = \alpha \frac{1 + e^\sigma}{1 + e^{\sigma - \beta G}}$$

background fitness (green arrow pointing to  $e^\sigma$ )

growth factor concentration (grey arrow pointing to  $\lambda(G)$ )

selection coefficient (red arrow pointing to  $\beta G$ )

good (local) (blue arrow pointing to  $\beta G$ )

producer growth:  $r_D = \lambda - \kappa$

free-rider growth:  $r_D = \lambda$

but the local concentration of  $G$  might differ

Multiple works by Marco Archetti. Gerlee, Kimmell, Brown & Altrock (2019).

Of note: Hauert, Michor, Nowak, Doebeli. "Synergy and discounting of cooperation in social dilemmas", JTB, 2006.

public good can be shared among a 'neighborhood' of size  $n$ ,  
 a producer cells experience a benefit-to-self

$$r_D = \alpha \frac{1 + e^\sigma}{1 + e^{\sigma - \beta \frac{n-1}{n} y_C}}$$

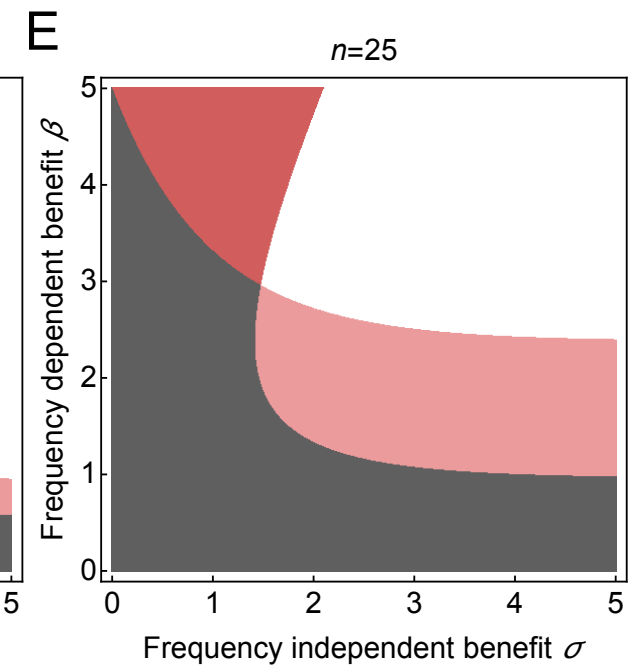
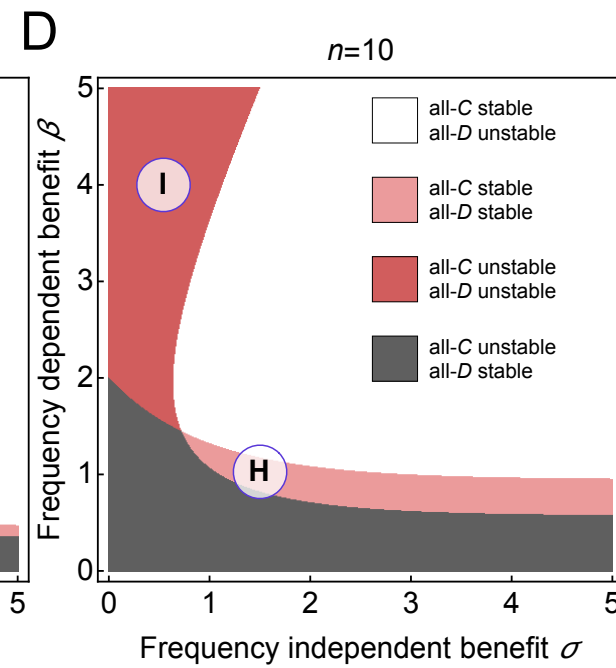
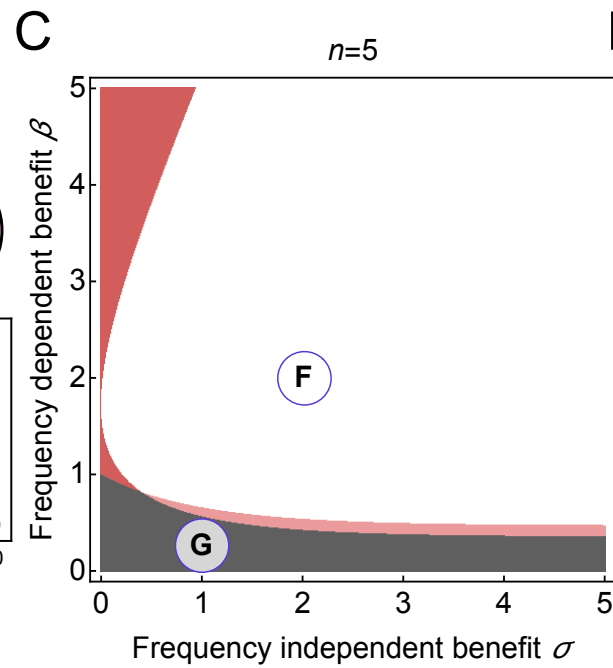
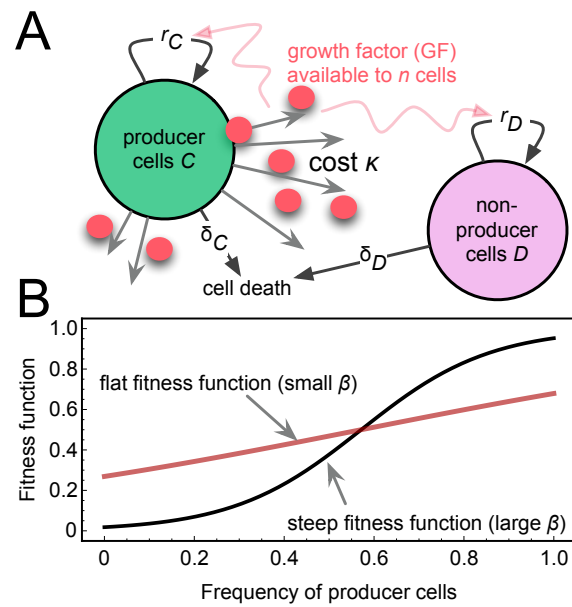
$$r_C = \alpha \frac{1 + e^\sigma}{1 + e^{\sigma - \beta \frac{1+(n-1)y_C}{n}}} - \kappa$$

population growth (eco-evolutionary dynamics)

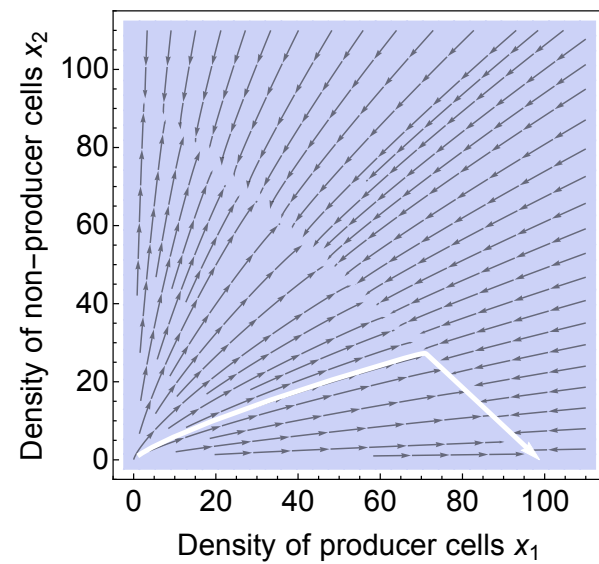
$$\dot{x}_C = r_C \left( 1 - \frac{x_C + x_D}{K} \right) x_C - \mu_C x_C$$

$$\dot{x}_D = r_D \left( 1 - \frac{x_C + x_D}{K} \right) x_D - \mu_D x_D$$

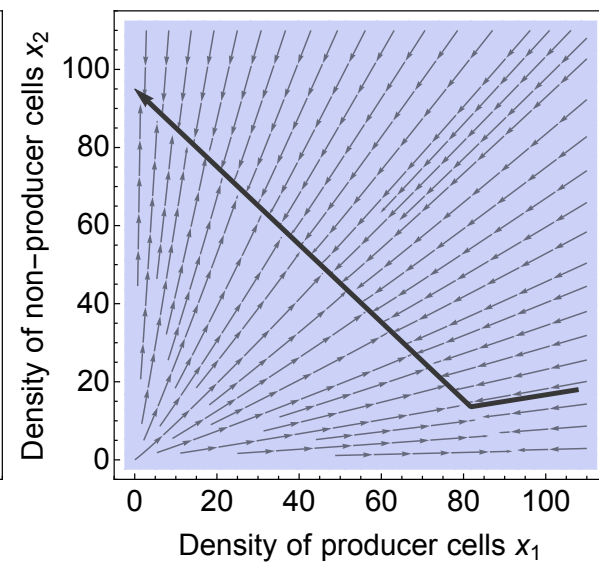




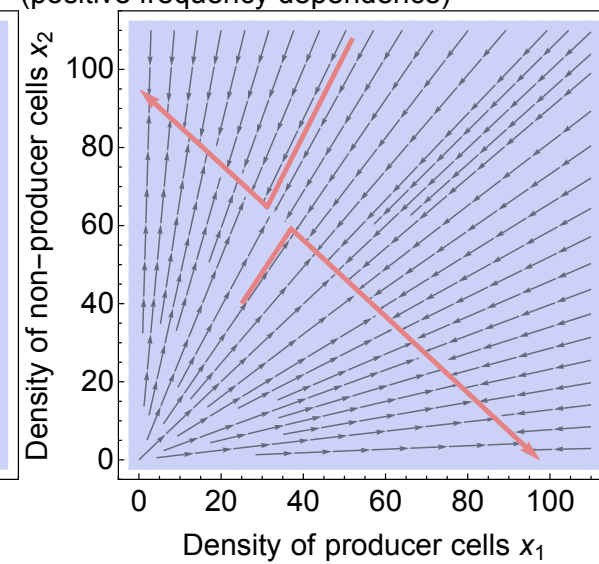
**F**  $\beta, \sigma \gg 1$ : producers win



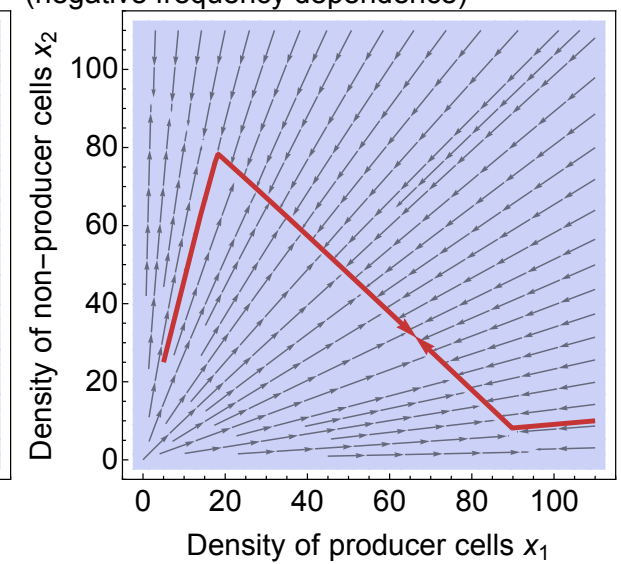
**G**  $\beta, \sigma \ll 1$ : non-producers win



**H**  $1-1/n < \beta < \sigma$ : bi-stable monomorphism (positive frequency dependence)

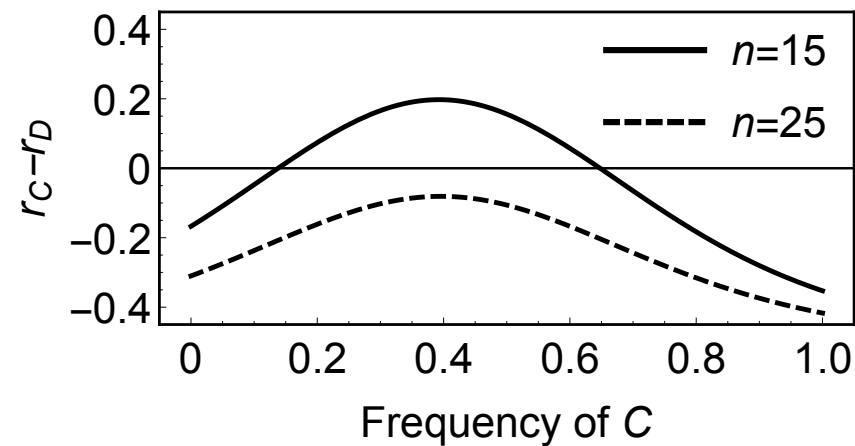


**I**  $\beta \gg \sigma$ : stable polymorphism (negative frequency dependence)

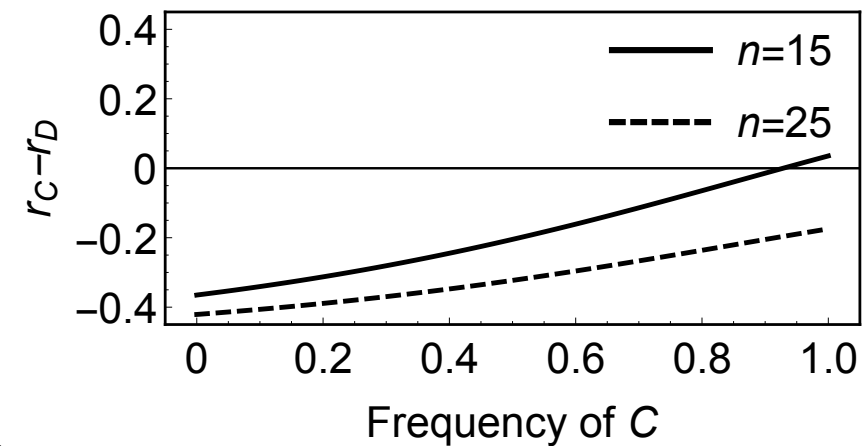


# bifurcations in nonlinear public good

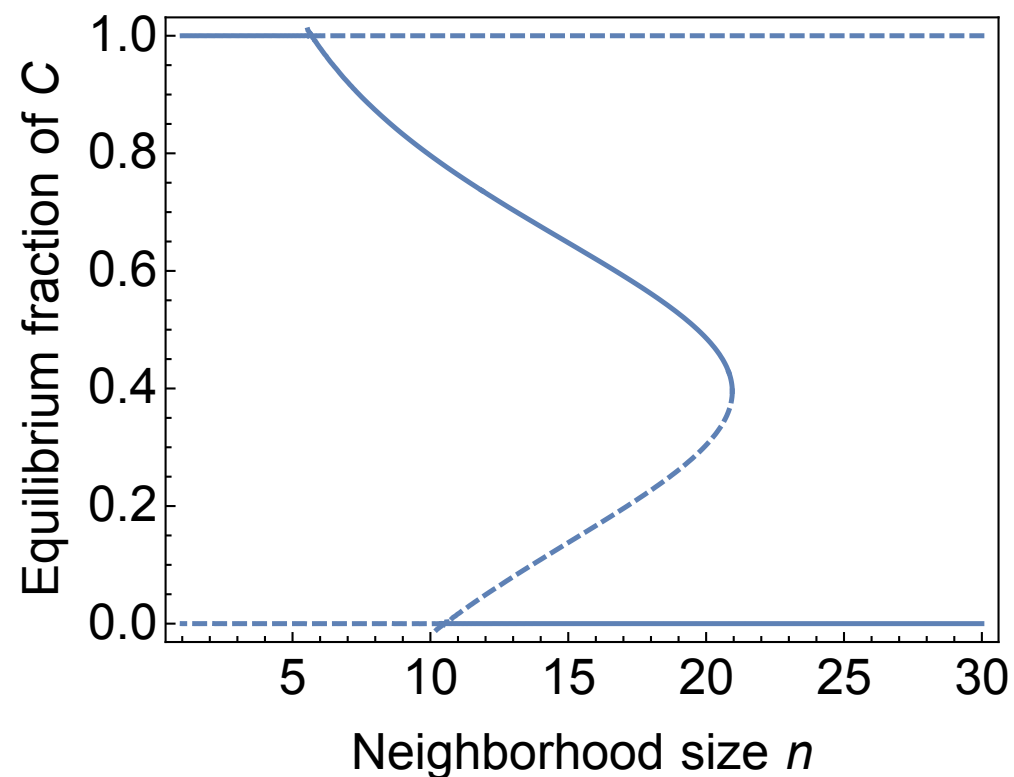
**A**  $\sigma=2, \beta=5$ : growth rate difference



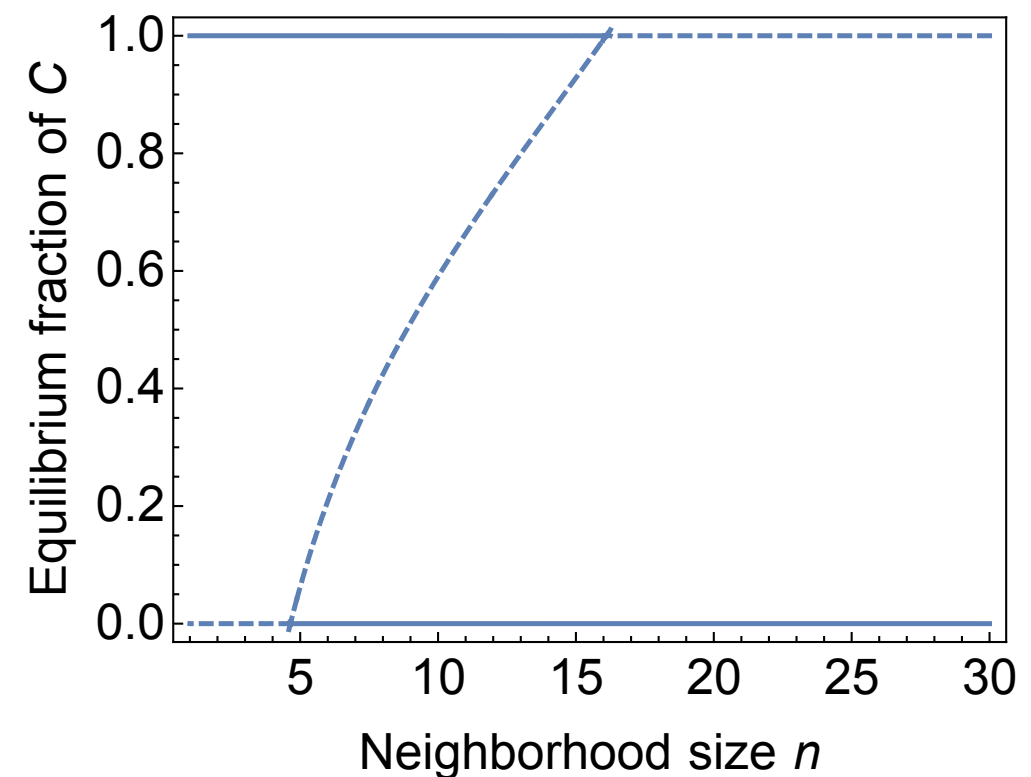
**C**  $\sigma=3, \beta=2$ : growth rate difference



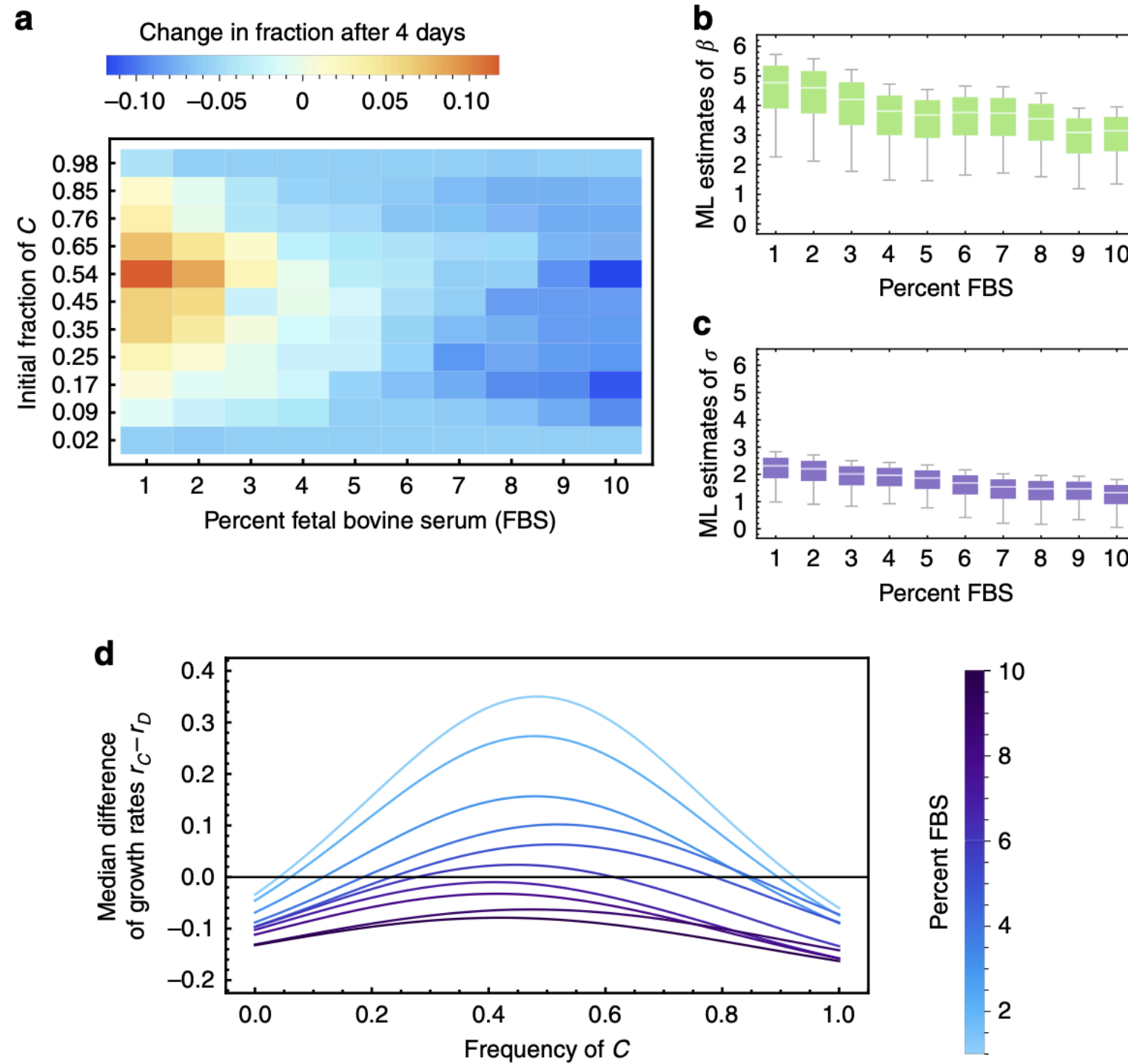
**B**  $\sigma=2, \beta=5$ : saddle-node bifurcation



**D**  $\sigma=3, \beta=2$ : stable state-switching

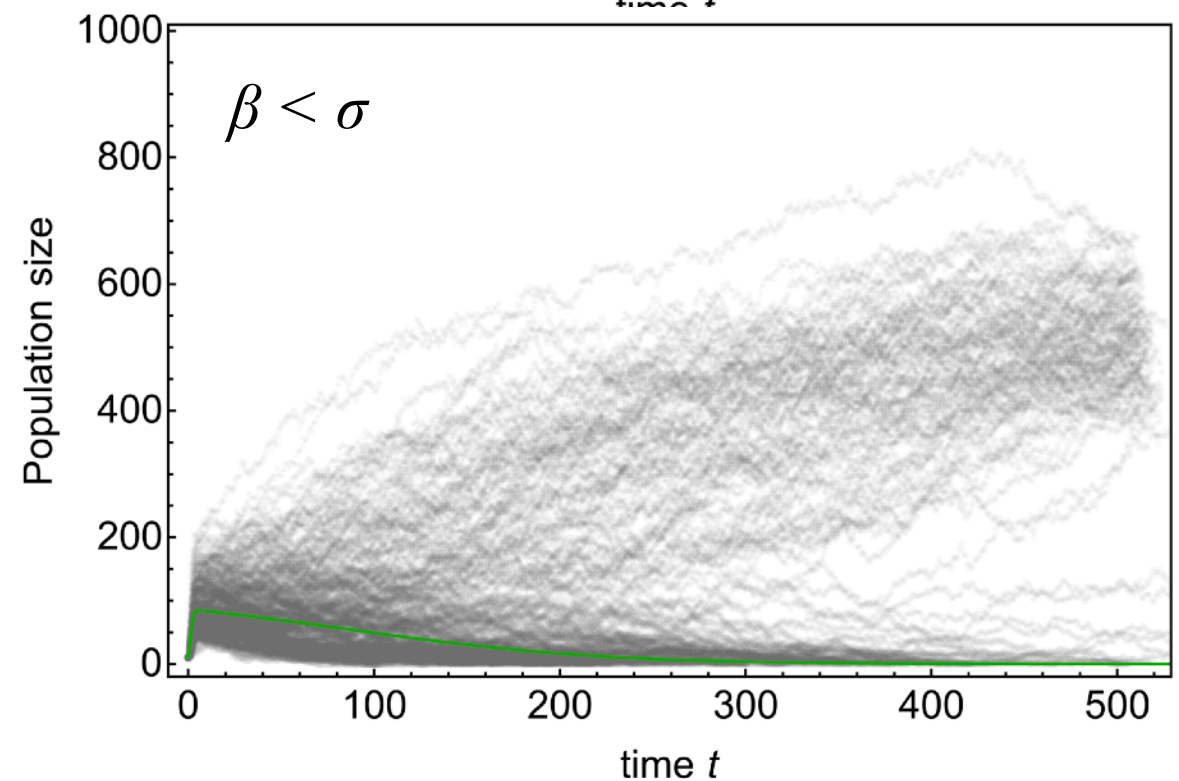
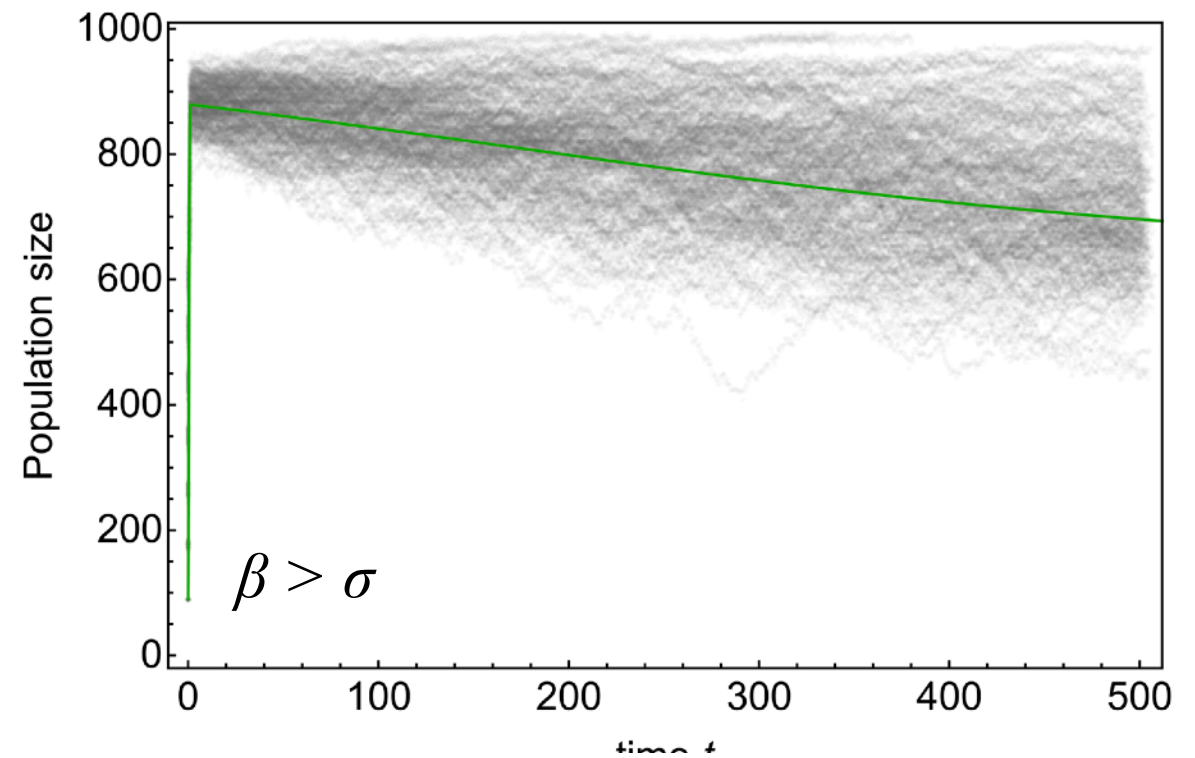
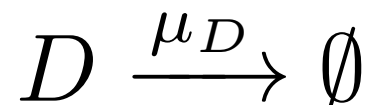
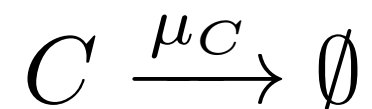
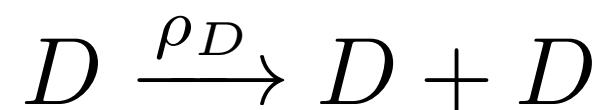
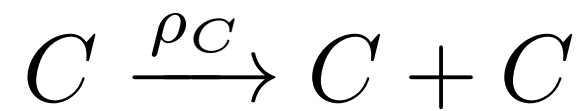


# our approach can be used to explain previously measured growth rate differences

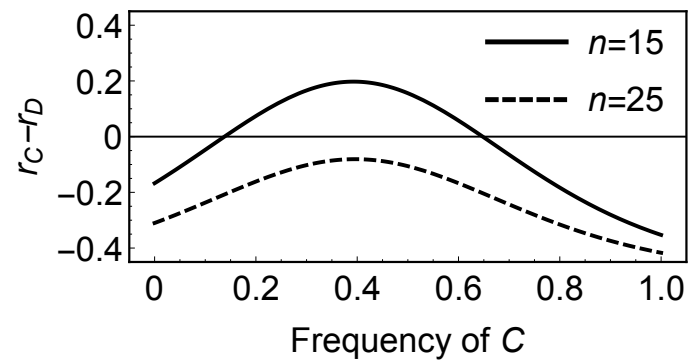




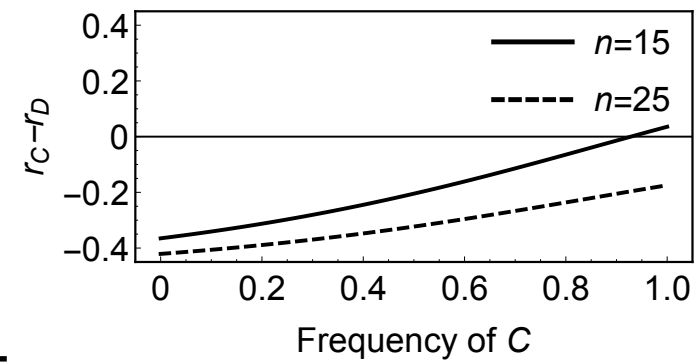
# stochastic dynamics due to demographic noise



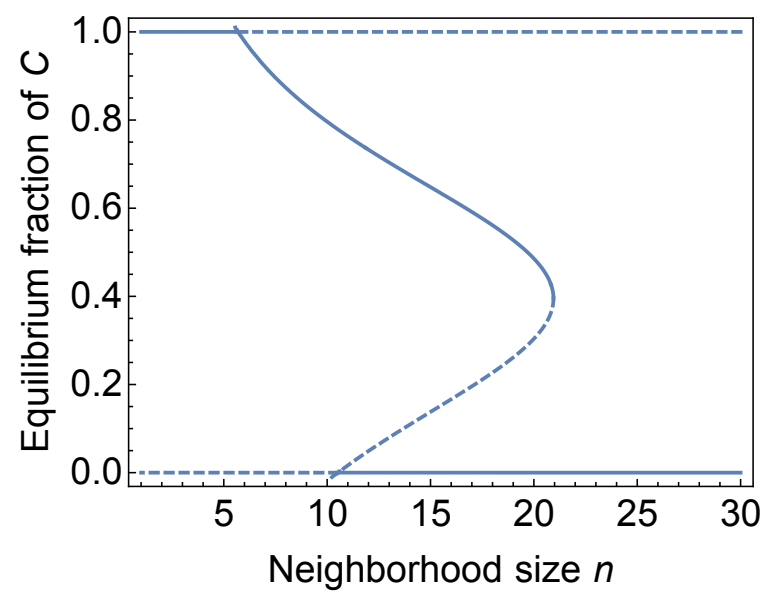
**A**  $\sigma=2, \beta=5$ : growth rate difference



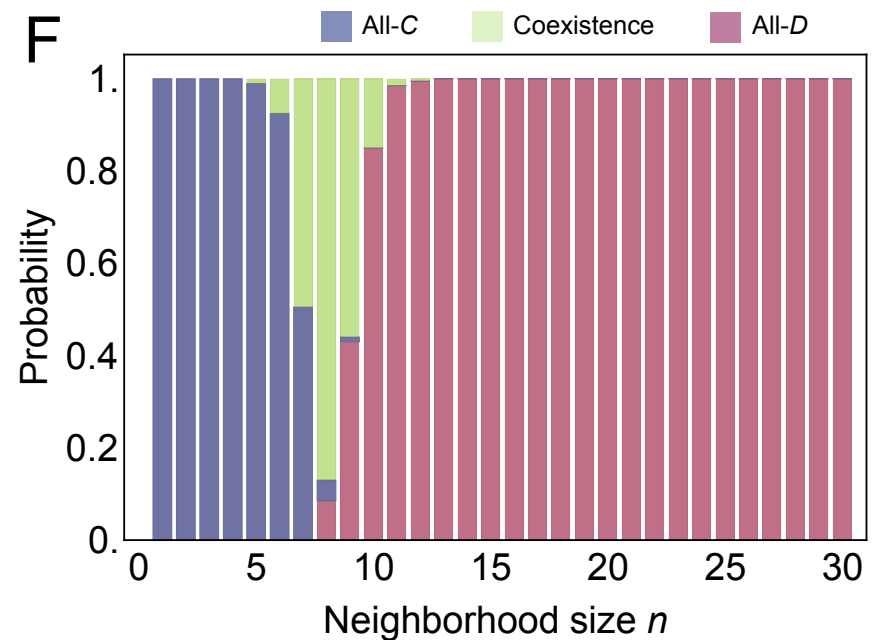
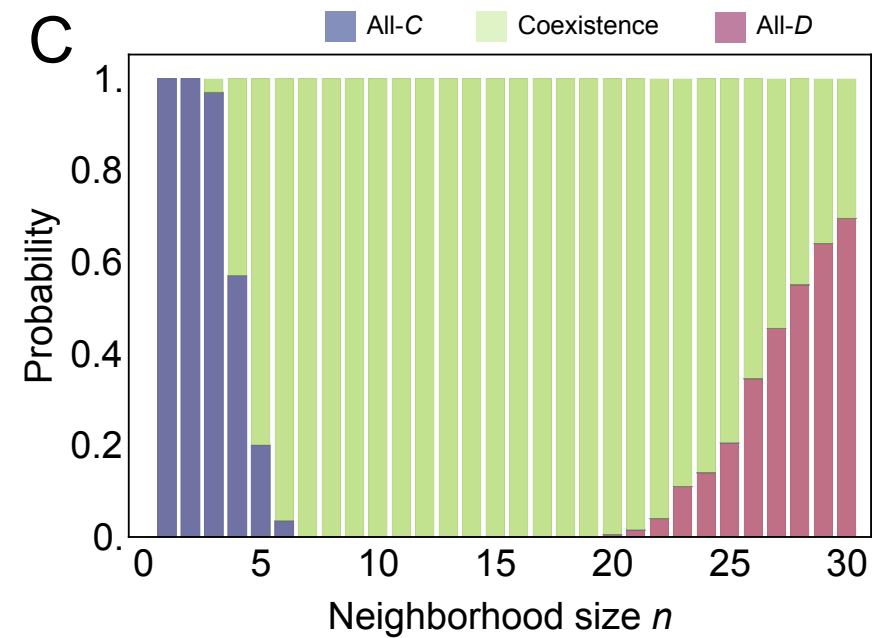
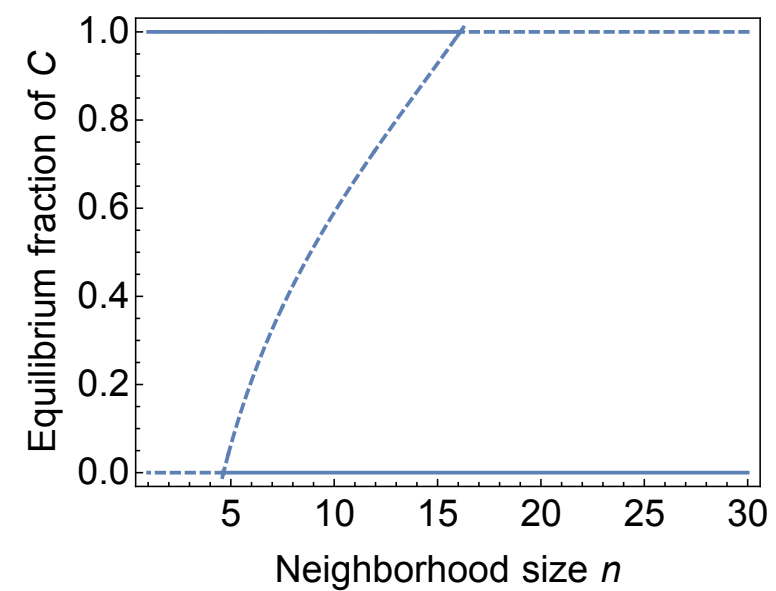
**D**  $\sigma=3, \beta=2$ : growth rate difference



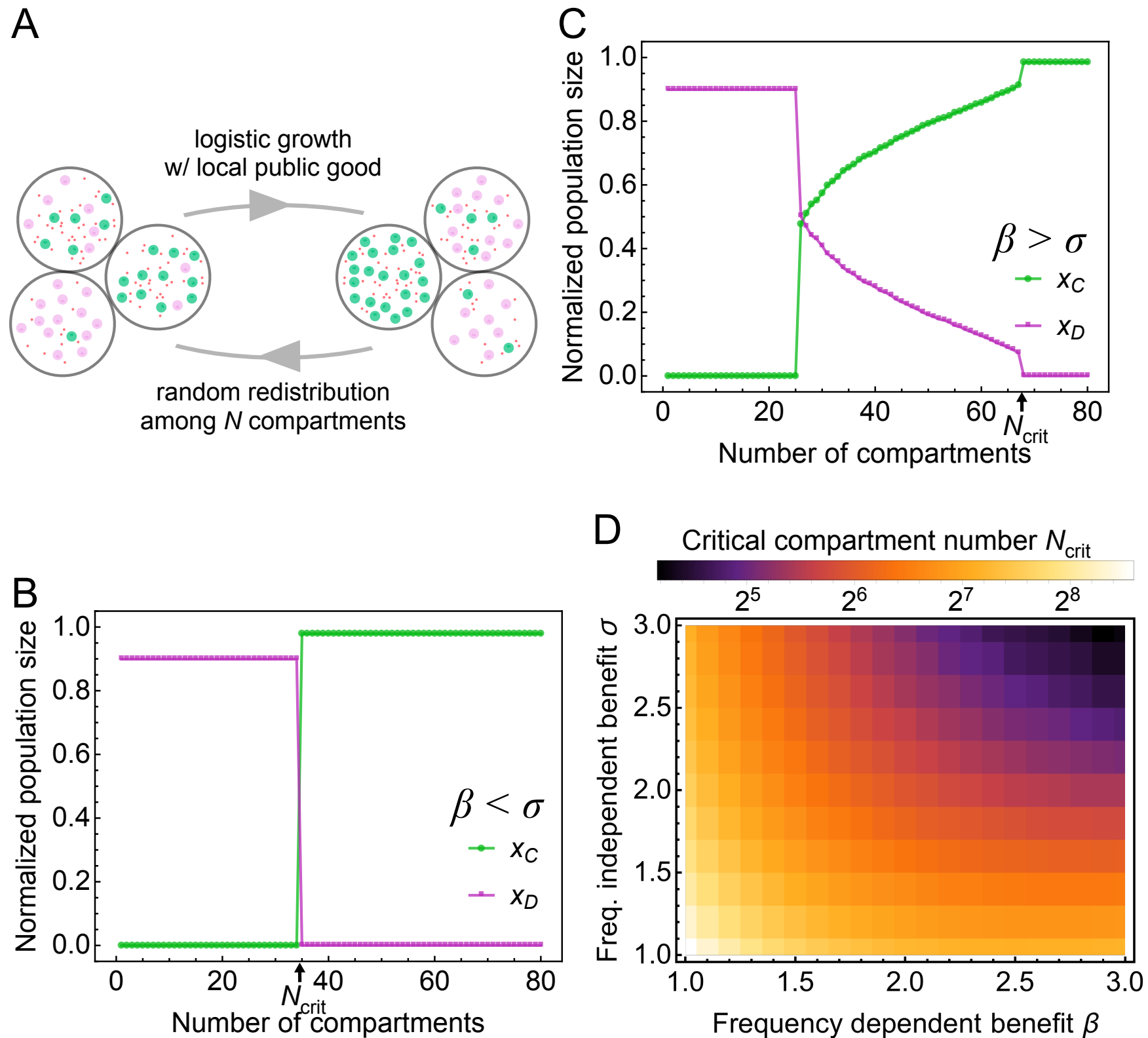
**B**  $\sigma=2, \beta=5$ : saddle-node bifurcation



**E**  $\sigma=3, \beta=2$ : stable state-switching

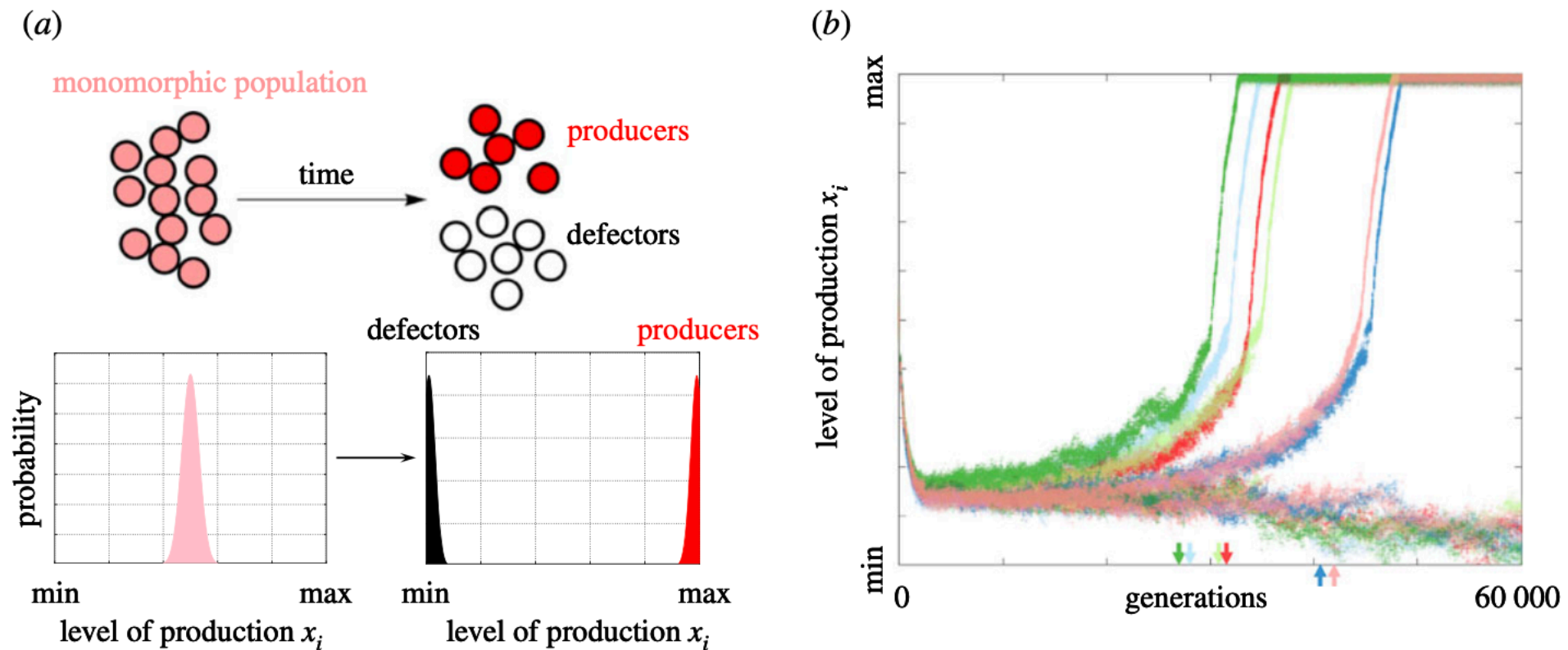


# the role of population partition: emerging neighborhood size



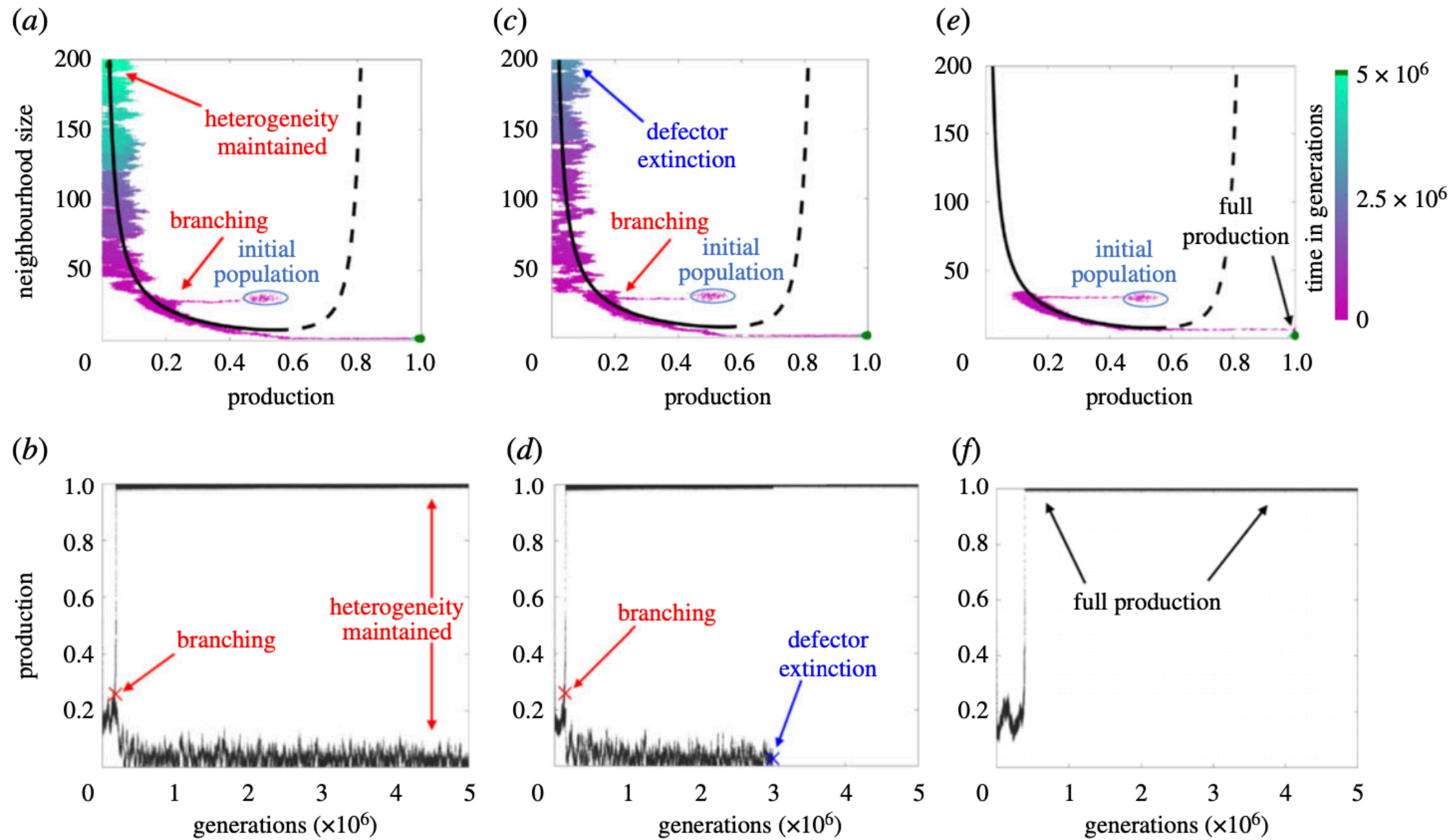
see also works by Cremer, Frey et al. on selection-redistribution models

# origins of producers and free-riders in nonlinear public goods games



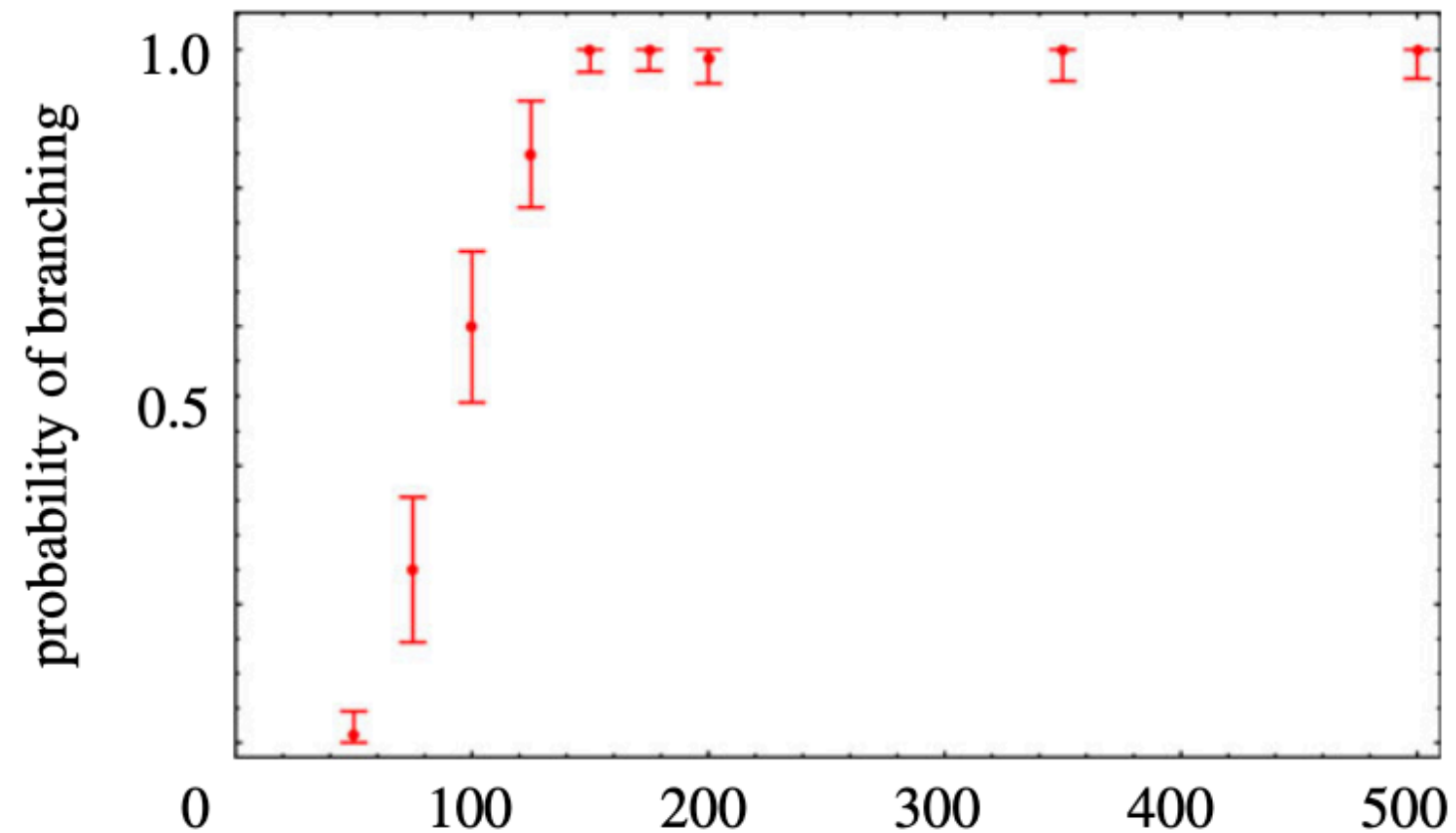
Johnson, Altrock & Kimmell, Royal Society Open Science 2021  
Doebeli, Hauert & Killingback, Science 2004

# adaptive dynamics in 2D trait space



Johnson, Altrock & Kimmell, Royal Society Open Science 2021

# evolutionary branching depends on population size



Johnson, Altrock & Kimmell, Royal Society Open Science 2021

# summary

Public goods games as a potential mechanism for cancer robustness/progression:  
non-autonomous expansion (context-dependent selection)

Shared (costly) resources can act as public goods in growing cell populations

Coexistence/producer success is impacted by nonlinearity

Population assortment and demographic noise can lead to producer invasion

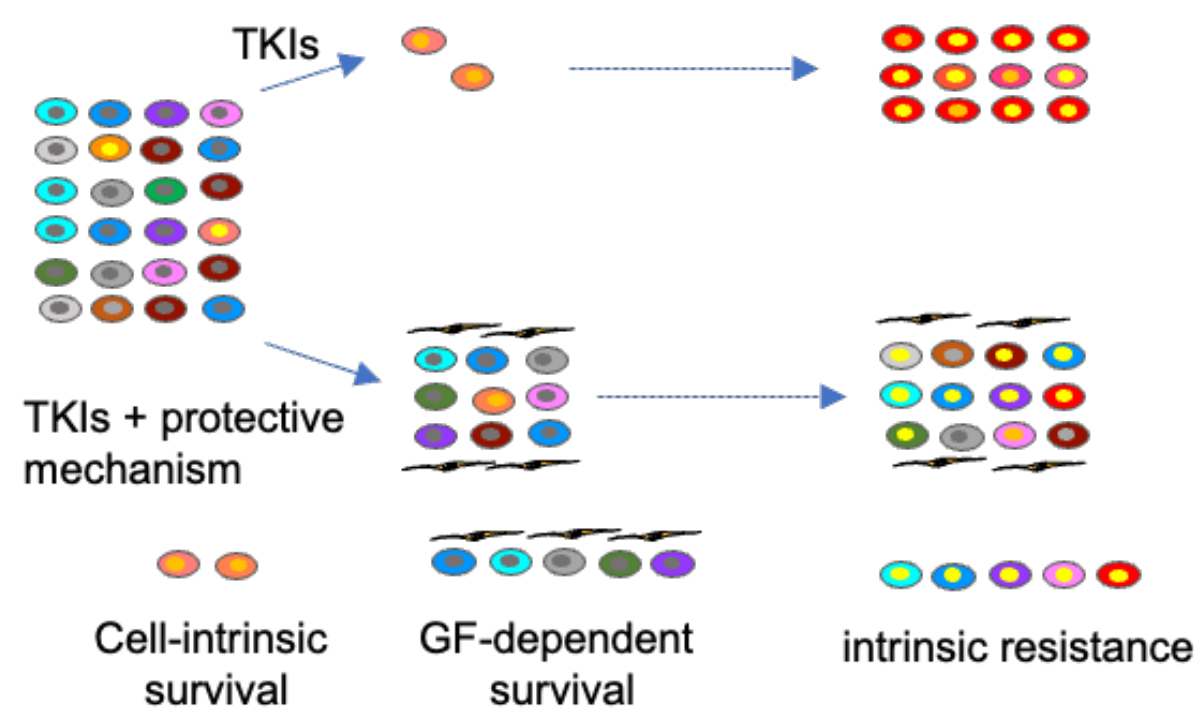
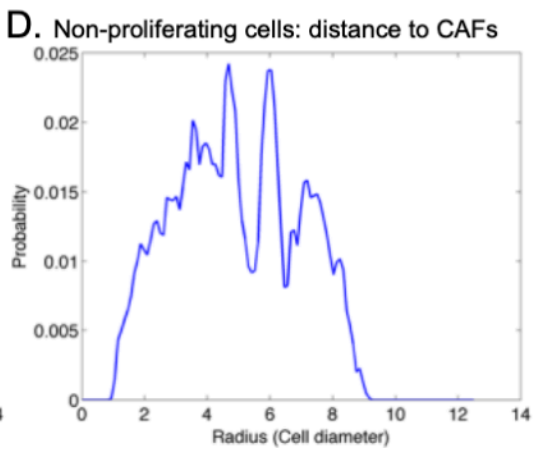
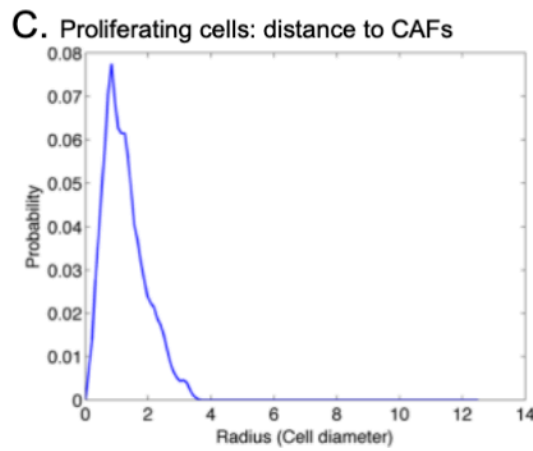
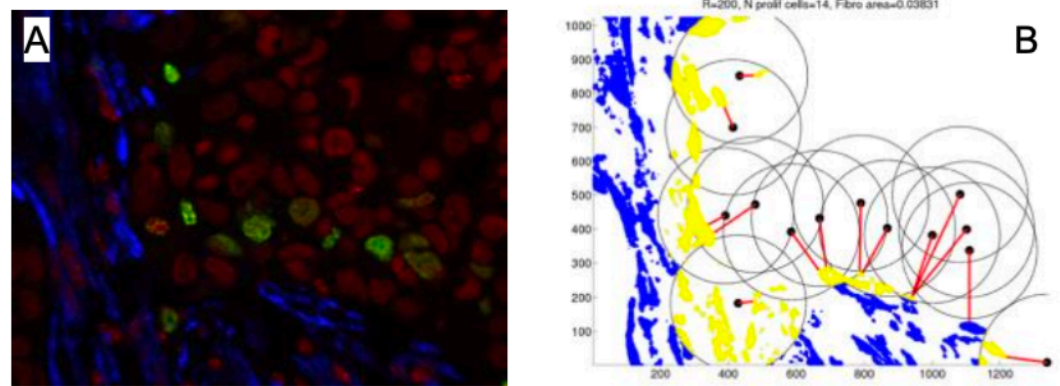
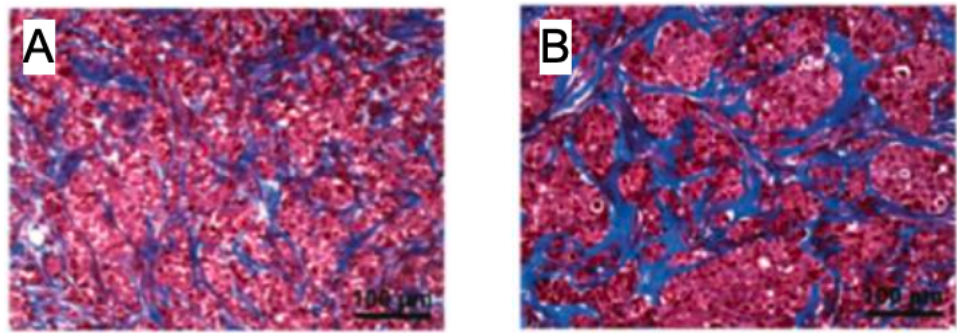
Current challenge: how to make these insights more useful for clinical applications?



# we want to understand diversity in outcomes based on understanding ecological tumor diversity

a) defining the “neighborhood” of interactions from clinical samples

b) protection from growth factors (GFs) during therapy

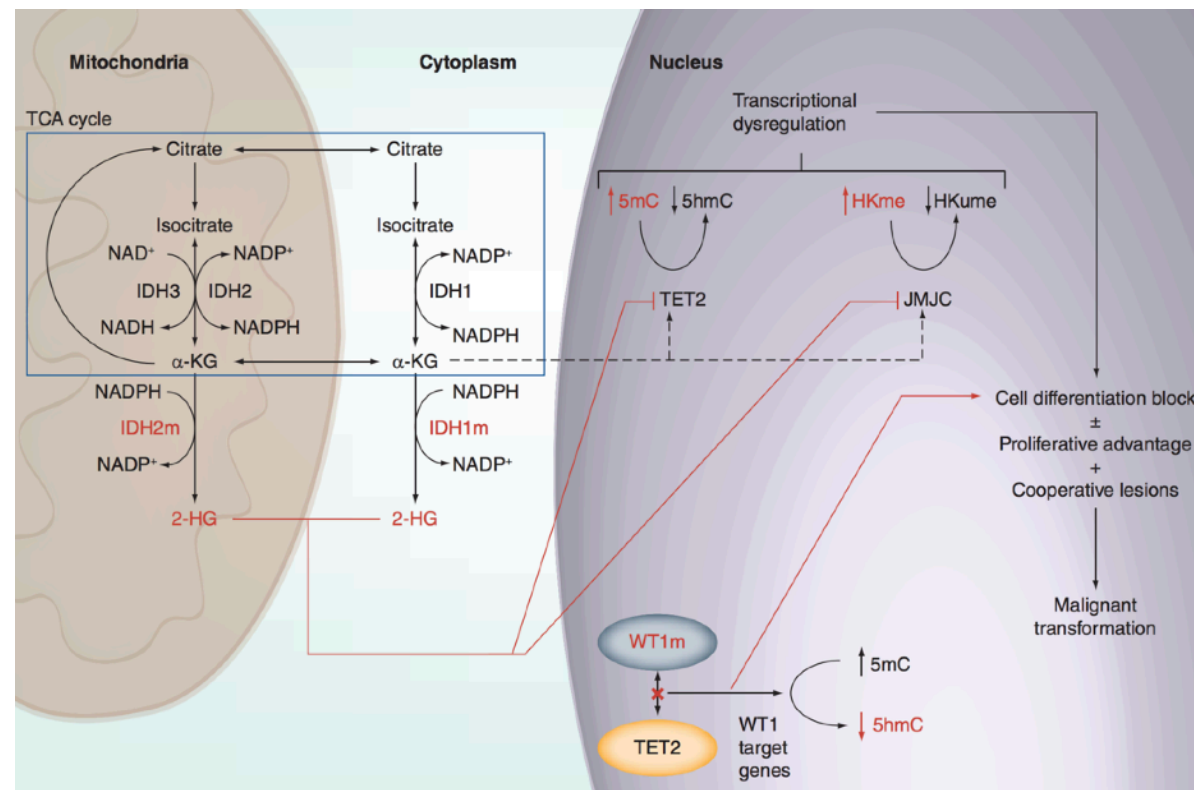


CAF: Cancer Associated Fibroblast. TKI: Tyrosine Kinase Inhibitor



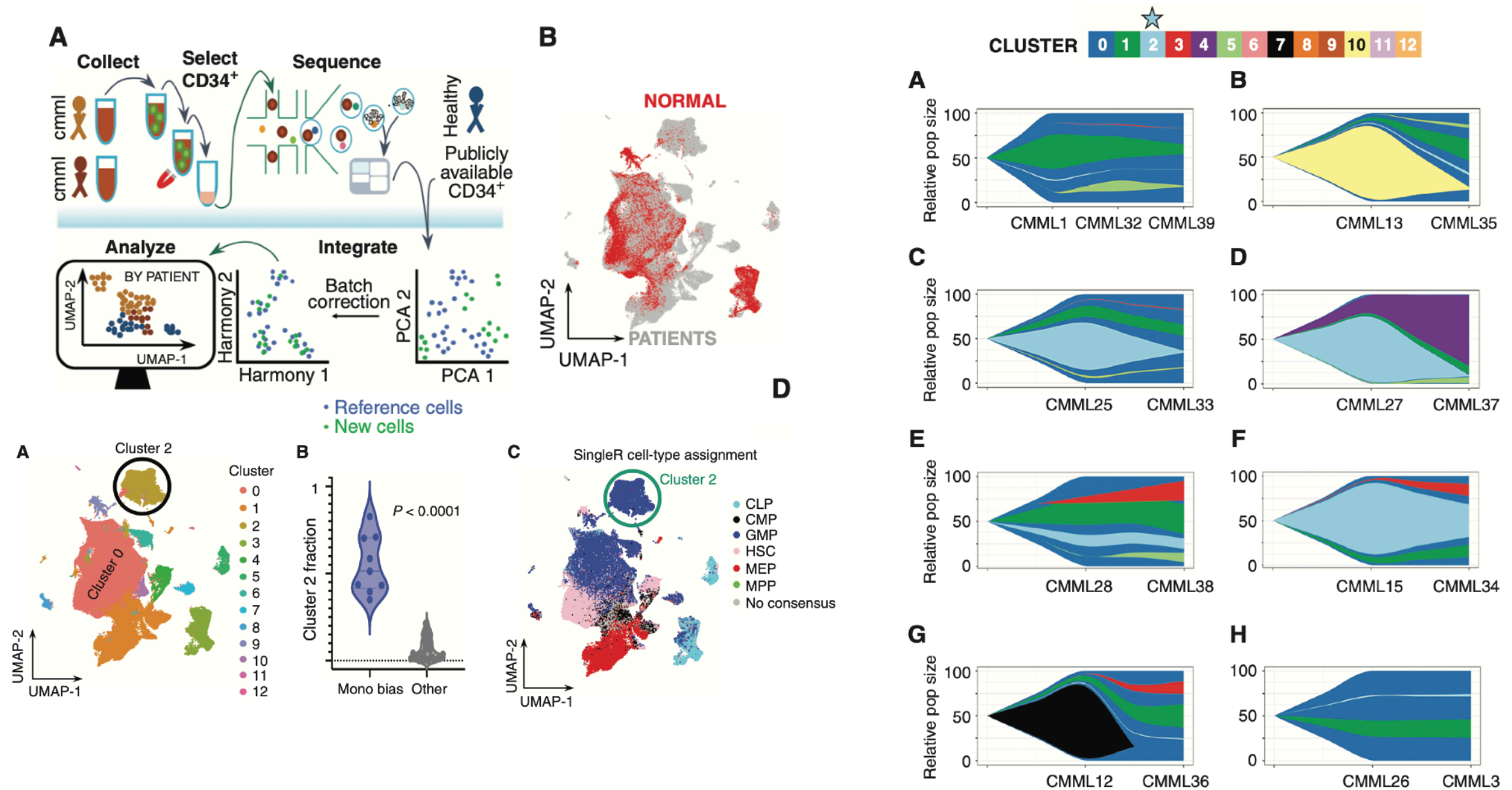
# leukemia evolution could be impacted by mutations that evoke public good-like properties

- ‘classical’ drivers: KRAS, NRAS,... may act cell autonomously
- transcription-modifying genes may act non-autonomously
- IDH 1/2 cells can alter self and other cells’ aberrant epigenetic programming (changes methylation, usually increasing), e.g., leading to higher fitness during inflammation



Garrett-Bakelman FE and Melnick AM. Mutant IDH: a targetable driver of leukemic phenotypes linking metabolism, epigenetics and transcriptional regulation. *Epigenomics*.8:945, 2016.

# selection dynamics could inform our understanding of inflammation-driven progression dynamics



Ferrall-Fairbanks *et al.*, Blood Cancer Discovery (2022).  
 UMAP: Uniform Manifold Approximation and Projection.  
 CMML: Chronic Myelomonocytic Leukemia.



Meghan Ferrall-Fairbanks

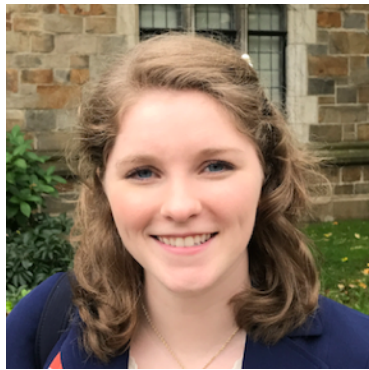


Brian Johnson

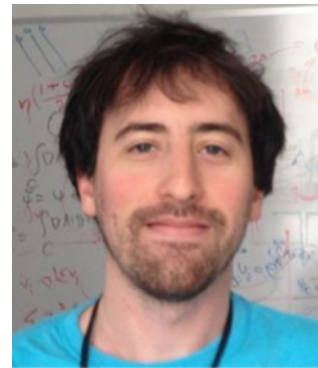


Eric Padron

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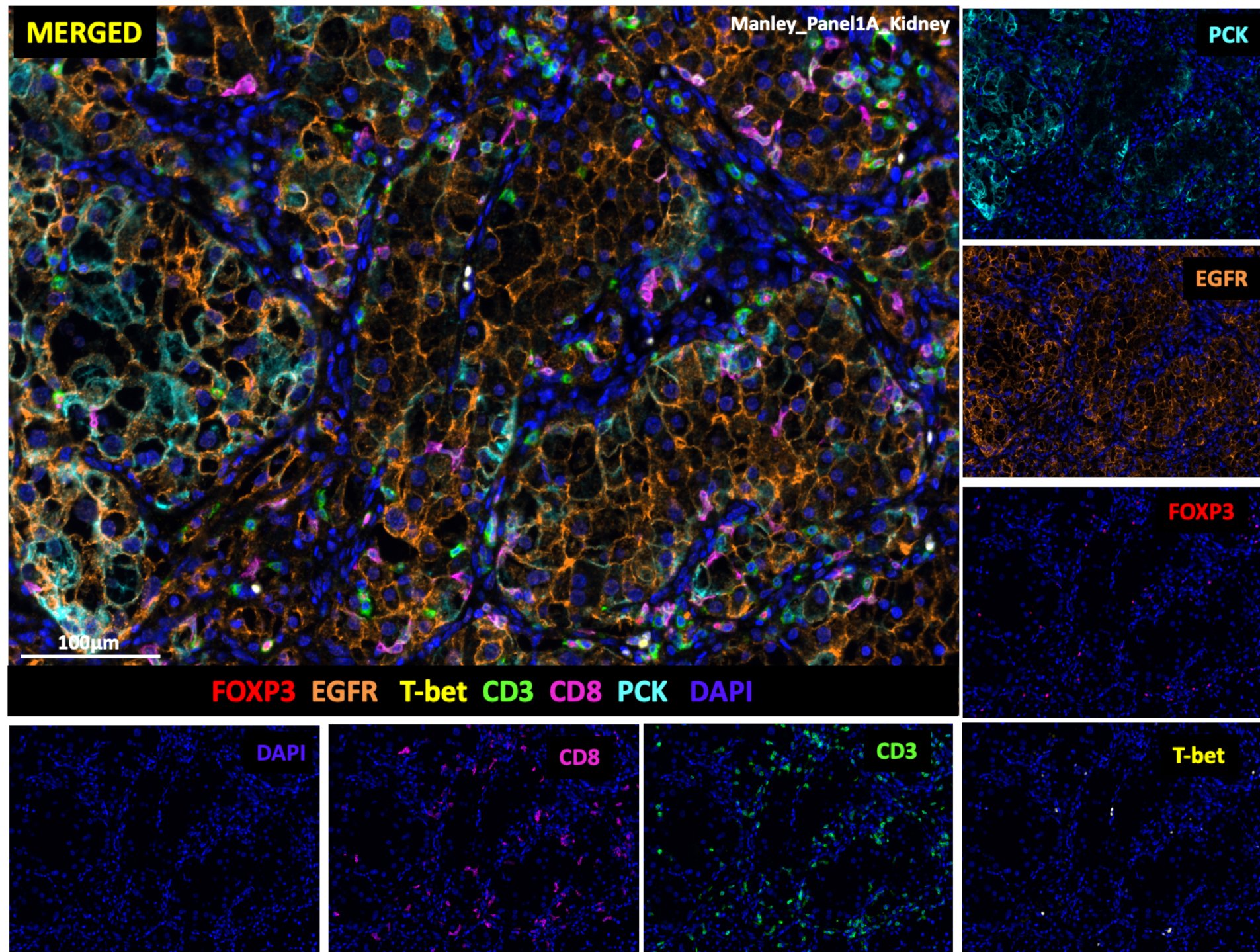
Brian Johnson, BSc  
(UCSD)



Noemi Andor, PhD  
(Moffitt)



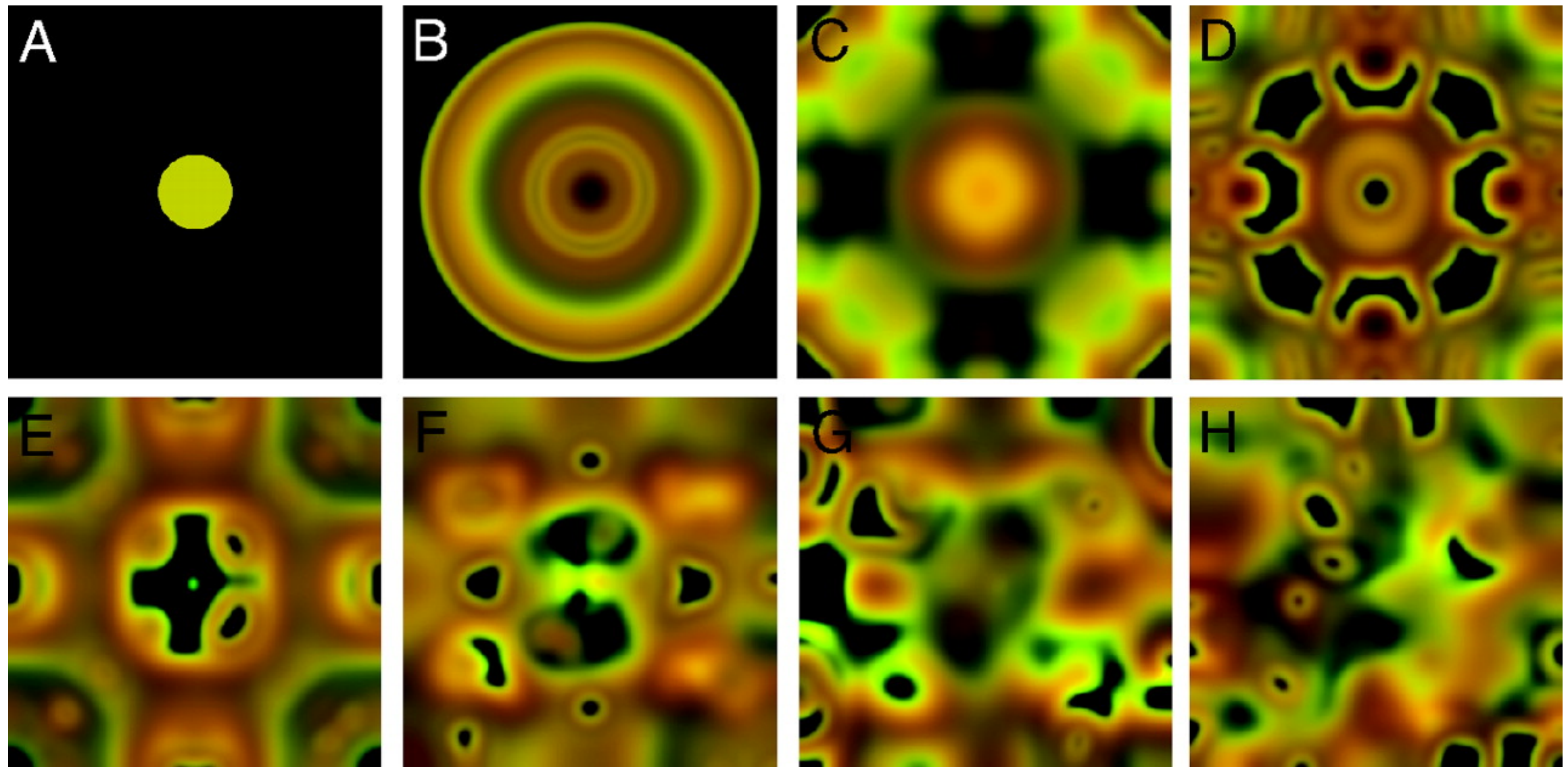




Courtesy of Brandon Manley, MD & Jonathan Nguyen (Moffitt)



# public goods in space



Wakano, Nowak & Hauert, PNAS 2009

# coupled equations for producers ( $U$ ), free-riders ( $V$ ), and public good ( $G$ )

$$\dot{U} = \Gamma_U \nabla^2 U + [\lambda(G) - \kappa][1 - (U + V)]U - \mu_U U,$$

$$\dot{V} = \Gamma_V \nabla^2 V + \lambda(G)[1 - (U + V)]V - \mu_V V,$$

$$\dot{G} = \Gamma_G \nabla^2 G + \rho U - \delta G(U + V).$$

**Table 1. Dimensional parameters used in the model given by Eqs (1)–(3).** The unit  $\text{cc}^{-1}$  means per cell cycle.

Parameter	Symbol	Typical ranges (values)	Reference
Producer's diffusion coefficient	$\Gamma_U$	$10^{-8} - 10^{-10} \text{ cm}^2/\text{s}$	[31]
Free-rider's diffusion coefficient	$\Gamma_V$	$10^{-8} - 10^{-10} \text{ cm}^2/\text{s}$	[31]
Public good's diffusion coefficient	$\Gamma_G$	$10^{-7} - 10^{-4} \text{ cm}^2/\text{s}$	[32, 33]
Cellular intrinsic growth rate	$\alpha$	$1 \text{ cc}^{-1}$	
Producer's death rate	$\mu_U$	$< 1 \text{ cc}^{-1}$	
Free-rider's death rate	$\mu_V$	$< 1 \text{ cc}^{-1}$	
Public good production cost	$\kappa$	$\ll 1 \text{ cc}^{-1}$	[34]
Public good production rate	$\rho$	$100\text{-}1000 \text{ cc}^{-1}$	[35]
Public good consumption rate	$\delta$	$100\text{-}1000 \text{ cc}^{-1}$	
Public good benefit (conc. independent)	$\sigma$	1-3	[27]
Public good benefit (conc. dependent)	$\beta$	$2\text{-}6 [\text{conc.}]^{-1}$	[27]
Characteristic length of spatial domain	$L$	1-10 cm	

<https://doi.org/10.1371/journal.pcbi.1007361.t001>

# rescaling to dimensionless quantities

$$\dot{U} = \gamma_U \nabla^2 U + (\lambda(G) - 1 + a)[1 - (U + V)]U - cU,$$

$$\dot{V} = \gamma_V \nabla^2 V + \lambda(G)[1 - (U + V)]V - crV,$$

$$\epsilon \dot{G} = \nabla^2 G + U - G(U + V).$$

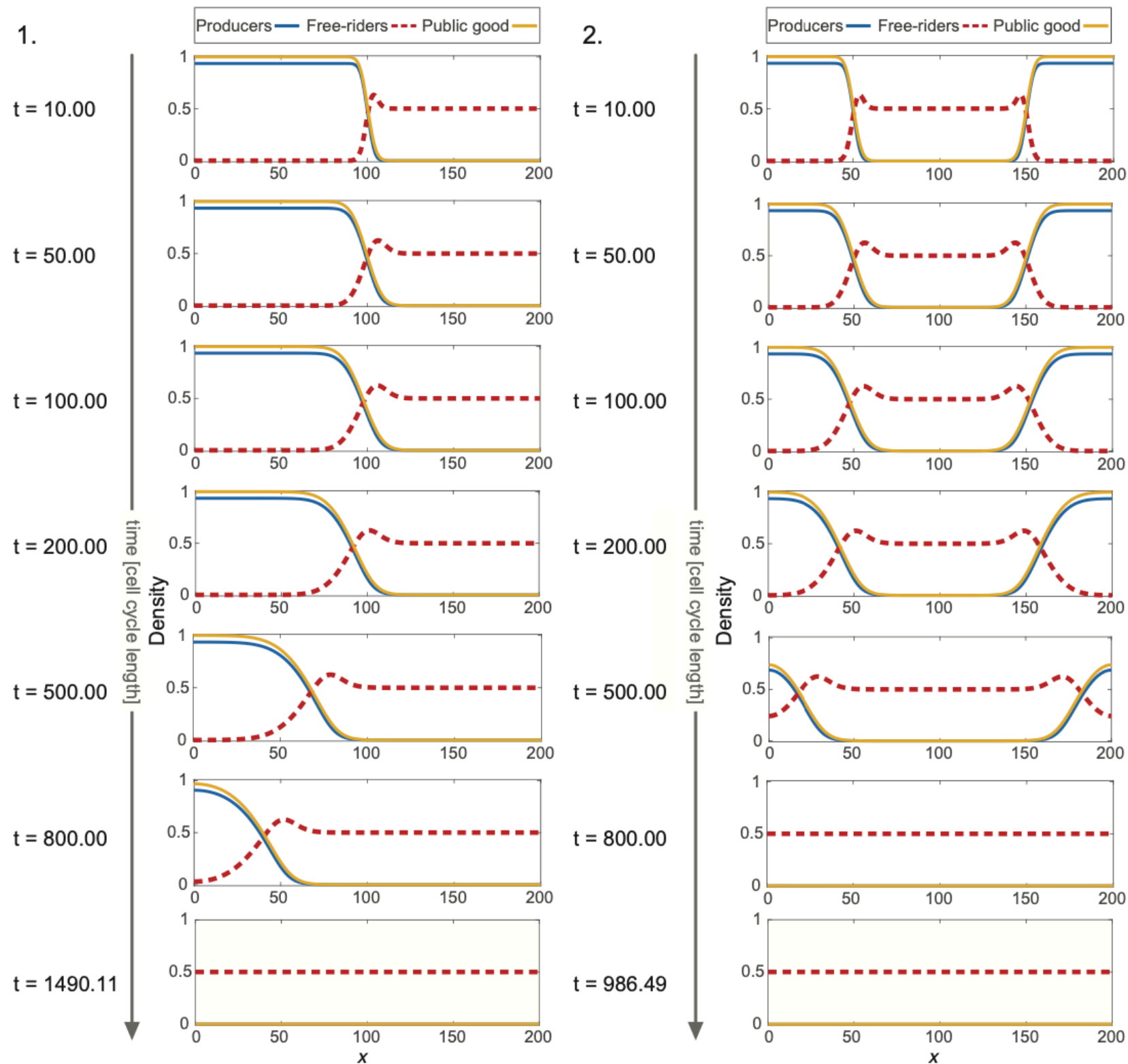
**Table 2. Definition of non-dimensional parameters used in the model given by Eqs (5)–(7).** Ranges are given as well as the typical values used throughout the text.  $\epsilon_{\text{exit}}$  is used to determine the  $\epsilon$ -extinction or fixation events.

Dimensionless parameter	Symbol	Identity	Range	Typical value
Producer's diffusion coefficient	$\gamma_U$	$\frac{\Gamma_U \delta}{\Gamma_G \alpha}$	$10^{-4} - 10^2$	0.5
Free-rider's diffusion coefficient	$\gamma_V$	$\frac{\Gamma_V \delta}{\Gamma_G \alpha}$	$10^{-4} - 10^2$	0.5
Producer (PG independent) birth rate	$a$	$1 - \frac{\kappa}{\alpha}$	0.75-0.9	0.9
Producer death rate	$c$	$\frac{\mu_U}{\alpha}$	0-1	0.5
Ratio of free-rider to producer death rate	$r$	$\frac{\mu_V}{\mu_U}$	$> 0$	1.0
Ratio of cell birth rate to consumption rate	$\epsilon$	$\frac{\alpha}{\delta}$	$10^{-3} - 10^{-2}$	$2 \times 10^{-3}$
Neighborhood of a fixed point		$\epsilon_{\text{exit}}$		$10^{-8}$

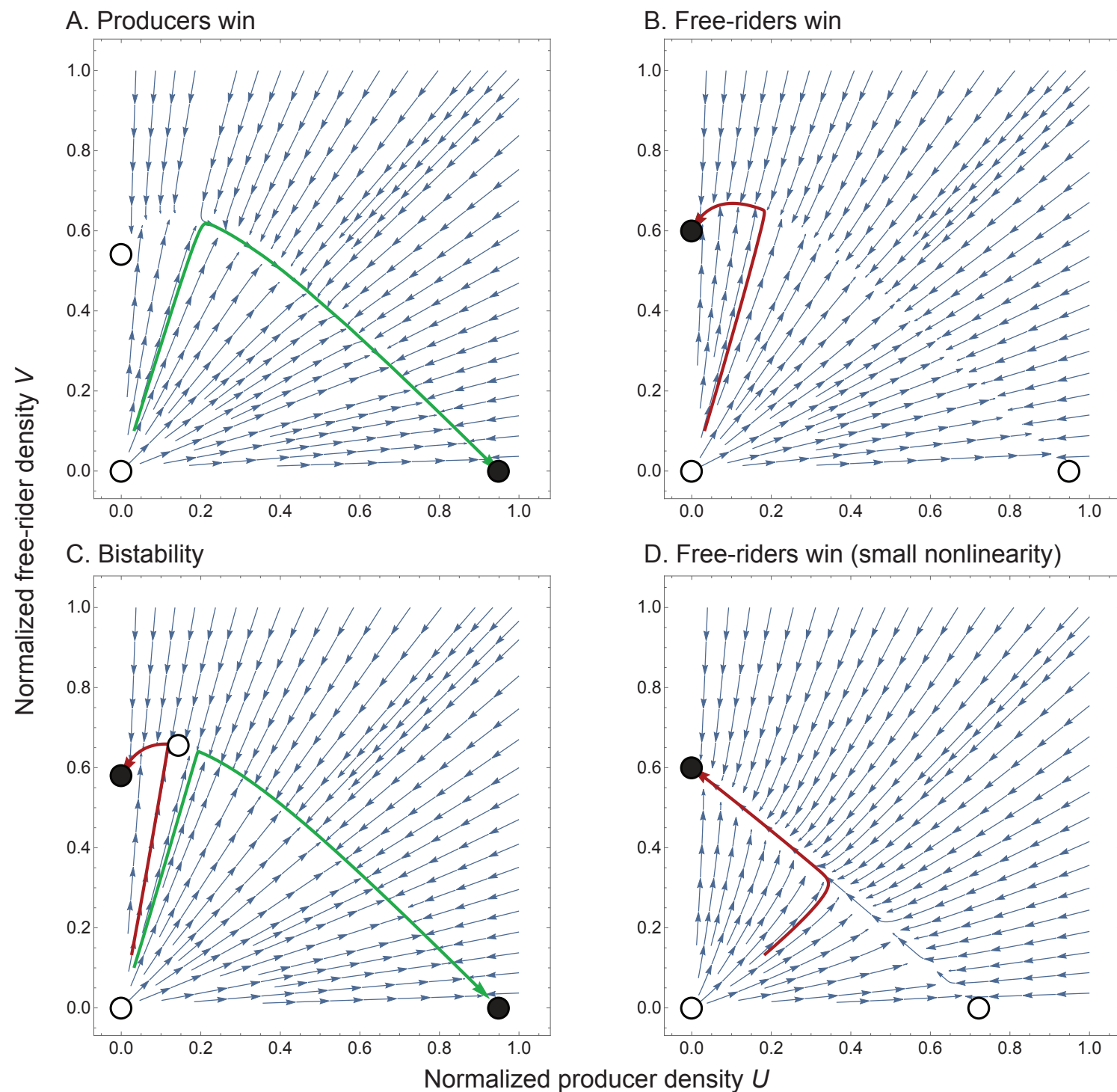
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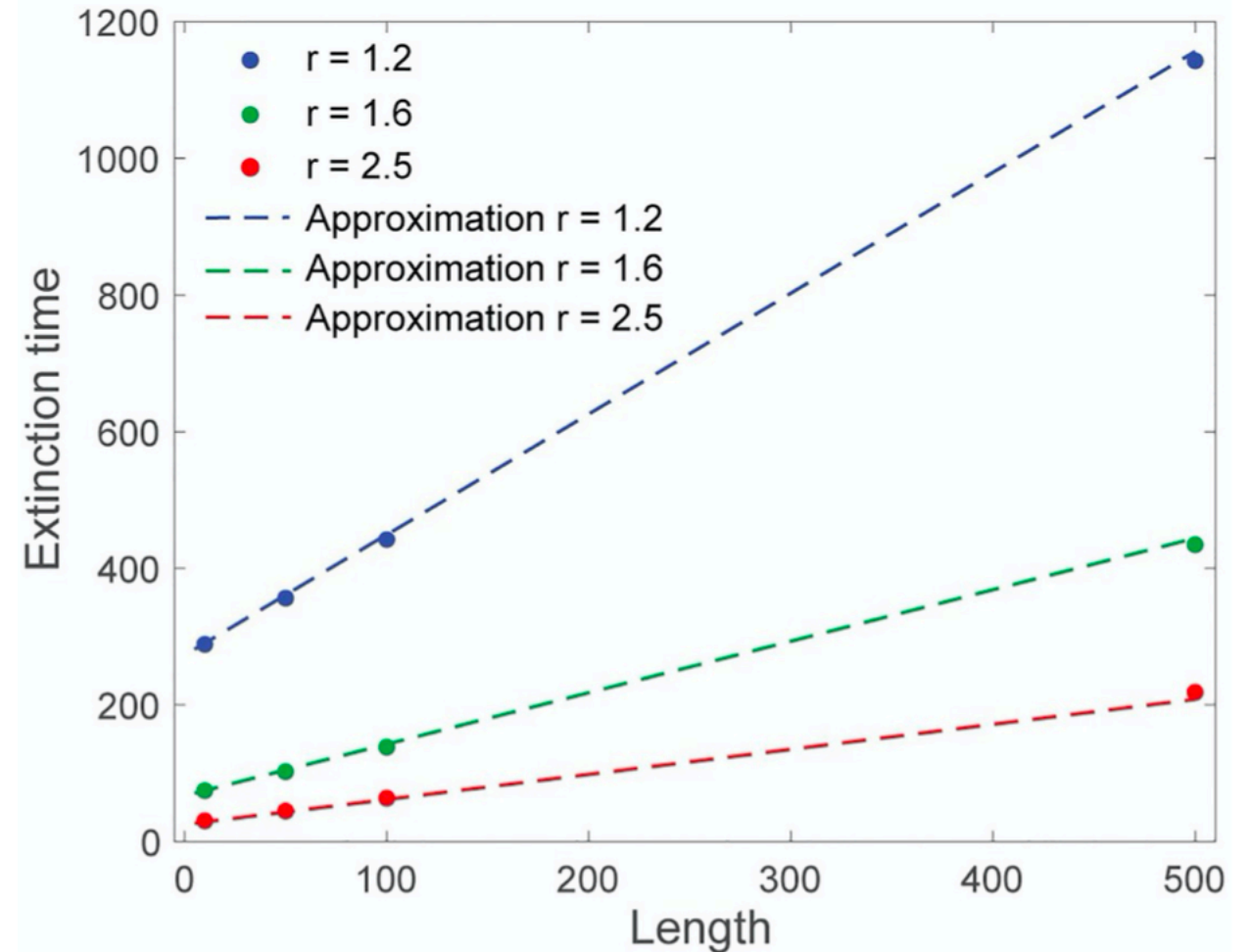
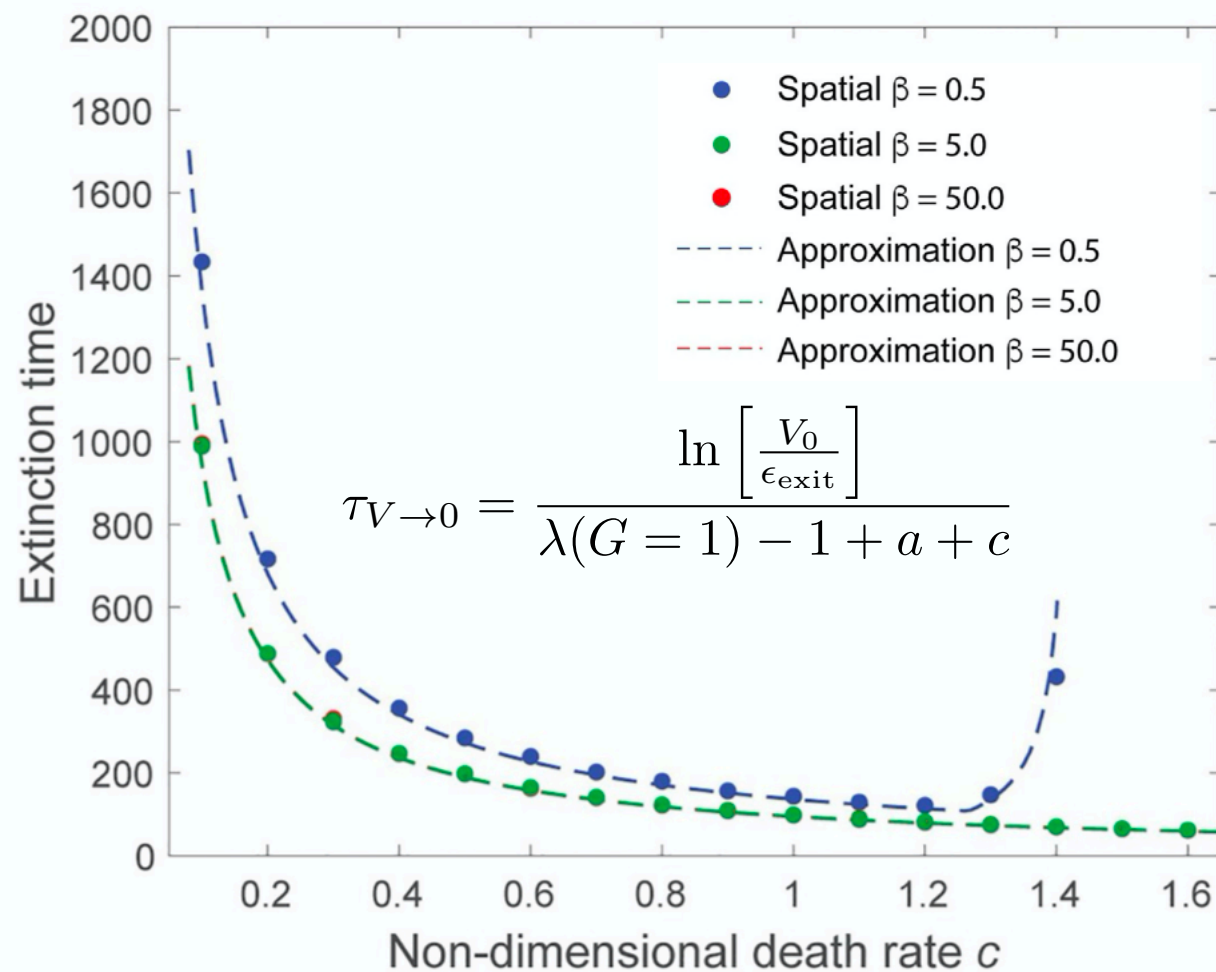
# dynamics in highly structured initial populations (domain of free-riders taking over producers)



the system is dominated by a slow manifold,  
for which we can neglect space



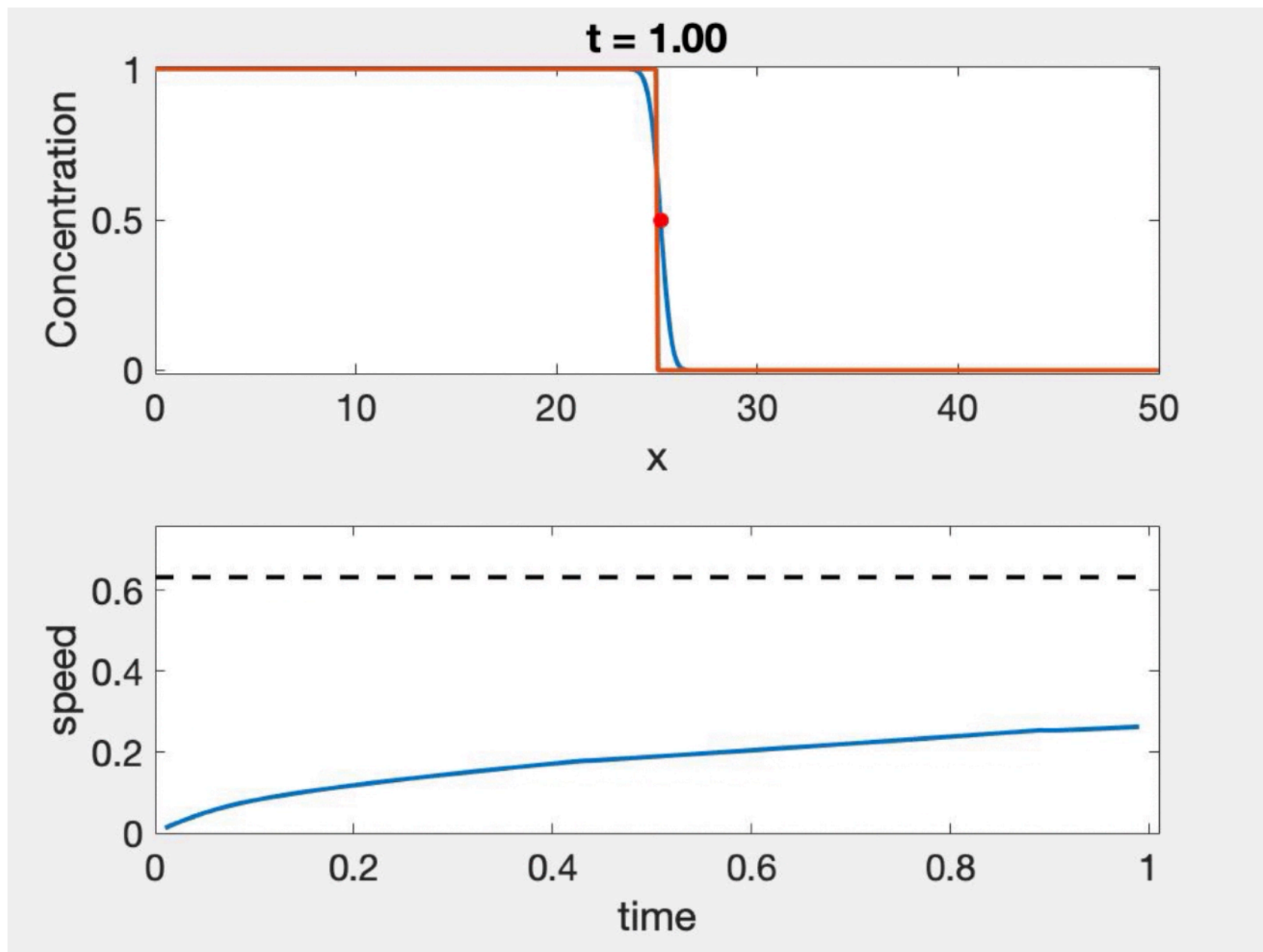
the system is dominated by a slow manifold  
and extinction times can be calculated



Analytical approximation of  $\epsilon$ -extinction time for producer cells to take over.  
 $r$  = ratio of death rates.  $c$  = death to birth ratio.  
For weak nonlinearity  $\beta$ , another singularity emerges at higher  $c$ .

what can we say for highly structured initial conditions?

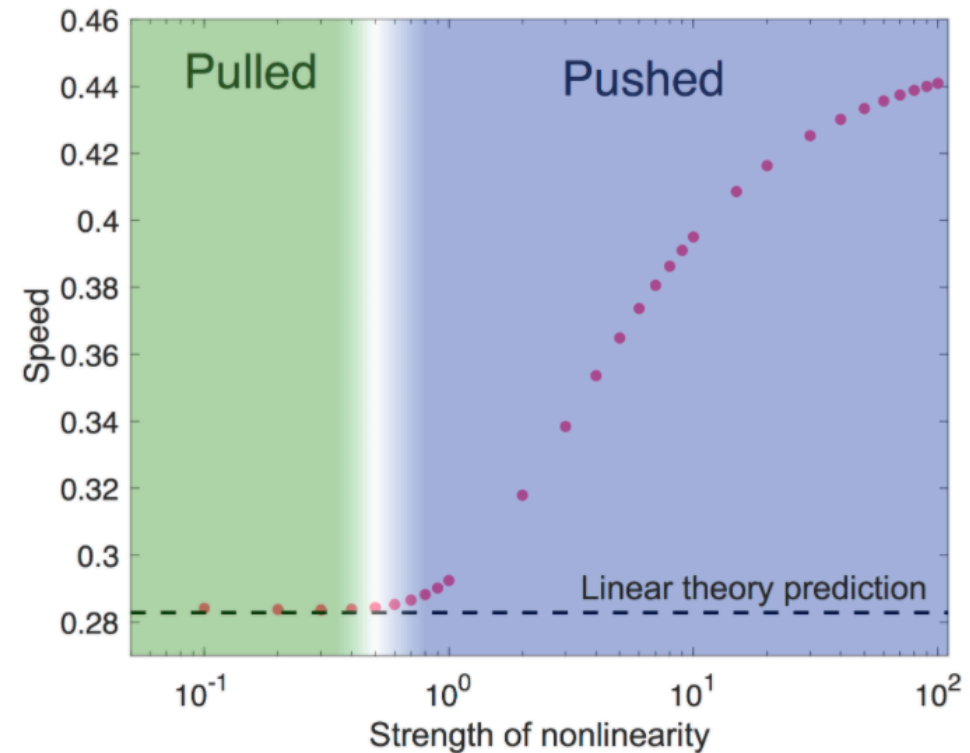
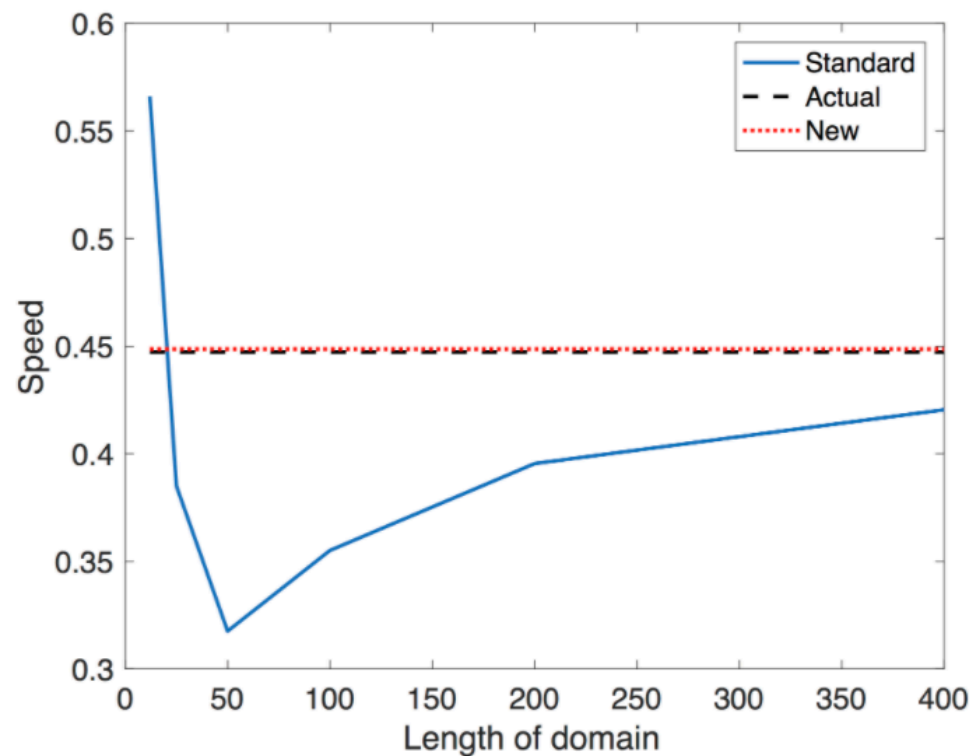
$$\dot{u} = D \nabla^2 u + r u(1 - u)$$



# estimating the speed leveraging that only propagation times change

$$t_{\text{total}} = t_{\text{formation}} + t_{\text{propagation}}(d) + t_{\text{boundary}}$$

$$\text{speed} \propto \frac{d_2 - d_1}{t_2 - t_1}$$



The degree of nonlinearity in the public good drives wave speeds that can be predicted using a simple numerical approach, using same IC (say a wall at L/2) but two domain lengths.

# Stability of the non-spatial system

- Extinction state:  $(0, 0, G^0)$  where  $G^0 \in [0, 1]$ . This state is stable if  $\lambda(G^0) < \min(cr, 1 - a + c)$ .
- Producers win:  $\left(1 - \frac{c}{\lambda(1) - 1 + a}, 0, 1\right)$ . This state is stable if  $a > 1 - \lambda(1) + \max\left(\frac{\lambda(1)}{r}, c\right)$ .
- Free-riders win:  $(0, 1 - cr, 0)$ . This state is stable if  $\frac{1}{r} > \max(a, c)$ .
- Isolated coexistence point (always unstable):

$$\left(G^* \left(1 - \frac{c(r-1)}{1-a}\right), (1 - G^*) \left(1 - \frac{c(r-1)}{1-a}\right), G^*\right), \quad (1)$$

where

$$G^* = \frac{1}{\beta} \left[ \sigma - \ln \left( \frac{(1 + e^\sigma)(r-1)}{r(1-a)} - 1 \right) \right]$$

- Non-isolated coexistence line:  $(G^*, 1 - G^*, G^*)$ . At least some finite part of this interval containing  $G^* = 0$  is stable.



# public good evolutionary games among cancer cells, collective benefit (at individual cost?)

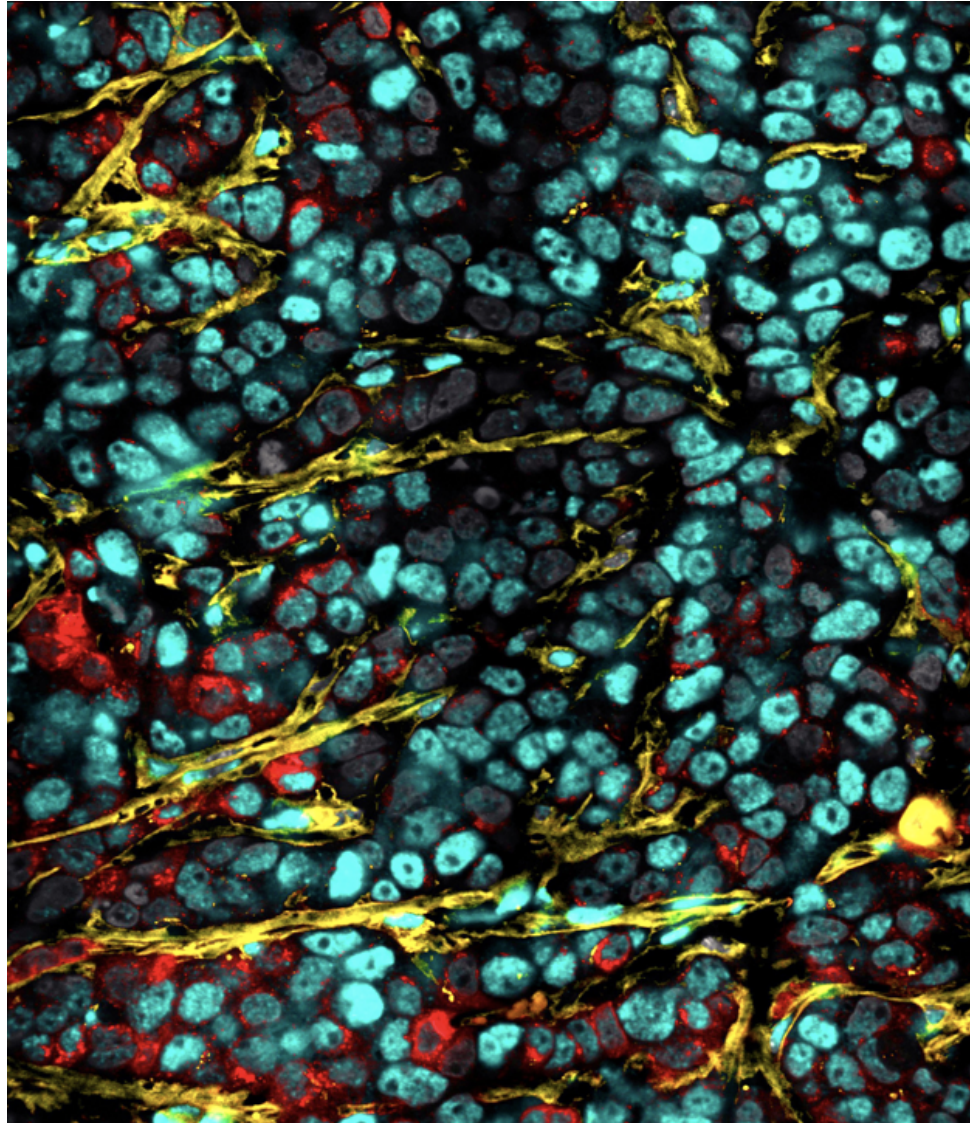


Image credit: Doris Tabassum

In heterogeneous MDA-MB-468 xenografts, IL11+ cells (secreted IL11 in red) act as non-cell autonomous driver of tumor growth by influencing pSTAT3 signaling (cyan) in the neighboring carcinoma and stromal (yellow) cells; nuclei are marked in grey. (Montage of 6 images on a 60X objective).